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SOME PROBLEMS OF ALTERNATIVE STOCHASTIC PLANNING MECHANISMS

The author examines basic assumptions and makes evaluations of some alternative stochastic planning mechanisms for optimal equilibrium plans of an abstract economy in the context of a simple two-period and two-sphere optimization model [1—3]. All the alternative mechanisms are the Benassy's competitive markets [4] with mixed coordination signals (prices and limits). Also, two problems of managerial risk aversion are discussed, and some rules to diminish aversion are introduced.

To deal with all these problems complexly in a short paper like this, the author had to sacrifice some strictness, cut off many corners and use heuristic proofs.

1. Introduction

The mechanism version in economics is clearly stronger than the equilibrium-theoretical and planning-theoretical ones [5]. Accordingly, it can be used to make more general and stronger assertions, and the last two theories could be seen as fragments or elements of the mechanism theory.

These contentions are especially viable under uncertainty or in the case of indeterministic approaches. On the basis of this paradigm the central planning theory and the decentralized equilibrium theory are untolerably limited to describe real phenomena. In the former case every agent is directly pushed into optimality by the omnipotent centre. In the latter case there is nobody to push prices into equilibrium, and everybody must have omnipotent intellect to figure out the right prices.

Of course, the extent stochastic mechanisms theories do not provide an adequate explanation for many problems. As a matter of fact, various theories ignore more or less at least four issues. First, the treatment of mixed or combined coordination problems (prices and limits or rations) is lacking. Second, the alternative problems of working out equilibrium coordination parameters, especially alternative pricing rules, are overlooked. Third, the problems of risk-sharing in mechanisms are unstudied. And fourth, frequently the initial social choice problem is not specified clearly enough. The paper fills the gaps only in introductory explanations of the issues. The author has attempted to classify the problems and sketch the solutions in a heuristic treatment.

In [1] we have presented a fully centrally coordinated planning mechanism under uncertainty with mixed signals where both future prices and limits (rations) are state-dependent. This approach is the basis to the theoretical analysis of centrally planned economies under uncertainty although it is unrealistic because it requires immense central state-dependent coordination work. This paper aims to provide some coordination schemes that will reduce the burden of the centre. Here we assume that the centre deals only with planning the wealth or endowment allocation of the system while future prices and quantities are determined decentrally. In other words, we assume that optimal state-dependent wealth rations for pricing mechanisms are determined exogenously, and only prices and quantities are endogenous. So we shall somehow draw a bridge between fully centrally coordinated economies and the recent non-Walrasian and Walrasian economies under uncertainty.

It should also be noted that there is already quite a lot of literature on the problem of optimal pricing under uncertainty and some major results have been achieved. However, the evaluation of these results is still a bottleneck. One aim of this paper is to try to contribute to the evaluation.

In this model we assume that the welfare allocation has been optimized by the centre (by the initial resources and lump-sum profit allocation). This assumption allows us to forget about this distribution and concentrate on the issue of allocatively efficient pricing-limiting. It also eliminates the problems of profit sharing (production theory).

First we shall study a competitive strategic market game where agents set state-dependent future prices and quantities in the two-period model. This falls in the theoretical framework of J. Benassy's competitive market mechanisms theory [4]. In this approach all the immense work of determining state-dependent prices and limits is now done by the agents, and state-dependent contingent planning offices are needed. So the approach is interesting theoretically but not practically because it is impossible to organize all planning offices state-dependent.

To move closer to realistic coordination mechanisms under uncertainty, we shall study models in the spirit of J. Green [2] and J. Grandmont [3]. In such an approach the central role is played by an agent's expectation about the state-dependent future spot prices in his decision-making. It is postulated that prices move fast enough in each period to match supply and demand. In the framework of this simple model the sufficient conditions for the existence of a competitive equilibrium on some assumptions are met. But here agents are assumed to be able to associate correctly future prices with states (correct expectations). Clearly, if they cannot do this, the equilibrium will not be optimal (effective).

In the end of this paper we shall deal with some problems of managerial risk aversion connected with the stochastic mechanisms. First we shall study some issues of insurance and contingent contracts to diminish unimplied risk aversion, and then we shall deal with an example of implied by the mechanism risk aversion, and possibilities of diminishing it.

But to start with, we still need some more introductory remarks, then we shall describe the initial central social choice model, and transform it into a decentralized setting. Next the problem of alternative pricing mechanisms will be discussed.

There are two aspects of mechanisms investigation [6]. The traditional equilibrium theory takes a mechanism or *modus operandi* (e. g. perfect competition) as given, and examines its properties (e. g. Pareto-optimality) and performance correspondence. In the theory of planned economies the reverse problem has become to be investigated. Given a correspondence regarded as a social desideratum (e. g. optimization correspondence), are there mechanisms that implement it [7]?

In particular, in the case of incomplete information the important problem is whether there are any decentralized (privacy-preserving) mechanisms realizing the social desideratum, mechanisms pushing the decentralized economy to the optimum state? Namely, privacy-preserving is specially important in the case of incomplete information, and it is next to impossible to get this incomplete private information into one centre without any manipulation and distortion or, in other words, to enforce truth-telling [8].

The same argument tells us that it is reasonable to organize the choice of divisional strategies in a game-theoretic setting played noncooperatively [9]. There are two polarized concepts of these schemes. The classical claim is that the information repeatedly exchanged among members (strategies) consists of prices and excess demands (price or competitive mechanisms). The alternative is a centralized revelation-command mechanism where all the information is gathered in the centre where individual consumptions and productions are computed and then issued as commands to the agents. As we have already claimed, because of the stochastic nature of the information it seems unthinkable to organize revelation of private information to the centre in all details. Moreover, this information would exceed the computational capacity of the centre.

On the other hand, the ideal price mechanisms in the stochastic case are unthinkable as well. In this case the ideal prices are state-dependent and unpractical (as there are infinitely many possible states in reality). So, in reality some mechanism should implement approximate prices, and thus cannot guarantee the realization of optimum and stability. It means that a sound stochastic mechanism is a mixed one [10, 11]. In this mechanism parallel to prices coordination with quantities is applied limiting or rationizing the agents' actions more severely, and achieving better stability than prices alone could do in a stochastic situation. So a mixed mechanism can achieve better approximation to equilibrium.

In this paper it is assumed that in the center state-dependent endowment allocations for consumers are computed. They are computed on the basis of an aggregated central model. These endowments are issued to the consumers as constraints in which they can implement the price-limit equilibrium. The central allocation of endowments allows us to avoid complicated problems of the stock market and private ownership of fixed assets.

2. The basic model and state-dependent price-limit equilibrium

In the basic model (described in [1]) there are two periods or stages $t=1,2$: the present for the final fixed plans and the future for the preliminary contingent (or state-dependent) plans. The number of possible states of economy $s \in S$ is finite. The true state is unknown at present, but will be known in the future. Production activities are denoted by $j=1, \ldots, k$, and non-production activities (consumption and the like) by $j=k+1, \ldots, n$. There are $i=1, \ldots, m$ systemic (global) resources (goods, stocks, etc.). Let $x_{tj}(s) \in X_{tj}(s)$ denote the plan of the activity j at the stage t, where $X_{tj}(s)$ are non-empty closed convex sets with the probability 1.0, while $x_{1j}(s)=x_{1j}$.

Let the effects of the non-productive activities be strictly concave functions $f_{tj}(x_{tj}(s), s)$ with the probability 1.0. Let the results of all activities in the input and output of the resource i be also strictly concave functions $g_{tij}(x_{tj}(s), s)$ with the probability 1.0, where $g_{ij}>0$ denotes output and $g_{ij} \leqslant 0$, input. The transfer of the resource i by an activity j from the first stage into the second one is denoted by a strictly concave function $g^1_{2ij}(x_{1j}, s)$. Let the limits of exogenous resources on the system be $b_{ti}(s)$, where $b_{ti}(s)>0$ denotes the output of the resource i and $b_{ti}(s) \leqslant 0$ its input.

The mathematical form of the initial problem is the following. Maximize on the basis of the plan $x(s)=(x_{1j}, x_{2j}(s)), j=1, \ldots, n$ the objective function

$$E \sum_{j=k+1}^{n} [f_{1j}(x_{1j}, s)+f_{2j}(x_{2j}(s), s)], \tag{1a}$$

where E stands for mathematical expectation, subject to

$$\sum_{j=1}^{n} g_{1ij}(x_{1j}, s) \geqslant b_{1i}(s), \quad i=1, \ldots, m, \tag{1b}$$

$$\sum_{j=1}^{n} [g^1_{2ij}(x_{ij}, s)+g_{2ij}(x_{2j}(s), s)] \geqslant b_{2i}(s), \quad i=1, \ldots, m, \tag{1c}$$

$$x_{tj}(s) \in \mathfrak{X}_{tj}(s), \quad j=1, \ldots n, \quad t=1, 2, \tag{1d}$$

where the constraints (1b) and (1c) are satisfied with the probability 1.0. A solution $x(s)^0$ to the problem is assumed to exist.

Let us decompose the initial problem (1) by activities $j=1, \ldots, n$ using mixed coordination by prices and limits.

To derive the rules for mixed coordination we combine the Lagrangian and Kornai-Liptak relaxations. First the Lagrangian function with perturbed (relaxed) initial systemic constraints is used. The economic content of this construction is the following. The objective function of the initial problem (1a) is modified by means of resource prices and resource constraints so that the maximizing plan of the modified objective function would satisfy the initial resource constraints (1b)—(1c). However, if the prices are not good enough, i. e. not exactly Walrasian, the perturbed constraints will be active. So the Walras law takes place with perturbed constraints and there will be perturbed equilibrium.

On the basis of the above-said the following Lagrangian problem equivalent to problem (1) is obtained.

$$\min_{y(s)} \max_{x(s)} L(x(s), y(s)) = E\{ \sum_{j=k+1}^{n} [f_{1j}(x_{1j}, s) + f_{2j}(x_{2j}(s), s)] + \\ + \sum_{i=1}^{e} y_{1i}(s) [\sum_{j=1}^{n} g_{1ij}(x_{1j}, s) - b_{1i}(s)] + \sum_{i=1}^{e} y_{2i}(s) [\sum_{j=1}^{n} g^1_{2ij}(x_{ij}, s) + \\ + \sum_{j=1}^{n} g_{2ij}(x_{2j}(s), s) - b_{2i}(s)]\} \tag{2a}$$

subject to

$$\sum_{j=1}^{n} g_{1ij}(x_{1j}, s) \geqslant b_{1i}(s) - \varepsilon, \quad i=1, \ldots, m, \tag{2b}$$

$$\sum_{j=1}^{n} [g^1_{2ij}(x_{1j}, s) + g_{2ij}(x_{2j}(s), s)] \geqslant b_{2i}(s) - \varepsilon, \quad i=1, \ldots, m, \tag{2c}$$

$$x_{tj}(s) \in \mathfrak{X}_{tj}(s), \quad j=1, \ldots, n, \quad t=1, 2, \tag{2d}$$

where $y(s) = (y_{ti}(s)) \geqslant 0$, $i=1, \ldots, m$, $t=1, 2$, and $\varepsilon > 0$, $\varepsilon \to 0$.

The systemic constraints (2b) and (2c) of the obtained problem (2) are decomposed by means of limits. Economically it means that the resources given with systemic constraints $i=1, \ldots, m$ are distributed between activities. No doubt, the optimal limits are those in case of which the value of the objective function (2a) is the highest, or, in other words, optimum limits do not set additional constraints on the Lagrangian function (2a) in comparison with the constraints (1b)—(1c).

The above-said yields the following problem equivalent to problem (2):

$$\max_{d(s)} G(d(s), s) \tag{3a}$$

subject to

$$g_{1ij}(x_{ij}, s) \geqslant d_{1ij}(s), \tag{3b}$$

$$g^1_{2ij}(x_{1j}, s) \geqslant d^1_{2ij}(s), \tag{3c}$$

$$g_{2ij}(x_{2j}(s), s) \geqslant d_{2ij}(s), \tag{3d}$$

$$d(s) \in D(s), \tag{3e}$$

$$x_{tj}(s) \in \mathfrak{X}_{tj}(s), \tag{3f}$$

$$i=1, \ldots, m, \quad t=1, 2, \quad j=1, \ldots, n,$$

where $d(s)=(d_{1ij}(s);\ d^1_{2ij}(s),\ d_{2ij}(s))$, $i=1,\ldots,m$ and $j=1,\ldots,n$ and

$$D(s)=\{d(s)/\sum_{j=1}^{n} d_{1ij}(s)=b_{1i}(s)-\varepsilon,$$

$$\sum_{j=1}^{n} d_{2ij}(s)=b_{2ij}(s)+\sum_{j=1}^{n} d^1_{2ij}(s)-\varepsilon,\quad \exists x_i(s),\quad j=1,\ldots,n\}$$

(here the last condition means that the constraints (3b)—(3d) should not make problem (3) contradictory) and

$$G(d(s),s)=\min_{y(s)}\max_{x(s)} L(x(s),y(s)). \tag{4}$$

Let optimum coordination parameters $y^0(s)$ and $d^0(s)$ be given. Then problem (3) breaks into coordinated subproblems. The subproblems of the production activities $j=1,\ldots,k$ are the following:

$$\max E\{\sum_{i=1}^{m} y^0_{1i}(s)\,g_{1ij}(x_{1j},s)+\sum_{i=1}^{m} y^0_{2i}(s)\,[g^1_{2ij}(x_{1j},s)+$$
$$+g_{2ij}(x_{2j}(s),s)]\}=k^0_j \tag{5a}$$

subject to

$$g_{1ij}(x_{1j},s)\geqslant d^0_{1ij}(s), \tag{5b}$$

$$g^1_{2ij}(x_{1j},s)\geqslant d^{1,0}_{2ij}(s), \tag{5c}$$

$$g_{2ij}(x_{2j}(s),s)\geqslant d^0_{2ij}(s), \tag{5d}$$

$$x_{tj}(s)\in X_{tj}(s), \tag{5e}$$

$$t=1,\ 2,\quad i=1,\ldots,m,\quad j=1,\ldots,k.$$

Thus, the coordinated objective function of the subproblems of production activities is the mathematical expectation of the difference between incomes and expenditures or profit over both stages. The profit is calculated at optimum prices $y^0_{ti}(s)$, $i=1,\ldots,m$ which are state-dependent.

The objective function is maximized on condition that the optimum systemic limits $d^0_{tij}(s), i=1,\ldots,m$ be prescribed to the activities and direct activity constraints $X_{tj}(s)$ be satisfied. The limits are likewise state-dependent.

It is easy to see that problem (5) consists in its turn of independent unconnected problems of stage I and stage II of the activity j.

The coordinated subproblem of the non-productive sphere can be written as follows:

$$\max E\sum_{j=k+1}^{n}[f_{1j}(x_{1j},s)+f_{2j}(x_{2j}(s),s)] \tag{6a}$$

subject to

$$E\sum_{j=k+1}^{n}\sum_{i=1}^{m}[y^0_{1i}(s)\,g_{1ij}(x_{1j},s)+y^0_{2i}(s)\,g_{2ij}(x_{2j}(s),s)-$$
$$-y^0_{1i}(s)\,b_{1i}(s)-y^0_{2i}(s)\,b_{2i}(s)]+\sum_{j=1}^{k}k^0_j=0 \tag{6b}$$

$$g_{1ij}(x_1,s)\geqslant d^0_{1ij}(s), \tag{6c}$$

$$g_{2ij}(x_{2j}(s),s)\geqslant d^0_{2ij}(s), \tag{6d}$$

$$x_{tj}(s)\in X_{tj}(s), \tag{6e}$$

$$t=1,\ 2,\quad i=1,\ldots,m,\quad j=k+1,\ldots,n.$$

Here the constraint (6b) ensures a balance of the mathematical expectation of the budget of the whole system, the proof of the application of this constraint is presented in the Appendix of [1].

Thus, the objective function of the non-productive sphere is the maximization of the sum of the expectations of the goals of the activities $j=k+1, \ldots, n$ within the boundaries of budget constraints and resource limits and with satisfying direct activity constraints.

The credit side of the budget consists of the difference between the values of inputs and outputs assigned to the system (limits) at equilibrium prices and the profit from the production sphere. The debit side comprises expenditures of non-productive activities at equilibrium prices.

Problem (6) does not consist of independent subproblems of activities. In order to break it up, an additional mechanism involved in allocating the system's incomes between non-productive activities should be arranged by the centre.

In [1] all the optimal state-dependent coordination parameters are determined by the center. Although this approach is basic in the theoretical analysis of totally centralized planned abstract economies, it is practically unrealistic as it requires immense central coordination work. In this paper we try to clarify some coordination schemes where optimal coordination is conducted decentrally, and the center deals only with the allocation of the system's incomes between non-productive activities.

For the following note that models (5) and (6) can be easily reformulated in the form of pure exchange models. This can be done because the optimal lump-sum profit is exogenously and optimally allocated between the consumers by the center, and we also assume one-to-one mappings between the plans and net demands to avoid technically complicated representation problems [12].

The central allocation enables us to reduce our treatment to only pure exchange models and also to avoid technical difficulties connected with special problems of production firms' stock markets, etc. This line of research is conducted under uncertainty by P. Diamond [13], J. Dreze [14], R. Radner [15], et al. The results of these works indicate that the firms with shares can be fitted into the stochastic mechanisms theory, but in this paper we avoid the additional problems connected with stock market.

3. Benassy's strategic market game and Nash equilibrium

Here we shall describe on the example of the models of activities (5) and (6) the strategic market game where strategic outcome functions satisfy Benassy's conditions [4] leading the Nash equilibrium of the game to optimal Walrasian coordination outcomes.

In this game each activity or agent $j=1, \ldots, n$ sends price and quantity messages to $i=1, \ldots, m$ markets. Let $\hat{y}_j(s) \in R^m_+$ be the vector of agent j's price messages, and $\hat{d}_j(s) \in R^m$ the vector of agent j's quantity messages. We call $\hat{y}(s) = \{\hat{y}_j(s) \mid j=1, \ldots, n\}$ and $\hat{d}(s) = \{\hat{d}_j(s) \mid j=1, \ldots, n\}$ the set of all agents' price and quantity messages.

The plans (contracts) of exchange $d_{ij}(s)$ and prices $y_{ij}(s)$ actually realized for the agent j on the market i are described by strategic outcome functions:

$$d_{ij}(s) = F_{ij}(\hat{y}(s), \hat{d}(s)),$$

$$y_{ij}(s) = \Phi_{ij}(\hat{y}(s), \hat{d}(s)).$$

We shall assume that these functions satisfy Benassy's assumptions [4]. These assumptions will lead to competitive outcomes. First, we assume voluntary exchange. That means that no agent can be forced to take more contracts than he plans, and trade at prices less favourable than the one he has quoted. Secondly, frictionless planning mechanism is assumed, i. e. agents do not miss obvious opportunities for trade. And the third assumption is that of price priority. It says that demanders will give preference to the suppliers announcing the lowest prices, and conversely, the supplier will want to supply in priority demanders announcing the highest prices. This assumption automatically means that the market is competitive, i. e. there are at least two active demanders and two active suppliers for every resource. A consequence of this assumption is that the supplier who quotes higher prices will be rationed and conversely, the demander who quotes lower prices will be rationed.

Under these assumptions Benassy has demonstrated [4] that Walrasian (optimal or Lagrangian) equilibrium is also Nash equilibrium. Indeed, everybody announcing Walrasian (optimal) prices $\hat{y}_{ij}(s)=y^0_i(s)$ and quantities $\hat{d}_{ij}(s)=g_{ij}(x^0_j(s), s)$, $i=1,\ldots,m$ will also get Walrasian outcomes, as no agent will be rationed and there will be no bargaining problem with prices. And no agent can improve his situation by changing his strategy. Indeed, assume that agent j deviates and announces non-Walrasian prices while other agents maintain their Walrasian strategies. First consider the case where agent j is announcing higher supply and lower demand prices. In this case he will be rationed, and he is forced to make an uneffective plan. Another case would be announcing lower supply and higher demand prices, but he would be ineffective again. Thus the Walrasian equilibrium is Nash equilibrium, and according to [4] Nash equilibrium is also Walrasian.

Although this approach is sound for the theoretical analysis of decentralized pricing-limiting, unrealistic assumptions are made again. First it is assumed that there are current future markets for every state $s\in S$ (Arrow-Debreu market). And, secondly, every agent is assumed to be able to determine Walrasian prices and quantities for every state $s\in S$, i. e. every agent has perfect foresight. In the next section we shall drop the assumption of the existence of state-dependent forward markets. Note that in this approach coordination is performed with mixed signals (prices and quantities).

4. A modification of the price-limit mechanism in J. Green's and J. Grandmont's style

The problem considered in this section arises from the fact that in reality complete current forward markets in state-dependent claims cannot exist because the number of the states is enormous. So we must reduce this ideal mechanism to a more practical one. For that purpose we use the model studied by J. Green [2] and J. Grandmont [3]. In this model an agent exchanges at futures market fixed contracts (plans) at fixed prices that implies the fixed delivery or the receipt of commodities at later dates. And future spot markets are assumed to be active at these dates (the dates of the delivery or receipt of these commodities). So the poossibility of future spot contracts on forward markets is open to the agent. But the prices on these markets are understandably unannounced and there must exist the agents' expectation concerning future spot prices and quantities.

To simplify matters still more, assume that the first period is deterministic. So in period 1 each agent knows his limits (endowment) and other parameters with certainty, but does not know yet what will happen in period 2. Each trader knows that in period 2 there will be a spot market for goods available at that date, but the spot prices are not announced. So he must also have expectations concerning the future.

Consider a representative agent in period 1. His Benassy strategy is $(\hat{y}_{1j}, \hat{d}_{1j}, \hat{y}^2_{1j}, \hat{d}^2_{1j})$. It represents the prices $\hat{y}_{1j}$ of the current goods (spot) and the future prices $\hat{y}^2_{1j}$ of planned purchases of goods to be delivered in the second period. It also represents the respective planned quantities of goods $\hat{d}_{1j}$ and $\hat{d}^2_{1j}$ for sure delivery or receipt.

In period 2 the agent will receive the signal of the state and announce the signal $(\hat{y}_{2j}, \hat{d}_{2j})$, describing his spot prices and the respective quantities of goods at date 2. At date 1 the agent forms his rational expectations about $(y^0_{2j}(s), d^0_{2j}(s))$.

Now our actor's decision problem is similar to the one described by J. Grandmont [3], and according to the assumptions made there the necessary and sufficient conditions for the existence of an equilibrium are present. Note that in this approach for the implementation of optimal plans, and the assumption of rational (correct) expectations of future spot prices and deliveries are necessary.

In 1987 R. Wright [16] proved that under certain conditions the complete market (Arrow-Debreu market in which all commodities for all dates and states are traded simultaneously) and the described recursive market systems are equivalent in a stochastic economy. In order for this result to go through a restriction is needed. Recursive competitive equilibria can never exist without some side conditions to prohibit from running up and rolling over arbitrarily large debts. This result is helpful in understanding the types of markets necessary to support competitive equilibria in dynamic economies.

It remains to be said that if this model satisfies J. Benassy's conditions the Nash equilibrium will be Walrasian. So the agent's plans are not coordinated about forward spot markets, but they are still compatible. This was achieved with the help of the perfect foresight approach postulating that all the agent's expectations are correct. The perfect foresight approach is very convenient, however, it is surely an improper tool for describing the reality [17].

5. Some problems of the reduction of managerial risk-aversion

We distinguish two kinds of managerial risk aversion problems: unimplied and implied by mechanism. The former problem is as follows. It is reasonable to assume the risk neutrality of the global objective function of the social choice rule. But in a decentralized setting this means that the objective functions (normatives) of firms (5a) are also risk neutral. However, in reality the managers of the firms are risk-averse. To stimulate the risk-averse managers to make risk-neutral decisions, there must be some kind of risk-sharing enforcement mechanism enabling managers to transfer incomes between the states. Below some rules for reducing this kind of risk aversion are explained.

Further we demonstrate the problem of implied by mechanism risk aversion by the example of quantity risk and linear incentive mechanisms. As a matter of fact, the incentive mechanisms themselves could imply additional managerial risk aversion [18]. We study the incentive mechanisms with interval plans, and demonstrate that the replacement of a point plan by an interval plan will reduce the rate of implied risk aversion.

In the case of direct unimplied managerial risk aversion insurance could be introduced to diminish aversion [19]. But the manager's ability to obtain an insurance contract is limited by his information privacy. For example, an insurance contract which pays the manager $a>0$ in state 1, and taxes the manager $b>0$ in state 2 cannot be implemented if the insurer must rely on the manager to report which of the two states has accrued (the manager will always report state 1). However, the manager's incentive to report the wrong state can be lessened, setting the condition that if he reports state 1 he must choose a production plan (observable by the insurer) that is inefficient in state 2, and vice versa [19].

The above model is good to emphasize the idea of a risk-averse manager trying to get insurance against fluctuations in his net income, and so reduce his risk aversion and make his decisions more effective from the point of view of the whole economy. But it is easy to see that in the case of more than two states it becomes much harder to describe the many truth-telling constraints. Also, it will be prohibitively costly to make contracts with the distinction of many states.

In the sense of real contracts it is of greater interest to analyze the contracts in simple terms of formulae linking some kind of publicly observable indicator (not states) with the insurance premium.

A good practical approximation is the describing of risk-sharing contracts on the basis of some publicly observable indicators characterizing the states aggregatively. We shall demonstrate such a possibility by discussing cost-sharing as an example.

In [20] a formal model of optimal contracting of competing offers to select the efficient contracts is presented. Two main results are established there. First, under general conditions, an «incentive» (risk-sharing) contract is superior to either the cost-plus or fixed-price extremes. Second, there exists a «signalling» contract which is strictly superior. Under this contract dual-signalling or dual-component bids are used: risk-sharing rates and cost rates are signalled.

This model is directly applicable in the case of Benassy mechanism. In this case the price announcements of the agents are dual-component, and so are the realized market prices.

As has already been mentioned, the use of some incentive mechanisms brings about the so-called implied risk aversion which may induce the agents to choose relatively conservative and globally not optimal plan alternatives. Mechanisms with interval plans are of interest for soothing this phenomenon since the replacement of a point plan by an interval plan seems to reduce the rate of implied risk aversion. The mechanisms with a point plan can be regarded as special cases where the interval converges into a point.

The aim of the following is to study the rate of the risk aversion implied by a piece-wise linear incentive mechanism with an interval plan and its dependence upon the width of the interval. In so doing both the mechanism and the agent are extremely simplified (risk neutral agent is maximizing the expectation value of the incentive) to allow the author to concentrate his attention on the main problem.

Let there be a risk neutral agent who maximizes the expectation value, Ew, of his incentive w. The incentive depends upon a random result (e. g. profit) described by a random normal variable y with a density function $h(y)$, mean value μ and standard deviation σ. The incentive also depends upon a piece-wise linear mechanism using an interval plan. Let the end points of the interval plan be y_1 and y_2, and let they be defined as follows:

$y_1=\mu-k\sigma$ and $y_2=\mu+k\sigma$ where $k>0$ is given and determines the width of the interval.

As we have already said, for clarity's sake an extremely simple mechanism has been chosen, namely

$$w=\begin{cases} ey-p(y_1-y), & \text{if} \quad y\leqslant y_1, \\ ey, & \text{if} \quad y_1\geqslant y\geqslant y_2, \\ ey-q(y-y_2), & \text{if} \quad y\geqslant y_2. \end{cases} \tag{7}$$

Here e is the share the agent will get of the result y if the latter is within the planned interval. The parameters p and q are penalty rates for unit under- or overfulfilment of the plan that are calculated with the parameter e.

For calculating the expectation value of the incentive the following notations will be used: s — standard random variate ($\mu_s=0$ and $\sigma_s=1$), $n(s)$ — density function of standard random variate, $N(t)=\int_{-\infty}^{t} n(s)\,ds$ and $\hat{M}(t)=\int_{t}^{\infty} sn(s)\,ds$.

Now, taking example of [17], the expectation value of the incentive can be derived as follows:

$$Ew=\int_{-\infty}^{y_1}[ey-p(y_1-y)]h(y)\,dy+\int_{y_1}^{y_2} eyh(y)\,dy+$$
$$+\int_{y_2}^{\infty}[y-q(y-y_2)]h(y)\,dy=e\mu-c(k)\sigma,$$

where $c(k)=(p+q)[\hat{M}(k)-kN(-k)]$.

As we can see, the expectation value of the incentive to an initially risk-neutral agent is a linear combination of the result's mean value μ and its standard deviation σ. Consequently, because of mechanism (7) the initially risk-neutral agent starts to consider the risk connected with the results (described by σ here), and we can speak of implied risk aversion. It can induce the agent to choose such alternatives which are not optimal from a global point of view. For example, if the initial problem (without decentralization and thus without an incentive mechanism) is aimed at maximizing the results' expectation values, its decentralized solution (with the help of the agents who are given incentive) may yield a more conservative result and a lower expectation value. Relying on [17] the rate of implied risk aversion can be described as follows:

$$r(k)=c(k)/e=(p+q)[\hat{M}(k)-kN(-k)]/e, \tag{8}$$

where k describes the width of the plan interval. If $k=0$, the interval will converge into a point or a fixed plan, and if k increases, the interval will likewise grow.

On the basis of the rate of implied risk aversion (8) a number of conclusions can be drawn. At that it is assumed that $e,p,q>0$ which means that the agent is interested in a high result and its falling within the plan interval. From the definition it is known that

$$\hat{M}(k)=\begin{cases} 0 & \text{if} \quad k=\infty \\ >0 & \text{if} \quad 0<k<\infty \\ 1/\sqrt{2\pi} & \text{if} \quad k=0, \end{cases}$$

and

$$N(-k)=\begin{cases} 0 & \text{if } -k=-\infty \\ >0 & \text{if } -\infty<-k<0 \\ 1/2 & \text{if } k=0. \end{cases}$$

Now it is easy to see that the maximum of the rate of implied risk aversion is $(p+q)/e\sqrt{2\pi}$, and it corresponds to the point plan ($k=0$). If the plan interval is expanded, the rate of risk aversion will decrease, it may even disappear. Thus, the wider the plan interval, the more boldly the agent chooses the alternatives, the less he considers the dispersion of the results of the alternative and the more attention he pays to the expectation value of the result. If, however, the alternative is given, the expansion of the interval will raise the expectation value of the agent's incentive, being thus useful to the agent. But too wide plan intervals may be unsuitable for the center or for the other contractor. Therefore we can speak of the rates of plan intervals optimal for the system, this, however, is a separate complicated problem.

We can also see in (8) that the rate of implied risk aversion will decrease if $(p+q)/e$ decreases. In other words, the lower the penalty rates and the higher the share of the incentives in the plan interval e, the more the agent considers the result's expectation value when deciding which alternative to choose.

6. Final remarks

The purpose of this paper was to clarify alternative planning mechanisms of abstract economies under uncertainty. The paper examines them mainly in the context of two optimal future price-limit coordination mechanisms. In the case of the first mechanism the basic assumption is that there exists a complete set of current (pre-event) state-dependent future goods markets. The second version says that the agents have current correct expectations about the optimal future (post-event) state-dependent prices and quantities. In the context of the initial model used in this paper the elaboration of optimal price and quantity plans in both mechanisms is achieved.

Also, issues of risk allocation in the mechanisms are clarified from two aspects. First, the possibilities of diminishing unimplied managerial risks are demonstrated. Second, the rate of risk aversion implied by the incentive mechanism and an interval plan is studied, and, as a result, the following assertion is made: the use of an interval plan instead of a point plan will lessen the risk aversion induced by the incentive mechanism and stimulate the agents to choose riskier alternatives.

However, the mechanisms studied are based on highly oversimplified assumptions. So the problem remains to be studied on more realistic assumptions and not oversimplified models. Significant open problems include the implementation of a correct expectation equilibrium (information acquisition), imperfectly competitive markets with expectations, bidding for long-term contracts with relationship-specific investments under uncertainty, etc.

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Ülo ENNUSTE

ALTERNATIIVSETE STOHHASTILISTE PLANEERIMISMEHHANISMIDE MÕNINGAID PROBLEEME

Artiklis on selgitatud alternatiivsete stohhastiliste planeerimismehhanismide lähteeeldusi ja hinnanguid. Mehhanismid on koostatud abstraktsele optimeerimismajandusele, mis koosneb kahest tegevussfäärist ja hõlmab kahte perioodi.

Alternatiivsed planeerimismehhanismid rakendavad kombineeritud koordineerimist (hinnad ja limiidid), kuid põhitähelepanu on pööratud stohhastiliste plaaniliste hindade mehhanismile. Samuti on vaadeldud nende mehhanismidega seotud planeerijate täiendavat riskikartust ja võimalusi selle leevendamiseks.

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Юло ЭННУСТЕ

НЕКОТОРЫЕ ПРОБЛЕМЫ АЛЬТЕРНАТИВНЫХ СТОХАСТИЧЕСКИХ МЕХАНИЗМОВ ПЛАНИРОВАНИЯ

В статье проанализированы основные предпосылки и оценки некоторых альтернативных стохастических механизмов планирования для абстрактной оптимизационной экономики, охватывающей два периода и две сферы деятельности. Альтернативные механизмы планирования применяют комбинированную координацию (цены и лимиты). Здесь же основное внимание уделено механизму плановых цен в условиях стохастики. Обсуждаются также неизбежные при таком механизме планирования проблемы риска и возможности снятия боязни риска среди плановиков.

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