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ON PARAMETRICALLY AND STRUCTURALLY STOCHASTIC OPTIMUM PROBLEMS WITH ADAPTIVE PLANS

The author suggests that the concept of stochasticity of the structural elements of optimum planning problems be applied in addition to stochasticity of the parameters of the problems. The concept of the expected value of structural and parametrical information in such problems is explained.

1. So far, stochastic optimum planning problems have been formulated mainly with the assumptions that the parameters of the problem are stochastic independent continuous variables, and the structure of the problem is fixed. Though their solving and analysis is complicated, such problems are still oversimplified to satisfactorily model typical planning problems, above all certain socio-economic problems.

For example, in the latter problems the following phenomena are quite common. Potential applications of activities are not certain; some activity may (for technological reasons) in certain cases prove to be inapplicable. Likewise, some resources may be available within certain limits only, or not at all. Frequently there exists a strong stochastic interdependence between the parameters of a planning problem. Moreover, it often happens that the classes of the interconnections (either linear or exponential, and the like) of the elements of the problem are not known for sure, etc.

The modelling of all the above-described phenomena with the help of the parameters is either extremely complicated or even impossible; moreover, such problems would be extremely complex and difficult to solve. The author suggests that the concept of stochasticity of the structural elements of problems be applied for the modelling of the described phenomena. This means that a method is suggested, where, besides the stochasticity of the parameters of the problem, the sets of its activities and resources and the classes of the functions describing it are stochastic.

2. To explain this idea, let us proceed from the following deterministic initial problem:

$$\max f_0(x, c)\}f_i(x, a_i) \geqslant b_i,\quad x=(x_j),\ j=1,\ldots,n,\ i=1,\ldots,m, \tag{1}$$

where c, a_i and b_i are the parameters of the problem, and n, m, f_i and f_0 are structural characteristics of the problem (here f_i describes a certain class of functions). In short, the parameters will be denoted by d, and structural characteristics by k.

As already mentioned, in case of common stochastic optimum problems it is assumed that the parameters are indeterministic and modelled by random variables, and complete information is given about the structural characteristics. Presently, an additional assumption is made that complete information about the characteristics is also lacking, and this phenomenon is modelled with the help of discrete random variables. In other words,

$k \in K$, where K is a finite set of probable structural variants; and the probabilities $p(k) \geqslant 0$, $\sum p(k) = 1$ are given. It is further assumed that the parameters d of problem (1) are given continuous random variables δ.

It is not difficult to see that on these assumptions various probable possibilities of both sets of activities and sets of resources as well as possible classes of connections can be described with ease, and an approximate modelling of the dependence of parameters becomes possible. To describe the latter, the existence of probable characteristic sets of parameters is assumed. At that, each characteristic set comprises parameters connected to each other to a certain extent, and the set of parameters stands also for changes in the classes of functions here (to put it differently, each set corresponds to the realization of a so-called significant event [1, 2]).

3. In stochastic optimum problems it is possible to look for optimum plans independent from as well as dependent (adaptive) on the realizations of random events, and for combinations of the two [3, 4]. In problems with a stochastic structure it is possible to adapt optimum plans not only with respect to parametric information but also with respect to structural information. Thus, we can describe here the following classes of plans:

		Dependence of a plan on structural information	
		Independent	Dependent
Dependence of a plan on parametric information	Independent	x	x_k, $k \in K$
	Dependent	$x(\delta)$	$x_k(\delta_k)$, $k \in K$

With the modelling technique suggested it seems to be natural to form three-stage problems: there is complete information and an independent plan at stage I; at stage II the information about parameters is incomplete and the plan is parametrically dependent; while at stage III complete information on parameters as well as the structure is lacking, and the plan is dependent both parametrically and structurally. By means of such problems it is possible to describe both dynamic and hierarchic systems [5] or combinations of the two.

4. As a simple example, let us present two one-stage problems in a general form. Firstly, a problem with a parametrically and structurally adaptive plan $x_k(\delta_k)$, $k \in K$:

$$\max_{x_k(\delta_k)} f_{0k}(x_k(\delta_k), \gamma_k)\} f_{ik}(x_k(\delta_k), \alpha_{ik}) \geqslant \beta_{ik}, \tag{2}$$

where $\delta_k = (\gamma_k, (\alpha_{ik}), (\beta_{ik}))$, $i = 1, \ldots, m$ are independent stochastic parameters in case of the structural variant $k \in K$. Thus, here each structural variant has its own plan that is adaptive with respect to the parameters δ_k of this variant.

Secondly, an expected payoff problem with compensation and an independent plan x:

$$\max_x \sum_k p(k) E[f_{0k}(x, \gamma_k) - \sum_i q_i(f_{ik}(x, \alpha_{ik}) - \beta_{ik})] \equiv$$
$$\equiv \max \sum p(k) E g_k(x, \delta_k), \quad (3)$$

where q_{ik} is a compensation function and E is the operator of the expectation.

A combination of the two problems would yield a two-stage (stage I and II) problem with a plan $(x_{\mathrm{I}}, x_{\mathrm{II}k}(\delta_k), k \in K)$. It may happen that the dependence of the plan $x_{\mathrm{II}k}(\delta_k)$ on the structure is significantly stronger than its dependence on the parameters, thus, in the first approximation the latter can be abandoned, and the plan $x_{\mathrm{II}k}$ sought for at stage II.

5. Consideration of the structural indeterminacy of a problem enables to assess the value of structural information and differentiate it from the value of parametric information. Let us describe the expected value of complete information in case of a one-stage problem as follows:

$$v = \sum_k p(k) E[f_{0k}(x_k^0(\delta_k), \gamma_k) - g_k(x^0, \delta_k)], \quad (4)$$

where $x_k^0(\delta_k)$ is the adaptive optimum plan of problem (2) and x^0 is the optimum plan of problem (3).

The value of complete structural information is described as follows:

$$v_s = \sum_k p(k) E[f_{0k}(x_k^0(\delta_k), \gamma_k) - g_k(x_k^0, \delta_k)], \quad (5)$$

where x_k^0 is the solution to problem

$$\max_{x_k} E[f_{0k}(x_k, \gamma_k) - \sum_i q_i(f_{ik}(x_k, \alpha_i) - \beta_{ik})]. \quad (6)$$

Now the value of complete parametric information can be written as

$$v_p = v - v_s = \sum_k p(k) E[g_k(x_k^0, \delta_k) - g_k(x^0, \delta_k)]. \quad (7)$$

This shows that the value of the parametric information of the structural variant k is taken into account with the weight $p(k) \leqslant 1$. Thus, the expenditures that are justified for the purpose of specifying the parameters of the variant k are lower than those required under the circumstances where no structural variants exist.

Note also that the consideration of structural indeterminacy increases the indeterminacy of the whole treatment, thereby apparently shortening optimal plan horizons and increasing optimal aggregatedness of the treatment.

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PARAMEETRILIS-STRUKTUURSELT STOHHASTILISTEST ADAPTIIVPLAANIDEGA OPTIMUMÜLESANNETEST

Stohhastilisi optimumplaneerimisülesandeid on siiani püstitatud peamiselt eeldusel, et ülesande parameetrid on pidevad sõltumatud juhuslikud suurused ning ülesande struktuur on fikseeritud. Kuigi nende ülesannete lahendamine ja analüüsimine on keerukas, on nad siiski liialt lihtsustatud, et rahuldavalt modelleerida mitmeid planeerimis-, eeskätt sotsiaal-majanduslikke probleeme. Näiteks on üsna tavaline, et neis ei ole kindlad tegevuste potentsiaalsed rakendusvõimalused; mingi tegevus võib (tehnoloogilistel põhjustel) teatavatel juhtudel osutuda üldse mitte rakendatavaks. Samuti võib mingi ressurss olla saadaval teatavates kogusevahemikes või üldse mitte. Sagedane on ka planeerimisülesande parameetrite tugeva omavahelise stohhastilise sõltuvuse olemasolu. Sellele kõigele lisandub veel asjaolu, et sageli ei ole ka kindlalt teada ülesande elementide omavaheliste seoste klassid (lineaarsed, eksponentsiaalsed vms.) jne.

Kõiki kirjeldatud asjaolusid on kas äärmiselt keerukas või isegi võimatu modelleerida planeerimisülesande parameetrite abil, pealegi kujuneksid sellised ülesanded ülikeerukaiks ja raskesti lahendatavaiks. Artiklis soovitatakse sellistel juhtudel modelleerimiseks kasutada ülesannete struktuurikarakteristikute stohhastilisust, seega võtet, kus peale ülesande parameetrite on stohhastilised ka tegevuste ja ressursside hulgad ning ülesannet kirjeldavate funktsioonide klassid. Ülesande struktuuri indeterminismi arvestamine võimaldab ühtlasi selgitada struktuurilise informatsiooni väärtust ning eristada seda parameetrilise informatsiooni väärtusest. Ka seda probleemi on artiklis selgitatud.

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О СТРУКТУРНО-ПАРАМЕТРИЧЕСКИХ СТОХАСТИЧЕСКИХ ЗАДАЧАХ ОПТИМИЗАЦИИ С АДАПТИВНЫМИ ПЛАНАМИ

Постановка стохастических задач оптимального планирования до сих пор основывается главным образом на предположении, что параметры задачи — это непрерывные независимые случайные величины и структура задачи зафиксирована. Хотя решение и анализ этих задач сложны, такие задачи все же слишком упрощенны для удовлетворительного моделирования многих проблем планирования, прежде всего — некоторых социально-экономических проблем.

Например, для таких проблем весьма обычны следующие явления. Допустимое множество (число) применяемых технологий не зафиксировано; в отдельных случаях какая-либо технология может быть в принципе неприменимой (по техническим причинам). Четко не определено и множество (число) используемых ресурсов. Нередко наблюдается взаимная стохастическая зависимость параметров задачи планирования. Кроме того, зачастую четко не определены виды взаимосвязи элементов (линейные или экспоненциальные, или т. п.). Такого рода явления крайне сложно или даже невозможно моделировать при помощи параметров задач планирования, к тому же подобные задачи оказались бы чрезвычайно сложными и трудно решаемыми. В настоящей статье для моделирования описанных явлений рекомендуется использовать понятие «стохастичность структурных характеристик», т. е. подход, при котором множество технологий и ресурсов задачи, а также виды функций, описывающих задачу, — стохастические при условии стохастичности параметров задачи.

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