EESTI NSV TEADUSTE AKADEEMIA TOIMETISED. ÜHISKONNATEADUSED
ИЗВЕСТИЯ АКАДЕМИИ НАУК ЭСТОНСКОЙ ССР. ОБЩЕСТВЕННЫЕ НАУКИ
PROCEEDINGS OF THE ACADEMY OF SCIENCES OF THE ESTONIAN SSR. SOCIAL SCIENCES

1986, 35, 2

https://doi.org/10.3176/hum.soc.sci.1986.2.03

Ülo ENNUSTE

TOWARDS STOCHASTIC MATHEMATICAL ECONOMICS

Rather similar turning points can be observed in the development of different branches of science which consist in the transition to a consideration of the object's indeterministic characteristics and approaches based on probability theory. In physics the changeover took place in the 1920s—1930s in connection with the formulation of Unbestimmtheitsrelationen (uncertainty principles) by W. Heisenberg. Now we are witnessing the same phenomenon in mathematical economics. Both in physics and economics the turn has been causing great controversy since it is not only a new attire of the theory that is involved but primarily a further development of numerous methodological and essential principles as well as a revision of several standpoints.

The present paper is an attempt at a brief survey of the developments and changes in mathematical economics, both in the methodology of planning and the methodology of the economic mechanism. We shall begin with some introductory remarks on the essence of the problem and the sources of indeterminacy and incomplete information in economy, and the methods of modelling incomplete information.

1. Introductory remarks

A lot of data used in economic calculation and particularly in planning are inaccurate or presented with incomplete information. That phenomenon is so unpleasant that for a long time we have simply tried to neglect it. Such unpleasantness seems to result mainly from the fact that the consideration of the incompleteness of information makes the reasoning more complicated, with problems of mathematical economics becoming especially complex. It is interesting to note here that even J. M. Keynes who has written a work on probability theory could not apply that concept in his general economic theory.[1]

Another reason for unpleasantness is the following. Sometimes the introduction of incomplete information into economics is misunderstood. It is claimed that this would mean taking a slippery road, allowing disorganization gain the upper hand as if stochasticity and disorganization were one and the same thing. The opponents are forgetting here that in dialectal materialism determinacy is not treated mechanically but that it also incorporates the concept of randomness. From the standpoint of managing socialist economy this means that in planned management of the economy the existence of incomplete information must be consciously taken into account. Unfortunately, in economic theory such considerations are so far insufficient, and the elimination of that shortcoming would help to raise the adequacy of the theory.

Up to the 1960s it was mainly the general simplified deterministic

[1] *Coddington, A.* Deficient foresight: a troublesome theme in Keynesion economics. — The American Economic Review, 1982, **72**, N 3, 480—487.

economic theory that was developed (with the exception of some specific theories treating insurance against economic risk, exchange, queues, etc.). This theory assumed that all the data and relations were exactly known. However, a number of important practical issues lack any contents within the framework of general deterministic economic theory and are thus either not treated at all or are not duly considered. Among them the following might be mentioned: development stability and adaptability, stocks, diversification, flexibility of plans and prices, value and cost of economic information, etc. Moreover, several so-called essential problems were not solved adequately in deterministic theory as it was still assumed that everything was exactly known beforehand, and all economic agents possessed equally complete information free of charge. No doubt, such simplifying assumptions distort the results of the theory both in the sphere of economic processes and economic mechanisms.

During the last two decades, however, the indeterminative economic theory has been making headway, and especially in the field of mathematical economics. One could easily produce several thousands of respective mathematical investigations dating from that period. In recent years works by Soviet researchers have occupied a noteworthy position among them.[2] But as common during revolutionary periods, these investigations contain sometimes quite controversial standpoints as well as precipitate conclusions.

In economic practice, however, the need for theoretical treatments considering incomplete information seems to become increasingly pressing as the share of the latter is growing. It is advisable to distinguish three types of sources of incomplete information.

In the first place, a number of processes related to economy are in principle indeterminate. For example, indeterminacy is characteristic of scientific and technological progress whose course of development is prognosticated with great difficulties. Likewise, it is hard to foresee climatic changes, changes in fashion and customers' preferences, etc.

Secondly, we can speak of economically optimum incompleteness of information. It is often wiser to operate with incomplete information than to obtain full information at an exorbitant price. As an example, the estimation of the volume of mineral resources might be cited.

Thirdly, there exists the so-called organized incompleteness of information. This results from the fact that for political, economic or other reasons some economic agents find it useful to suppress or to distort certain information. Thus, it is extremely hard to forecast the potentialities of foreign trade, sometimes higher authorities find it difficult to estimate the abilities and efforts of the units subordinated to them, etc.

For the modelling of data with incomplete information mainly the following constructions are used: random variables, fuzzy variables and intervals. There exist respective optimum plan models as well as models for efficiency estimates and analysis for each of them. Among them stochastic models based on random variables are the most highly elaborated and the most general ones. Below only these models will be discussed.

However, it is hard to describe various phenomena vital in socio-economic processes by means of the existing traditional stochastic models. Such phenomena are, for example, the stochastic interdependence of the model and the discontinuous changes in some parameters. For a simplified description of these phenomena it is sometimes comfortable to use the

---

[2] See e. g. *Ермольев Ю. М., Ястремский А. И.* Стохастические модели и методы в экономическом планировании. М., 1979; Моделирование в процессах управления народным хозяйством. Под ред. М. П. Федоренко и Н. Я. Петракова. М., 1984.

concepts of stochasticity of the structure of the model.[3] Its idea consists in the assumption that the model's structure depends on random events, being thus probabilistic (coupled with the probabilistic character of the parameters of the model).[4]

It should be added here that in case of such an approach the expected value of the complete information of the model is divided into the value of the information on the structure of the model and the value of the information on the model's parameters. Consequently, as compared to the conventional approach (determinacy of the structure), the expected value of the information on the parameters of the model becomes reduced here.

2. On methodological problems of mathematical forecasting and planning

The increasing importance of forecasts in economic management and planning seems to attest to a growing consideration of probabilistic factors in economy. From the standpoint of the present contribution it is of significance to stress that forecasts should not be simplified so as to become determinative point forecasts, but, on the contrary, they must characterize the possible variance of the value of the indicator studied, and more fully to characterize the incompleteness of information they are based on. As to the practical extrapolational forecasting, here the following recommendations should be made.

First, as a rule in practical forecasting the form of the forecasting models and their parameters are determined only on the basis of statistical time series. And here it is advisable to supplement this information by external information, which is often non-statistical. Such information may, for example, consist of directive instructions which are not yet reflected in statistical data on the object, or some asymptote that may limit or adjust the further course of the process.

Secondly, it is often forgotten here that the form of the regression model is not determined. In practice complete information on the form of the model is not available. This fact should be considered in forecasts, and this means that the regression functions with a stochastic structure (i.e., the form of the function is random) are the most adequate ones.

Thirdly, forecast intervals or zones are often replaced by point forecasts, but this is an oversimplification. To justify such practice, it is usually claimed that the decision-maker has no time to concern himself with interval information and altogether he cannot stand vague answers. However, it often happens that no other answers can be given about the future, and this is to be taken into account.

In deterministic planning theory a plan was something inflexible and final, but now inflexibility and finality are not always the only and the best properties of a plan. Assuming that additional information is constantly received both in time and space (in the management hierarchy), it is expedient to use this additional information, and this possibility must be taken into account already when the plans are being compiled.[5] Such problems as optimum preliminariness and flexibility or in one word adaptivity of plans, optimum delay of final adoption of a plan, optimum rate of flexibility, etc. crop up. Analogous problems of optimum adaptivity

---

[3] *Ennuste, Ü.* On parametrically and structurally stochastic optimum problems with adaptive plans. — ENSV TA Toim. Ühisk., 1983, 32, N 1, 28—31.

[4] Let us note that in certain cases it is possible to describe also some economic theory with the help of a model, thus a model with a stochastic structure enables to describe the probability of different theories. In other words, a situation can be described where this or that economic theory holds with a certain probability.

[5] *Эннусте Ю.* О принципе определенно-вероятного планирования, моделях и функционировании экономики. — Изв. АН ЭССР. Обществ. н., 1974, № 1, 3—21.

are connected also with several economic coordination parameters — prices, rations, etc.

Proceeding from the assumption that additional information will be received in the course of time, it is advisable to fix the plan definitely only for the beginning of the planning period, i.e., for some initial subperiod. Plan indices for the next subperiod should be given as preliminary ones (in respect to forthcoming additional information), i.e., in the form of functions of stochastic states of the system, or in the form of intervals, etc. At that the final plans x_{I} of the initial subperiod determine the possible domain $D(\ldots)$ of the adaptive plans of the next subperiod $x_{\mathrm{II}}(\alpha)$:

$$x_{\mathrm{II}}(\alpha) \in D(x_{\mathrm{I}}, \alpha),$$

where α denotes a random variable whose realization is not known when x_{I} is fixed.

The possibility of obtaining additional information and the consideration of the expenditures involved turn such characteristics of the planning model as the duration of the period of final plans and the slipping step (i.e. the time of the final actualization of the plans), the volume of additional information to be collected, etc. into characteristics which are to be optimized.

The determination of an optimal actualization horizon in an unsimplified way is an extremely complicated task since here the substitutability of economic stockpiles, actualization time, etc. must be taken into account. Therefore in practice it often becomes necessary to pursue the following rule: actualization of the plan is premature if it is not accompanied by the beginning of preparatory work.

The principle of the flexibility of a plan is connected with the aggregatedness of planning information on the highest levels of planning. On the basis of this principle we can distinguish fixed and flexible plans. A flexible final plan for the lower level of the planning system, for instance, is given by an interval or distribution or something of the kind. The flexibility of a plan, like its preliminary nature, is a parameter that is to be optimized. The corresponding optimization problem must take into consideration the expenditures or losses connected with the rate of flexibility, e.g., balancing difficulties on the higher level of the planning system on the one hand, and on the other — the effect gained from a freer determination of plans on the lower levels of the planning system and thereby a fuller use of local information.

On the basis of the characteristics formulated, four classes of plans can be described. The classes are presented in the table below.

Classes of plans

Flexibility	Finality: Final	Finality: Preliminary
Rigid	0 fixed plan	I preliminary plan
Flexible	II flexible plan	III preliminary flexible plan

The plan of class 0 is not adaptive, while the three others are. In the Table the adaption with regard to the information received in the future is carried out horizontally, and with regard to information losses at aggregation and the detailing of data in the hierarchic system of planning, vertically.

Thus, the planning by means of adaptive plans eliminates superfluous rigidity of planning, and this enables to decrease the effect of indeterminately forecast negative events, and to make fuller use of advantageous circumstances not exactly foreseen as yet and local information unknown on the higher levels of planning.

Here the concept of the expected value of an adaptive plan should be mentioned. This concept does not coincide with the concept of the expected value of complete information. Indeed, it is easy to give examples where the adaptive plan coincides with the fixed plan and the expected value of adaptive plan is zero but the expected value of complete information is positive, and vice versa.[6]

In the theory of planning of economic processes an active improvement of concepts is taking place in connection with the transition to probabilistic bases. Randomness intrudes into production economics through technology, prices as well as demand and supply. Studies indicate that the strategy of planning should depend on the relative risk rates of the elements mentioned here. Thus, no recipes valid in all cases can be presented, but in general the growth of indeterminacy seems to increase the variety of products or the diversification (several irons in the fire), decrease the volume of optimum output[7] and increase optimum stocks.

In the theory of consumption the consideration of stochasticity enables to profoundly treat the relationship of current consumption and savings, and find its connection with the rate of the consumers' risk aversion. However, the fact that indeterminacy affects incomes, prices, interest rates, etc. differently complicates the matters also here.

The economics of resources is especially closely connected with indeterminative factors. On the one hand, the stock of resources is indeterminate: the increment of renewable resources is usually random; for nonrenewable resources, however, the volume of reserves is not known exactly, etc. On the other hand, it is difficult to forecast general conditions concerning the use of resources such as technology, possibilities of replacement, import and export, etc. All these indeterminacies should be taken into account in pricing in the form of corresponding additions to prices, and also in optimizing the use of resources.

The situation is almost the same in case of environmental economics. Here the incomplete information is connected primarily with estimating environmental quality and its economic effect, and also with forecasting the performance of purification equipment.

The use of stochastic models is widespread in the modelling of financial flows as these flows are of a probabilistic nature. The same is also valid for insurance operations and shares of stock (portfolio theory), optimization of stocks, queues, maintenance and repairs, etc.

It being impossible to get additional information without extra expenditures, the problem of the expected value of information acquires

---

[6] *Birge, I.* The value of the stochastic solution in stochastic linear programs with fixed resource. — Mathematical Programming, 1982, **24**, N 3, 314—325.

[7] On the other hand, it is shown that in the case of learning from the experience on the output decisions, an experimenting firm tends to overproduce. See *Harpaz, G., Lee, W., Winkler, R.* Learning, experimentation and the optimal output decisions of a competitive firm. — Management Science, 1982, **28**, N 6, 589—603.

great importance. Hence the so-called concepts of optimum learning and experimentation can be derived. It is easy to see that the latter in their turn form complicated relations with the rate of economic adaptivity.

3. On methodological problems concerning economic mechanisms and systems of models

The theory of economic mechanisms seems to be occupying a more important place than earlier in economics thanks to probabilistic approaches. Namely, feedback in management and current regulation will become more important when indeterminacy is taken into account. The decentral nature of information (it is impossible to forward all incomplete information to the centre so that it will remain trustworthy) and the resulting phenomena of information asymmetry will become more evident. All these circumstances make scientists and decision-makers adopt a new approach to economic decentralization on the one hand and central coordination on the other, and show the invalidity of both completely centralized and decentralized systems in new aspects.

In the stochastic theory of mechanisms a new submechanism — the insurance system — comes to the fore. This system makes it possible to distribute the economic risk of an individual agent between a larger number of economic agents or units, and, as a result, the individual agent can undertake risky activities useful from the society's point of view which otherwise would be impossible.

Using stochastic equilibrium theory it is possible to show that pure market mechanisms are not sufficient for gaining an equilibrium.[8] Namely, in a stochastic approach, the economic units have no adequate information about prices; moreover, different agents have different information (rational expectations of prices). To gain an equilibrium, attempts have been made to include some additional mechanisms into the theory in such a way that they resemble in several senses coordinating planning bodies.

As to the theory of the mechanisms of centrally planned economies, the probabilistic approach brings about the terms of the adaptivity of plans and their coordination parameters. At the same time, the problems of information asymmetry, stimulation of honest data transfer and distribution of risk crop up here.

In investigating coordination principles and structures of the systems of centrally controlled indeterministic forecasting and planning models and mechanisms, methods of decomposition are of special importance. However, the coordination principles of such systems have so far been insufficiently studied. Today only the coordination principles of deterministic systems have more or less been developed. First steps are being made in the field of investigating coordination under incomplete information. More results have been here achieved in the field of stochastic competitive economic equilibrium.

The decomposition analysis of elementary two-stage stochastic optimization problems shows the advisability of applying adaptive coordination.[9] In this model it has been carried out by using the stochastic functions. At that it is elucidated that the optimal adaptivity of coordinating parameters exceeds that of planning indices. In the present

---

[8] *Grandmont, J. M.* Temporary general equilibrium theory. — Econometrica, 1977, **45**, N 3, 535—572.

[9] *Ennuste, Ü.* On stochastic equilibrium of centrally co-ordinated optimum plans. — Proc. ESSR Acad. Sci. Social Sciences, 1985, **34**, N 3, 237—245.

concrete case this means that even the coordination of final plans is carried out by means of adaptive coordinating parameters.

So we can see that a possibly complete description of the adaption of economic development in mathematical mechanisms models coincides to a large extent with an adequate description of incomplete information and related problems.

4. Concluding remarks

To sum up, we can say that an extensive consideration of indeterminate aspects of economic processes will substantially change the methodology of the mathematical economic theory. A number of new terms, e.g., incomplete information, stochastic models, stability, adaptivity, value and cost of information, asymmetry of information, etc. will be introduced. New issues and standpoints will be raised in the traditional chapters of economics.

In the methodology of planning, the optimal preliminariness and flexibility of plans as well as the optimal lag in adopting final plans will become crucial. In the theory of production, the problems concerning diversification and stocks will rise to the fore; in the theory of consumption, the relation between current consumption and savings will be of major importance while in the theory of resources, the problems of exhaustion and prices connected with the indeterminacy of stocks and new technologies will stand out.

In the theory of economic mechanisms an important role will be played by the problems of the adaptivity of prices, rations and quotas as well as those of providing incentives for the forewarding of information without falsification and risk distribution between economic units.

In the economic theory there will arise a need for new chapters dealing with general principles of the consideration of incomplete information, the value and cost of information, and the optimum volume of information, etc.

The importance of different chapters will likewise change. The consideration of incomplete information seems to increase the importance of feedback and thus that of current regulation, and decrease relatively the proportion of the chapters devoted to planning. At the same time the importance of forecasting will rise in the chapters dealing with planning. Abstract models describing either completely decentralized or centralized systems will not be as important as they are now, and more important will be the study of mixed systems.

Although there are still a number of unsolved problems as well as immature results in the field of considering indeterminate factors, there are already numerous standpoints here which deserve more attention in our economic research and practice and in the respective courses at higher educational establishments.

Presented by K. Habicht

Academy of Sciences of the Estonian SSR,
Institute of Economics

Received
July 11, 1985

Ülo ENNUSTE

TEEL STOHHASTILISE MAJANDUSMATEMAATIKA POOLE

Artiklis on kirjeldatud neid edasiarendusi ja muutusi matemaatilises majandusteaduses, mis on seotud majandusliku vaegteabe täielikuma arvestamisega eeskätt viimasel aastakümnel.

On kirjeldatud majandusliku vaegteabe allikaid, selle matemaatilise modelleerimise võimalusi, kerkivaid uusi probleeme ja mudeleid ning nende lahenduste tõlgendusi. Kõiki neid küsimusi on vaadeldud nii majandusprotsesside kui ka majandusmehhanismide teooria alal. Eriti on rõhutatud majandusliku adaptiivsuse modelleerimise vajadust seoses vaegteabe arvestamisega nii planeerimises kui ka majandusmehhanismide uurimises.

Eesti NSV Teaduste Akadeemia
Majanduse Instituut

Toimetusse saabunud
11. VII 1985

Юло ЭННУСТЕ

НА ПУТИ К СТОХАСТИЧЕСКОЙ МАТЕМАТИЧЕСКОЙ ЭКОНОМИКЕ

В статье поясняются те усовершенствования и изменения в математической экономике, которые связаны с более адекватным учетом неполноты экономической информации в последнем десятилетии. Рассматриваются источники неполноты экономической информации, возможности ее математического моделирования, появление новых проблем и моделей, а также экономических интерпретаций их решений. Все эти проблемы описываются на примере как экономических процессов, так и хозяйственных механизмов. Особенно подчеркивается потребность в более совершенном моделировании экономической адаптивности в связи с учетом неполноты информации. Это касается как планирования, так и исследования хозяйственных механизмов.

Институт экономики
Академии наук Эстонской ССР

Поступила в редакцию
11/VII 1985