https://doi.org/10.3176/hum.soc.sci.1990.4.02

Proc. Estonian Acad. Sci. Social Sciences, 1990, 39, N 4, 388—397

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GENERAL ANALYSIS OF PRODUCTION GROWTH

Introduction

The paper discusses the treatment of the economic analysis proceeding from the conception of synthetic components of production growth. Because of the limited space, no attention has been paid to the theoretical aspects of the conception. A survey of them has been given in the monograph [1]. Methodological questions connected with the application of the conception have not been examined either. Their discussion can be found in [2]. General analysis, the presentation of which is the aim of this paper, is an exception.

In the first section an introductory survey of the conception of synthetic components of production growth has been given. In the next section factor systems belonging to the conception have been discussed. In the third section the analysis of the factor systems has been dealt with. In the fourth section these factor systems have been generalized to one scheme of analysis which includes all the factors. In the fifth section the results of the general analysis have been presented. In the sixth section concluding remarks are presented.

The scientific result of this work is a more general treatment than the existing ones of the analysing technique which enables to consider primary factor systems as a special form of the secondary factor systems, and the methods of treatment of the primary systems (Section 3) as a special case of the treatment of secondary systems (Section 4).

1. Conception

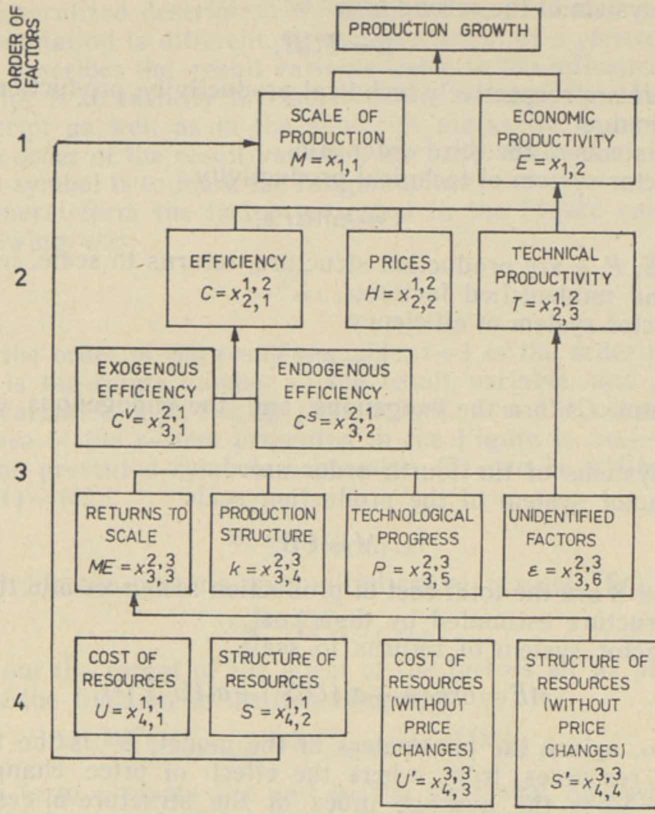
The conception of synthetic components of production growth (CSC) derives from a systematic treatment of production growth, the object of which is the complex of growth factors together with the inherent relationships.

The factor systems of CSC with the corresponding subordination are presented in the Figure. Four orders have been distinguished in the subordination of factors (the number of order shows the level of abstraction according to a concrete factor). The interdependence of the factors belonging to the system is clearly reflected in the Figure — a phenomenon of a concrete order is the result variable of the factors of the lower order, being at the same time a factor to higher order phenomenons.

The CSC in the given form consists of the factor systems depicted in the Figure.

The conception is deductive as to the way it was compiled. This means that the picture presented in the Figure is the result of the derivation of factors from higher levels of abstraction. The factors of lower orders have been developed from those of higher levels by decomposition.

The order of factors is indicated by the corresponding figure. The principle is that the bigger the figure, the lower the level of abstraction. For example, the level of abstraction of a third order factor, let it be returns to scale, is smaller than that of a first or a second order factor (e. g. production efficiency or productivity).



Factor systems of production growth.

2. Factor systems

It is possible to speak of two kinds of factor systems — primary and secondary — within the framework of the conception of synthetic components. The primary systems of factors which will be dealt with in the present section, represent expressions of dependence which are quite well-known in economic literature. In the present article attention has been focussed on developing their formal side — on reformulating the corresponding forms of relations, developing factor systems, establishing concrete specifications of indices, etc.

Secondary factor systems represent combinations of primary systems. The number of the latter is relatively large. In the fourth section a possibility of compiling secondary factor systems will be examined in some detail.

Below the factor systems are presented in accordance with their order (the order of a factor system is determined by the order of factors belonging there).

A factor system of the first order is:

$$Q=ME, \tag{1}$$

where Q, M, E are respectively the amount of output, production scale, and economic productivity (specifications of economic indices belonging to the present as well as the next factor systems are presented in [2]).

A factor system of the second order is:

$$E = TCH, \tag{2}$$

where T, C, H are respectively technical productivity, production efficiency, and price change.

Factor systems of the third order are:

(a) a factor system of technical productivity

$$T = kMEP\varepsilon, \tag{3}$$

where k, ME, P, ε are production structure, returns to scale, technological progress, and unidentified factors;

(b) a factor system of efficiency

$$C = C^v C^s, \tag{4}$$

where C^v and C^s are the exogenous and the endogenous efficiency of production.

Factor systems of the fourth order are:

(a) a factor system of the production scale

$$M = US, \tag{5}$$

where U and S are the total cost of production resources and their average index of structure estimated by their cost;

(b) a factor system of returns to scale

$$ME = \exp(a_0 + a_1(U'S') + a_2(U'S')^2), \tag{6}$$

where a_0, a_1, a_2 are the parameters of the model; U' is the total cost of production resources from where the effect of price change has been eliminated; S' is the average index of the structure of resources from where price change has been excluded.

3. An analysis of factor systems

Index systems can be developed from factor systems for analysing with their help the relative influence of one or another phenomenon in the dynamics of the result variable. As the economic analysis based on the theory of indices does not need special comments thanks to its extensive spread we will examine below the determination of the absolute extent of the effect of factors on the increment of the result variable. The outcomes show the increment of the result variable due to a concrete factor. To make this kind of analysis the approximation method of the total differential [3, p. 188—191] can be applied. The corresponding program has been realized at the Institute of Economics of the Estonian Academy of Sciences.

To find out the extent of the effect of factors several comparison bases are suitable. As we are going to deal with time-series data, the data of the previous year will occur as the comparison basis of the concrete observation (the comparison basis can also be the first or some other year of the period of analysis, and economic measures of an analogous enterprise or production). Therefore, the extents of the effect reflect the increment of the result variable (as compared with the previous year) caused by the increment of a factor occurring in a concrete year (also as compared with the previous year).

The technique of finding the extent of the effect is based on a rather simple train of thought. Below it will be presented in a simplified form, so as not to repeat it for every single factor system.

For a generalized description we use symbols presented in the Figure. Only the denotation is different. In the denotation of a concrete factor the superscript describes the result variable liable to the influence of a factor. The subscript is to identify the factor itself. Besides, the first symbol in the superscript as well as in the subscript marks the order (the former reflects the order of the result variable, the latter the order of the factor). The second symbol is to mark the range number.

In a general form the factor presented in the Figure can be written in the following way

$$x^{ij}_{i+1,\ j+k},$$

where i is the order of the result variable, $i+1$ is the order of the factor variable, j is the range number of the result variable, and $j+k$ that of the factor variable, $i \in [0, 1, 2, 3]$, $j \in [1, 2, 3, 4]$, k is the correction factor which in case of the system presented in the Figure is $k \in [-2, -1, 0, 1]$.

Using the presented symbols expression (7) can be written from any system of (1)—(6).[1]

$$x_{ij}=f(x^{ij}_{i+1,\ j+k}). \tag{7}$$

As a rule, factor systems are multiplicative (except (6)), thus

$$x_{iy}=\Pi\, x^{ij}_{i+1,\ j+k}. \tag{8}$$

To find out the extent of the effect of the factors let us put down the increment of the function in the following way:

$$\Delta x_{ij}=f(x^{ij}_{i+1,\ j+k}+\Delta x^{ij}_{i+1,\ j+k})-f(x^{ij}_{i+1,\ j+k}). \tag{9}$$

Our aim is to estimate the part of the increment Δx_{ij} which is contingent on the increment of a concrete factor $\Delta x^{ij}_{i+1,\ j+k}$. The corresponding phenomenon will be denoted as $\Delta[x^{ij}_{i+1,\ j+k}]x_{ij}$.

Expression (9) represents the total differential of function (7). Therefore, we can answer the question of interest for us using the approximation method of total differential [3, p. 188—191] to treat expression (9). This enables to estimate $\Delta[x^{ij}_{i+1,\ j+k}]x_{ij}$.

Using this technique the extents of the effect (separated by a line in Table 1) can be found from the primary factor systems of CSS. They are:

$\Delta[C^{v}]C$ — the increment of production efficiency resulting from exogenous efficiency,

$\Delta[C^{s}]C$ — the increment of production efficiency resulting from endogenous efficiency,

$\Delta[k_j]T$ — the increment of technical productivity resulting from the change in production structure,

$\Delta[ME]T$ — the increment of technical productivity resulting from returns to scale,

$\Delta[P]T$ — the increment of technical productivity resulting from technological progress,

$\Delta[\varepsilon]T$ — the increment of technical productivity resulting from the effect of unidentified factors,

$\Delta[S]M$ — the increment of scale of production resulting from the change in the structure of resources estimated on the basis of their cost,

$\Delta[U]M$ — the increment of the scale of production resulting from the change of total cost of resources,

[1] An exception is the scale of production whose order of factors is $i+3$ instead of $i+1$.

Table 1

Summary table of the analysis of production growth

Factors	Result variables (the figure marks the order of factors)											
	Output Q				Scale of production M	Economic productivity E			Efficiency C	Technical productivity T		Returns to scale ME
	1	2	3	4	4	2	3	4	3	3	4	4
M	$\Delta(M)Q$											
S				$\Delta(S)Q$	$\Delta(S)M$							
U				$\Delta(U)Q$	$\Delta(U)M$							
E	$\Delta(E)Q$											
H		$\Delta(H)Q$				$\Delta(H)E$						
C		$\Delta(C)Q$				$\Delta(C)E$						
C^V			$\Delta(C^V)Q$				$\Delta(C^V)E$		$\Delta(C^V)C$			
C^S			$\Delta(C^S)Q$				$\Delta(C^S)E$		$\Delta(C^S)C$			
T	$\Delta(T)Q$					$\Delta(T)E$						
k			$\Delta(k)Q$				$\Delta(k)E$			$\Delta(k)T$		
P			$\Delta(P)Q$				$\Delta(P)E$			$\Delta(P)T$		
			$\Delta(\varepsilon)Q$				$\Delta(\varepsilon)E$			$\Delta(\varepsilon)T$		
ME			$\Delta(ME)Q$				$\Delta(ME)E$			$\Delta(ME)T$		
U'				$\Delta(U')Q$				$\Delta(U')E$			$\Delta(U')T$	$\Delta(U')ME$
S'				$\Delta(S')Q$				$\Delta(S')E$			$\Delta(S')T$	$\Delta(S')ME$

$\Delta[T]E$ — the increment of economic productivity resulting from a change in technical productivity,

$\Delta[H]E$ — the increment of economic productivity resulting from a price change,

$\Delta[C]E$ — the increment of economic productivity resulting from a change of production efficiency,

$\Delta[U']ME$ — the increment of returns to scale resulting from a change in the total cost of resources from where the effect of price change has been eliminated,

$\Delta[S']ME$ — the increment of returns to scale resulting from a change in the structure of resources from where price changes have been excluded,

$\Delta[U']E$ — the increment of economic productivity resulting from a change in the total cost of resources from where the effect of price change has been eliminated,

$\Delta[M]Q$ — the increment of output resulting from a change in the scale of production,

$\Delta[E]Q$ — the increment of output resulting from a change in economic productivity.

4. An algorithm of the general analysis of production growth

In the above-presented analysis we explained the effect of the factors of the $j+1$th order on the result variable of the jth order (e. g., the increment of technical productivity resulting from technological progress). In addition, also the effect of a concrete factor on the variables of a higher order (e. g., the effect of technological progress on economic productivity or dynamics of output) is of interest.

The basis of the general analysis of production growth is secondary factor systems belonging to the conception of synthetic components. As mentioned above, the latter are obtained by combining the primary factor systems. For example, combining the factor systems (1), (2) and (5) we get

$$Q = TCHUS.$$

In using the secondary factor systems in economic analysis a technique analogous to the treatment of primary factor systems can (in principle) be used. In practice, however, another way must be chosen. The reason is that the estimation method based on the approximation of total differential is very labour-consuming in the case of more than four factors.

To find the effect of a concrete factor in the increment of the result variable of a higher order let us assume that it is proportional with the quota of the given factor effect in the increment of the result variable of the same order. If, for example, the shares of technical productivity and production efficiency in the increment of economic productivity are both 50 per cent, then the relation 1:1 must be also dominating in the output increment due to technical productivity and efficiency.

To present the above-mentioned algorithm the denotation analogous to the previous one will be used. The only difference is that the interval between the orders of the result variable and the factor need not be equal to one as it is, as a rule, in the case of primary factor systems. Therefore, the shift of the range number of a factor with respect to the result variable will be marked in the way presented in Table 2.

Table 2

The shift of the range number of the factor with respect to the result variale

Difference in order	Order	Difference in range number	Range number
0	i	0	j
1	$i+1$	k	$j+k$
1	$i+2$	l	$j+l$
1	$i+3$	m	$j+m$
1	$i+4$	n	$j+n$

We write out the corresponding formulas with respect to the difference between the orders of the factor and the result variable.

(1) The difference is equal to four:

$$\Delta[x^{ij}_{i+4,\, j+n}]x_{ij}=\Delta[x^{ij}_{i+1,\, j+k}]x_{ij}(\Delta[x^{i+1,\, j+k}_{i+2,\, j+l}]x_{i+1,\, j+l}/\Delta x_{i+1,\, j+k})\times$$
$$\times(\Delta[x^{i+2,\, j+l}_{i+3,\, j+m}]x_{i+2,\, j+l}/\Delta x_{i+2,\, j+m})\,(\Delta[x^{i+3,\, j+m}_{i+4,\, j+n}]x_{i+3,\, j+m}/\Delta x_{i+3,\, j+m}), \quad (10)$$

where $\Delta[x^{ij}_{i+4,\, j+n}]x_{ij}$ is the increment of the result variable x_{ij} caused by the increment of the factor $x^{ij}_{i+4,\, j+n}$;

$\Delta[x^{ij}_{i+1,\, j+k}]x_{ij}$ is the increment of the result variable x_{ij} caused by the increment of the factor $x^{ij}_{i+1,\, j+k}$;

$\Delta[x^{i+1,\, j+k}_{i+2,\, j+l}]x_{i+1,\, j+k}$ is the increment of the result variable $x^{ij}_{i+1,\, j+k}$ caused by the increment of the factor $x^{i+1,\, j+k}_{i+2,\, j+l}$;

$\Delta[x^{i+2,\, j+l}_{i+3,\, j+m}]x_{i+2,\, j+l}$ is the increment of the result variable $x_{i+2,\, j+l}$ caused by the increment of the factor $x^{i+2,\, j+l}_{i+3,\, j+m}$;

$\Delta[x^{i+3,\, j+m}_{i+4,\, j+n}]x_{i+3,\, j+m}$ is the increment of result variable $x_{i+3,\, j+m}$ caused by the increment of the factor $x^{i+3,\, j+m}_{i+4,\, j+n}$;

$\Delta x_{i+1,\, j+k}$ is the total increment of the result variable $x_{i+1,\, j+k}$;
$\Delta x_{i+2,\, j+l}$ is the total increment of the result variable $x_{i+2,\, j+l}$;
$\Delta x_{i+3,\, j+m}$ is the total increment of the result variable $x_{i+3,\, j+m}$.

The following example is given to show the practical application of the presented algorithm:

$$\Delta[S']Q=\Delta[E]Q \quad \Delta[T]E/\Delta E \quad \Delta[ME]T/\Delta T \quad \Delta[S']ME/\Delta ME,$$

that reflects the increment of result variable Q resulting from the increment of the factor S' (the symbols used are presented in Table 1).

(2) The difference is equal to three (the symbols used are analogous to the above-mentioned ones):

$$\Delta[x^{ij}_{i+3,\ j+m}]x_{ij}=\Delta[x^{ij}_{i+1,\ j+k}]x_{ij}(\Delta[x^{i+1,\ j+k}_{i+2,\ j+l}]x_{i+1,\ j+l}/\Delta x_{i+1,\ j+k})\times$$

$$\times(\Delta[x^{i+2,\ j+l}_{i+3,\ j+m}]x_{i+2,\ j+l}/\Delta x_{i+2,\ j+m}). \qquad (11)$$

The following formula is an example of the practical application of the presented algorithm:

$$\Delta[ME]Q=\Delta[E]Q \quad \Delta[T]E/\Delta E \quad \Delta[ME]T/\Delta T,$$

that reflects the increment of the result variable Q caused by the increment of factor ME.

(3) The difference is equal to two:

$$\Delta[x^{ij}_{i+2,\ j+l}]x_{ij}=$$

$$=\Delta[x^{ij}_{i+1,\ j+k}]x_{ij}(\Delta[x^{i+1,\ j+k}_{i+2,\ j+l}]x_{i+1,\ j+1}/\Delta x_{i+1,\ j+k}). \qquad (12)$$

An example of the practical application of the presented algorithm is:

$\Delta[T]Q=\Delta[E]Q \quad \Delta[T]E/\Delta E,$

that shows the increment of the result variable Q caused by the increment of the factor T.

(4) The difference is equal to one:

$\Delta[x^{ij}_{i+1,\ j+k}]X_{ij}$ — the corresponding algorithms were given in the previous section.

The last statement refers to the fact that the algorithms of primary factor systems represent special cases of the above-presented algorithms of secondary factor systems.

5. Results of the general analysis of production growth

The results of the application of formulas (10)—(12) are the following:

$\Delta[C^V]E$ — the increment of economic productivity resulting from exogenous factors of efficiency,

$\Delta[C^S]E$ — the increment of economic productivity resulting from endogenous factors of efficiency,

$\Delta[C^S]Q$ — the increment of output resulting from endogenous factors of efficiency,

$\Delta[C^V]Q$ — the increment of output resulting from exogenous factors of efficiency,

$\Delta[k_j]E$ — the increment of economic productivity resulting from a change in production structure,

$\Delta[k_j]Q$ — the increment of output resulting from a change in production structure,

$\Delta[ME]E$ — the increment of economic productivity resulting from returns to scale,

$\Delta[ME]Q$ — the increment of output resulting from returns to scale,

$\Delta[P]E$ — the increment of economic productivity resulting from technological progress,

$\Delta[P]Q$ — the increment of output resulting from technological progress,

$\Delta[\varepsilon]E$ — the increment of economic productivity resulting from the effect of unidentified factors,

$\Delta[\varepsilon]Q$ — the increment of output resulting from the effect of unidentified factors,

$\Delta[S]Q$ — the increment of output resulting from a change in the structure of resources estimated by their cost,

$\Delta[U]Q$ — the increment of output resulting from a change of total cost of resources,

$\Delta[T]Q$ — the increment of output resulting from a change of technical productivity,

$\Delta[H]Q$ — the increment of output resulting from a price change,

$\Delta[C]Q$ — the increment of output resulting from a change of production efficiency,

$\Delta[U']T$ — the increment of technical productivity resulting from a change of the total cost of resources whence the effect of price changes have been eliminated.

$\Delta[U']E$ — the increment of economic productivity resulting from a change of the total cost of resources whence the effect of price changes has been eliminated,

$\Delta[S']E$ — the increment of economic productivity resulting from a change in the structure of resources whence the price changes have been eliminated,

$\Delta[U']Q$ — the increment of output resulting from a change of the total cost of resources from where the effect of price changes has been eliminated,

$\Delta[S']Q$ — the increment of output resulting from a change in the structure of resources whence the price changes have been eliminated.

When using the presented formulas factor systems of different orders are joined, and therefore the corresponding analysis can be called general analysis. To check the results of general analysis it is useful to group them in the way presented in the summary table (Table 1). Such a table characterizes production growth during a concrete period — a month, a quarter, a year, a decade. The rows in the summary table reflect the effect of a concrete factor on the result variables (the latter are placed in the columns). The columns of the Table, on the other hand, reflect the result of the effect of different factors on the change of a concrete result variable.

In general analysis the result of the effect of the factors can be presented in absolute as well as relative form (the latter is equal to the quotinent of the effect resulting from the influence of a certain factor on the increment of a result variable). The summary table can be read in two ways — in rows or columns. Let us first examine the information included in the rows of the Table. For example, let us consider the row of factor S' (this symbol designates the index of the structure of production resources from where the effect of price changes has been eliminated). In the last column the effect of S' on the increment of returns to scale has been given. As the returns to scale is a factor of technical productivity, then, consequently, S' affects the technical productivity as well. The result of this effect is presented in the column $\Delta[S']T$. Technical productivity, in turn, is one of the factors of economic productivity. Therefore, the extent of the effect of S' can also be distinguished in the increment of economic productivity (in the column $\Delta[S']Q$). And finally, the output depends on economic productivity. In this way, through returns to scale, the dynamics of technical and economic productivity, S', has an effect on the increment of output, too (column $\Delta[S']Q$).

Another possibility to examine the summary table lies in the examination of columns.

Observe the formation of the increment of output, ΔQ. ΔQ is formed from factors of first order as the effect of the scale of production and economic productivity. So $\Delta Q = \Delta[M]Q + \Delta[E]Q$. If an effect of second-order factors is of interest, the next column reflecting the effects of technical productivity, efficiency, and price change should be considered. In this connection note that $\Delta[H]Q + \Delta[T]Q + \Delta[C]Q \neq \Delta Q$ because the scale of production has no second-order factors in the conception of synthetic components.

The last column of the output increment characterizes the effect of fourth-order factors. And $\Delta[S]Q + \Delta[U]Q + \Delta[S']Q + \Delta[U']Q \neq \Delta Q$, as neither price change, efficiency nor technical productivity has factors of fourth order.[2]

6. Concluding remarks

The summary table presents 36 elementary measures of economic growth. They reflect different levels of the effect of factors. Therefore, the technique of general analysis based on the conception of synthetic components of production growth enables to get a good survey of the inner logic of the growth process.

As mentioned above, the summary table reflects the results of the comparative analysis of the data of two periods (the increment of the year investigated compared to the previous year). But often it is necessary to find out the growth tendencies during a longer period. For this purpose it is useful to join columns of summary tables of different years to form new tables where the extents of the effect that caused the increment of a concrete index are presented as time-series.

[2] This fact indicates a possibility of improving the conception of synthetic components. For that purpose an all-embracing system of factors where all factors have developments as low as the factors of the lowest order can be formed.

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Presented by Ü. Ennuste

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Received
Jan. 29, 1990

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TOOTMISE KASVU KOONDANALÜÜS

On esitatud tootmise kasvu sünteetiliste komponentide kontseptsioonist lähtuv majandusliku analüüsi käsitlus. Artikli piiratud mahu tõttu pole siin tähelepanu osutatud kontseptsiooniga seostuvatele teoreetilistele momentidele. Huviline saab neist ülevaate monograafias [1]. Samuti ei vaadelda kontseptsiooni rakendamisega seotud metodoloogilisi küsimusi. Nendega võib tutvuda brošüüris [2]. Erandiks on selles suhtes koondanalüüs, mille esitlemine ongi artikli eesmärk.

On antud lühiülevaade tootmise sünteetiliste komponentide kontseptsioonist; nimetatud kontseptsiooni kuuluvad tegurisüsteemid; peatutud nende süsteemide analüüsil; koondatud tähendatud tegurisüsteemid ühte, kõiki tegureid hõlmavasse analüüsiskeemi ja esitatud koondanalüüsi tulemused.

Töö tulemuseks on kontseptsioonil põhineva metoodika senisest üldisem käsitlus, mis võimaldab vaadelda esmaseid tegurisüsteeme teiseste tegurisüsteemide erikujuna ja esmaste tegurisüsteemide töötlemise metoodikat teiseste tegurisüsteemide metoodika erijuhuna.

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29. I 1990

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СВОДНЫЙ АНАЛИЗ РОСТА ПРОИЗВОДСТВА

В статье дается трактовка анализа экономического роста, основанная на концепции синтетических компонентов. Из-за ограниченного объема в статье не рассматривались теоретические проблемы концепции. О них можно получить представление в монографии [1]. Нетронутыми остались также методические вопросы применения концепции. С ними можно ознакомиться в [2].

Статья состоит из пяти разделов. В первом из них дается краткая характеристика концепции синтетических компонентов. В следующем разделе приводятся факторные системы концепции синтетических компонентов. В третьем разделе рассматривается анализ факторных систем. В четвертом разделе факторные системы обобщены в одну, сводную систему, охватывающую все факторы. В пятой части подводятся итоги.

Научным результатом работы является более обобщенная трактовка методики, которая позволяет рассматривать: а) первичные факторные системы как частный случай вторичных систем; б) методику обработки первичных систем (третий раздел) как частный случай методики обработки вторичных систем (четвертый раздел).

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Поступила в редакцию
29/I 1990