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ON A DECOMPOSITIONAL SYSTEM OF MODELS FOR NORMATIVE FORECASTING OF A REGION'S SOCIO-ECONOMIC DEVELOPMENT

1. Introductory remarks

1. Social and economic phenomena of a region being interconnected, an interconnection of or a systems approach to their forecasts is required. Such forecasting is in principle possible only within the framework of one unified mathematical model; however, in practice it would not be expedient to take that way. Namely, a unified model would be of extremely large dimensions, and thus it would be difficult to centralize trustworthy data for it, find its solution, and give an interpretation of its contents.

Therefore, a somewhat less rigid approach, realized with the help of systems of models, is considered to be expedient for forecasting socio-economic processes.

2. Two different principles exist for constructing systems of models: compositional and decompositional. In the former case, the system is formed by means of interconnecting (integrating) the separate models or submodels. In the latter case, however, a unified hypothetical initial model is formed first, and the submodels and instructions of their co-ordination are then derived on its basis.

In the field of applying the compositional principle for the construction of systems of mathematical models, a number of successful studies have been accomplished both by Soviet and foreign authors. Above all, we ought to mention the following scholars: N. Fedorenko et al. [1], R. Rayatskas [2], E. Baranov [3], V. Danilov-Danilyan and M. Zavelski [4], I. Adirim [5], D. Ray et al. [6], F. Archibugi [7]. Contributions by a number of Japanese authors also deserve notice (for review see [8]).

The compositional principle has found verbal treatment in several works by I. Bestuzhev-Lada [9], A. Köörna [10], a group of Finnish researchers [11], K. Hornback et al. [12], W. Swart et al. [13], R. Ackoff [14], etc.

An advantage of the compositional trend is that the existing separate models of various local processes are easily applicable. However, difficulties crop up in connecting or co-ordinating separate models. There do not exist any firm instructions for co-ordinating the models, and this tells on the strictness of the treatment, especially in case of normative forecasting.[1]

It is easier to solve the problem following the so-called decompositional principle, where the co-ordination instructions can be derived in a logico-formal way. For this reason the present work relies on this principle; however, considering the complicated nature of the problem, elements of composition have been used here and there.

We shall begin with a description of the contents of a unified hypothetical initial model, and then its mathematical description will be dealt with. Next, decomposition of the initial model and the construction of a system of models on its basis will be discussed. Before doing it, however, some more introductory remarks ought to be made.

3. In compiling mathematical models for the normative forecasting of a region's socio-economic development it is important to define the set of socio-economic activities and the set of the respective results.

As to socio-economic activities, there is no controversy of principle in literature. In general, they refer to both public and private activites (a more detailed discussion of these terms will be given at the beginning of § 2).

The problem of the indicators of the results arising as a consequence of the respective socio-economic activities, and especially of the development indicators, is more complicated. Various authors differ especially in classifying development indicators [see 6—14 and 16—22]. Indeed, the number of alternatives is big here; for example, these indicators can be chosen among private activities or their final results, or among public activities or their final results, etc. Without any discussion of the problem, an example of a tree (hierarchy) of development indicators is presented at the end of § 2, and its compilation principles that are important in the present paper are elucidated.

4. It is of interest to note that the necessity of a systems approach is intuitively not so clear at forecasting as at planning. It is clear that the planning of education must be linked to the planning of production. However, it does not seem wrong to forecast the development of education apart from that of production, and vice versa. And still it is possible to prove that if we know and consider the connections between the elements of the system, it will become possible to increase our knowledge about the system [23].

Thus, at forecasting a system's development, a consideration of inter-elementary connections is expedient, contributing to an improvement of our knowledge about the system.

5. In mathematical economic planning, the problem of uncertainty[2] is already known on the level of parameters or data, while on the level of theories or models uncertainty is not essential. However, in forecast models uncertainty is essential also on the level of theories or models. Here it is usually not evident what kind of connections prevail among phenomena, and, besides, there is uncertainty on the level of parameters. In socio-economic forecasting uncertainty is essential on the level of

[1] Forecasts are classified into descriptive and normative (prescriptive). The former give a passive description of the processes, giving no account to possibilities of control. The latter consider control and present also its forecast-plans. The forecasts of the plans being sought for are either optimal or effective. In case of optimality, there exists a general goal (actually); in case of effectivity, there exist several goals, and the general one, formed on their basis, serves only as a formal means of calculation.

[2] By uncertainty we refer here to indeterminateness, fuzziness, randomness, and the like, and it can be measured either by entropy or dispersion.

theories in the sense that it is not clear which indicators of socio-economic development should be used. It seems impossible to give generally acceptable solutions, and experts' «taste» determines which solutions are selected.

Below an attempt is made to develop such a forecasting principle which allows to consider, besides uncertainty on the data level, also uncertainty on the theory level. The principle is as follows. It is assumed that a subjective confidence coefficient can be ascribed to every alternative theory. The final forecast is found as a compromise solution of the alternative theories. To form the compromise solution, the confidence coefficients are used as weights.

6. In the following models, besides objective data, subjective data are likewise used. The values of the objective data can be found in the course of measuring socio-economic phenomena. To form composite indicators on the basis of measurable objective data, we use subjective data called here weighting or importance factors.

At that, weighting factors can be interpreted as formal or essential ones. In the former case an effective plan is being sought for (a plan in case of which it is impossible to improve the value of any development indicator without reducing the value of some other one), and the weighting factors serve here as computation means only. For instance, in this case the computation may be started with arbitrary values of weighting factors, and they may be corrected in the course of the computation to find the necessary solution.

If, however, the values of weighting factors are considered essential, i. e., if they are socio-economic value judgments, the forecast plan will be optimal. The values of the factors may be adapted on the basis of the preliminary computation results also in this case [24].

2. On the contents of the initial model and the choice of indicators

1. Our initial model is to be an applied (a quantitative) one, and therefore it may comprise, as its initial data, only such objective indicators whose values are measurable. However, for the sake of clarity let us first describe a somewhat wider or theoretical (qualitative) model, and derive the applied one from it.

2. To describe the theoretical model, the notions of socio-economic activity, socio-economic result and the general goal of socio-economic development (welfare) are used. At that the respective phenomena have the following causal connections: activities yield results, and the latter, in turn, determine the level at which the goal is to be attained.

Socio-economic activities can be divided into two large subgroups: public and private. Public activities, in turn, are divided into activities of material and non-material production. As examples of material production, we could mention manufacturing activities, agricultural activities, etc. Activities of non-material production are, for example, the system of education (primary schools, secondary schools, etc., as activities) and health care (hospitals, policlinics, etc., as activities), etc. Let us add that in case of every activity we distinguish three aspects: essential (value), technological and territorial. Thus, medical treatment (essential aspect) may take place either at hospitals or policlinics (technological) situated in different localities (territorial).

Private activities are, for example, the participation of the members of some population group in public activities as well as recreation, studies, family life and bringing up of children, entertainment, sports, etc. In case of these activities we likewise distinguish three aspects.

The results of public activities are divided into outputs and inputs of activities. The former comprise produced goods and energy, services delivered, available information, environmental pollution or its abatement, etc. The inputs comprise the necessary labour, the need for natural resources, expenditure of exogenous goods and services, need of means and funds, etc.

The results of private activities are divided in the same way. Inputs comprise here consumption of goods and services, utilization of natural resources, etc. The outputs are divided into two groups. Firstly, there are outputs into public activities, i. e., participation of individuals in the work of public activities (provision of labour force). Secondly, there are outputs for the individual himself, or private final results. They are, for example, the physical well-being, psychic well-being, public appreciation, feeling of security, recreation, etc.

All activities (public and private) must at any moment satisfy certain balance conditions given for the final results of all activities and activity groups.[3] Namely, those are the final results of the public activities that constitute the conditions under which private activities are performed. Private activities, in turn, provide labour for the functioning of public activities. Besides, the final results of all activities, both public and private, must satisfy certain conditions regarding the use of natural resources, imports and exports, population migration, etc. Finally, the volumes of activities may be directly constrained by such factors that are not discussed in the present model (ideological and moral, political, etc.).

Within the described bounds, the plans of activities are choosable. In order to forecast the best activity plan in such a theoretical model, it seems to be advisable to proceed from the general development goal whose arguments are private final results. The obtained forecast would then comprise the best distribution of the socio-economic goods produced in the public sphere among individuals or their groups, and the best forecast of private activities (mode of life).

3. In an applied quantitative model, however, the idea of describing the final results of private activities must be given up, since they are not measurable (we cannot measure peace of mind, public appreciation, etc.). Likewise, we cannot make a detailed description of private activities, as there are no data on the relations between private activities and their results (e. g., there are no data on the structures of private consumption at work, studies, sports, entertainments, etc.). These data exist only on an extremely aggregated level, such as the so-called expenditures or structures of the private consumption of social groups (on the basis of budget observations). In our terms these expenditures represent the expenditures of aggregated private activities. Thus, aggregated private activities may be included in an applied model as its ingredients and various public activities are then added. As to the latter, statistics on their volumes and the respective results are in general available.

Thus, the described applied model leaves untackled the problems concerning the structure of private activities (mode of life). Consequently, they must find a parallel and less quantified treatment. The applied model does not deal with questions concerning the distribution of

[3] Final results of an activity group are the inputs and outputs of this group, while intra-group or intermediate results do not belong among them.

socio-economic goods among individuals or their groups either. This question must likewise be treated separately.

To sum up, the applied model is as follows. By activities we refer here to public activities, with the only exception of one aggregate private activity called private consumption. Various kinds of socio-economic goods produced in the public sphere constitute the inputs of this activity. All the activities mentioned have results or input-output flows of socio-economic goods, dependent upon the prognosticated intensities (volumes per time unit) of those activities. The flows of all prognosticated socio-economic goods must meet balance conditions. In such a model it is expedient to forecast, among the plans that meet the constraints, the best ones with regard to the most important final results of public activities and the volume of private consumption. Such final result indicators that are important in making this choice are called goal results or development indicators, while all the others are indicators of balanced results.

4. The choice of concrete classifications of the indicators (their number and contents) for an applied model of a region's socio-economic forecast is, at the present time, one of the questions whose solution lacks a generally approved methodological basis [7—12, 17—22]. Thus, the choice of the classifications depends on the forecaster's taste, and generally acceptable solutions cannot be expected for the time being. Apparently, a concrete classification is to be worked out by experts collectively.

Let us mention some principles that should be considered in discussions of experts. The most responsible task is here the compilation of the classification of the socio-economic goal or development indicators. It is important to bear in mind that activity indicators should not be chosen as development indicators (without a special need). Furthermore, in its most detailed form the classification may comprise only measurable indicators (as initial indicators), or still better, indicators on whose values statistics are already available.

A model of practical value comprises a great number of initial development indicators, and a comparison of their importance or harmony assessment would be beyond the powers of the experts. Initial indicators should be joined to form composite indicators, and to this end subjective weighting factors are used. The obtained composite or derivative indicators are called indicators of a higher level. In their turn, they are aggregated. In such a way a tree or hierarchy of indicators is formed; at the top of it is one indicator called the indicator of the general development goal, or, in other terms, the indicator of the quality of life.

Now, the lowest level of the hierarchy is occupied by measurable indicators, while all the other levels contain derivative indicators which include also subjective data. In combining indicators of lower levels we have to see to it that the indicators belonging to one aggregate should be possibly close as to their contents and in harmony with the denomination of the respective composite indicator (to be more exact, with the essence of the phenomena corresponding to this denomination).

Table 1 presents an example of the fragments of a tree of applied indicators of a region's socio-economic development. Indicators of the final results of public activities and private consumption are used; the construction of the tree begins from its trunk (the top of the hierarchy), and then Level 1 and Level 2 are formed. A further description of the tree would have taken too much room, and it has not been tackled here.

Table 1

An example of the fragments of a tree of a region's socio-economic development indicators

Level 0	Level 1	Level 2	Level 3	. . .
		1.1.1. Efficiency of production	1.1.1.1 Labour productivity	. . .
		1.1.2. Dynamics of production	1.1.1.2. Output-asset ratio	
		1.1.3. The balance rate of production	. . . .	
	1.1. Economic development	1.1.4. Production assets		
		1.1.5. Infrastructure		
		1.1.6. Foreign trade		
		1.1.7. Supplies		
		1.1.8. Losses		
		. . . .		
		1.2.1. Private consumption	1.2.1.1. Short-term private consumption	. . .
		1.2.2. Public consumption	1.2.1.2. Long-term private consumption	
		1.2.3. Services	. . . .	
		1.2.4. Housing		
	1.2. Development of everyday life	1.2.5. Water supply and sewage system		
		1.2.6. Recreation possibilities		
		1.2.7. Municipal transportation		
		1.2.8. Car		
		. . . .		
		1.3.1. Population of the first level of education	. . .	. . .
	1.3. Development of education	1.3.2. Population of the second level of education		
		. . . .		
Indicator of the general goal of socio-economic development		1.4.1. Population of the first level of health	. . .	. . .
	1.4. Development of health	1.4.2. Population of the second level of health		
		. . . .		

Level 0	Level 1	Level 2	Level 3	...
		1.5.1. Science	...	...
		1.5.2. Literature		
		1.5.3. Fine arts		
	1.5. Cultural development	1.5.4. Theatre		
		1.5.5. Cinema		
		1.5.6. Physical culture		
		1.5.7. Tourism		
		....		
		1.6.1. Quality of natural	1.6.1.1. Forests	...
	1.6. Quality of the natural environment	1.6.2. environments and	1.6.1.2. Water	
		1.6.3. availability	1.6.1.3. Wild animals	
		... of natural resources	....	
		1.7.1. Working conditions	1.7.1.1. Jobs per able-bodied population	...
		1.7.2. Social insurance	1.7.1.2. Number of accidents at work	
		1.7.3. Kindergartens and creches	....	
	1.7. Socio-political development	1.7.4. Care for aged people		
		1.7.5. Political participation		
		1.7.6. Security		
		....		
		1.8.1. Population number	...	...
		1.8.2. Birth rate		
		1.8.3. Death rate		
	1.8. Demographic development	1.8.4. Migration		
		1.8.5. Age structure		
		1.8.6. Ethnic structure		
		....		
	...	...	...	...

Let us stress once more that the initial indicators at the lowest level must be measurable objective indicators. The values of the indicators (showings) may be given either as absolute numbers or as relative ones. The latter may be expressed either with regard to the initial state of the base or some desired minimum or maximum level.

Besides the classification discussed above, a classification of such results that are not concerned with the goals but are only balancing, and a classification of public activities must be constructed for the model.

Indicators that are only balancing comprise first and foremost various kinds of economic goods, such as kinds of products, kinds of services, fixed assets, current assets, capital investments, etc. At the same time, any development indicator can likewise be regarded balancing.

A classification of the public activities of material production comprises various industries (machine-building, foodstuffs, etc.), branches of agriculture, construction, transport, etc. Public activities of non-material production may be activities of health care, education activities, etc. A concrete compilation of these classifications should be carried out by the forecasters collectively, taking into account the number of forecasters, the time, statistics, etc., available for forecasting.

3. A schematic mathematical description of the initial model

1. First a schematic description of the theoretical model is given, and then, by means of simplifications, an applied model is derived. For the sake of clarity, only static and deterministic descriptions are discussed at first, and afterwards remarks on dynamic and stochastic models are made. Specific mathematical questions concerning the modelling (the modelling of collective goods and indices in a linear model, etc.) are treated at the end of the paragraph.

2. Let the set of public activities be denoted by $N=\{j|j=1,\ldots,n\}$, where n is the number of public activities and j the general index of a public activity (as already said, three aspects are described by an index j). The intensity of the public activity j is denoted by x_j. Thus, the whole forecast of these activities is a vector $x=(x_j)$, $j\in N$. Direct constraints on x are assumed: $x\in X$.

Let the set of results corresponding to the public activities be $M=\{i|i=1,\ldots,m\}$. Let the relation of an activity's intensity x_j and the result of the activity j be described by a result function $g_j:E^{n_j}\rightarrow E_m$, where n_j is the number of co-ordinates of the vector x_j. The result $z_j=(z_{ij})$, $i\in M$ of the activity j is determined by the activity's intensity $x_j: z_j=g_j(x_j)=(g_{ij}(x_j))$, $i\in M$.

It is assumed that the final result of the whole public activity $z=(z_i)$, $i\in M$ can be expressed additively by the results of the activities $j: z=g(x)=\sum g_j(x_j)$. A limit $b=(b_i)$, $i\in M: z\geqslant b$ is assumed to be imposed on the final result z. Thus, a schematic description of the sphere of public activities has been given.

Let the set of private activities be $R=\{l|l=1,\ldots,r\}$ and the respective intensity of an activity be described by $y=(y_l)$, $l\in R$. And let it be directly constrained: $y\in Y$. Let the set of the results of the public activities M and the set of the private final results $Z=\{k|k=1,\ldots,z\}$ be comprised in the set of the results of the private activities. Thus, the whole set of the results of private activities is $W=M\cup Z=\{r|r=1,\ldots,w\}$.

Let the intensity y_l of a private activity l be connected with a result function $d_l{:}E^{n_l}\rightarrow E_w$, where, n_l is the number of the co-ordinates of the vector y_l. The vector function d_l is regarded as consisting of two vector functions: $d_l=(f_l,\ h_l)$, where $f_l\in E^m$ and $h_l\in E^z$. Consequently, f_l represents the results on the set of indicators M, h_l, on the set Z. Let the final result of the private activities be again additive.

3. For a schematic description of a theoretical model, private final results are assumed to serve as arguments of the general goal of socio-economic development. This is expressed by the function $u=u(h)$, where $h=(h_l)$, $l\in R$.

Now the forecast of the best socio-economic development can theoretically be found from the model

$$\max u(h)\Big\}\sum g_j(x_j)+\sum f_l(y_l)=0,\quad h=\sum h_l(y_l),\quad \sum g_j(x_j)=z\geqslant b,\quad x\in X,\ y\in Y. \tag{1}$$

The forecast found provides the values of the following indicators: $h=(h_l)$, $f=(f_l)$, $l\in R$; $x=(x_j)$ and $g_j(x_j)$, $j\in N$, $z=(z_i)$, $i\in M$ and u.

4. For a schematic description of an applied model, aggregated private consumption is assumed to belong to the set of public activities as one of the activities. Next we assume that the set of the results of the public activities includes an indicator of private consumption. The result function of private consumption as an activity is assumed to have the same properties as the ordinary public activities.

5. Let the objective function of the applied model be represented by a sum weighted by the weighting factors of the values of the final results of the public activities $u=\sum u_i z_i$, where u_i is the weighting factor of an indicator i, $u_i\geqslant 0$, $\sum u_i=1$.

In describing an applied model we take account of essential uncertainty on the level of theories, i. e., there exist a number of alternative theories, and none of them is noticeably more trustworthy than the others. For a simplified consideration of this fact, the set of acceptable theories is assumed to be $Q=\{p\,|\,p=1,\ldots,q\}$. Differences of principle among these theories are assumed to exist in the choice of the set of results M_p and in the estimates of the values of the respective weighting factors u_i^p. In fixing the set of public activities, however, there are no essential differences among the theories.

Next we assume that a certain subjective confidence factor s_p, $p\in Q$, is attributable to every theory, serving as a means of comparing the values of the objective functions of different theories. The forecast must meet certain restrictive conditions for all theories (in a stochastic treatment this requirement may be worded more flexibly). Now we can write the applied model that takes account of q theories in the following form:

$$\max \sum_p s_p \sum_i u_i^p z_i^p \Big\} \sum_j g_j^p(x_j)=z^p\geqslant b^p,\quad x\in X^p,\ p\in Q \tag{2}$$

where the upper index p stands for a theory.

This model yields one activity forecast x, but there are q sets of respective results and goals.

6. A mathematical description of socio-economic problems gives rise to a great number of questions of mathematical modelling. They concern,

above all, the considerable uncertainty of data, the modelling of collective goods as well as the description of relative numbers in the model. As it is generally limited to linear approximations in applied problems, we shall confine ourselves to a linear treatment, too.

Consideration of uncertainty on the theory level was discussed in the previous sections. Here we discuss only the so-called questions of stochastic planning connected with uncertainty about data. We can say that, in general, the data of socio-economic problems are approximate (intervals). This, as well as the fact that the forecasts of plan indicators should also be approximate must be taken into account in formulating the problem. A solution to these questions is suggested in the theory of stochastic planning, and here that problem is not dealt with.

As to the modelling of collective goods [25—27], the following questions crop up. Collective goods have the property that they can be used by everyone, or, to put it differently, they form the conditions under which the activities are realized. Because of this property there arise difficulties in keeping the models additive. For example, the input of a resource i in an activity j depends on the production x_k of the collective goods k. Thus, the input function of the activity j is $g_{ij}(x_j, x_k)$, which does not involve additivity. In order not to lose additivity, we use, in the modelling of collective goods, such linear approximations where the input of a resource i in the branch j, x_{ij}, depends on a linear combination of the intensities of activity in the branches j and k, thus $x_{ij}=a_{ij}x_j+$ $+a^k_{ij}x_k$.

In case of socio-economic problems it is often necessary to model various relative numbers (efficiency indicators, growth rates, etc.); this is not customary in linear planning. However, this question is easy to solve by making use of additional (fictitious) activities. For example, let the production volumes to be determined by x_1 and x_2, and the respective production of the reference year $x_0=x^0_1+x^0_2$. Denote the growth rate by x_3. Now the latter can be found from the equality $x_1+x_2-x_0x_3=0$. As we can see, x_1, x_2 and x_3 can be found all together. Thus, using the respective fictitious activities, it is possible to describe indicators represented by relative numbers.

In concluding this paragraph, let us note that in order to change the initial problem into a dynamic one in case of a linear planning problem, only one supplementary index (time) must be added, and no formalization difficulties ensue. Moreover, in an applied problem it is often expedient to be confined to a pseudodynamic treatment, i. e., to such a treatment where the cumulative indicators of the previous year (e. g., funds) are connected with the results of the same year, only (e. g., the investments).

4. A decomposition of the initial model into a system of submodels

1. We start our decomposition of the initial model according to theories. We form a forecasting submodel on the basis of every theory, and, in addition, a general co-ordinating model. All the forecasting models of the obtained system of models are engaged in assessing the value of one and the same vector x_0. Thus we have a conjunctive system [15].

A conjunctive system has two co-ordination possibilities [15]: the method of activity penalties and that of activity limits. In case of the former, non-linear penalty functions must in general be used, and this may cause difficulties in the application of the method in practice. In case of the latter method, the co-ordination of the submodel of the theory p is carried out with the help of the so-called activity limits. In principle, this is done as follows: $\check{x}^p \leqslant x^p \leqslant \hat{x}^p$, where $\check{x}^p$ and $\hat{x}^p$ are instrumental co-ordination parameters, obtained from a co-ordinating model, and x^p is the plan forecast according to the theory p.

The task of the co-ordinating model in the iterative operation of the system of models is to effect the solutions of the forecasting submodels so that they would approximate one another as well as the solution to the initial model. As already said, in case of the method of activity limits, additional constraints on activities are used for that purpose. These constraints are corrected on the basis of the solutions to the dual problems of the submodels about the co-ordinating activity limits, which show the boundary efficiency yielded by a correction of constraints and serve as basis for deciding in which direction the co-ordinating constraints should be corrected in order to move towards the solution of the initial problem. To this end, the following heuristic principle may be applied at the correction: if the boundary efficiency is relatively high, the constraints must be loosened, and if the boundary efficiency is relatively low, the constraints must be approximated to the forecast $\bar{x} = \sum x^p/q$.

Thus, according to theories we can decompose a system where the forecasting submodel $p \in Q$ is the following:

$$\max s_p \sum_i u_i^p z_i^p \left.\right\} \sum g_j^p (x_j^p) = (z_i^p) \geqslant b^p, \quad x^p \in X^p, \qquad (3)$$

$$\check{x}^p \leqslant x^p \leqslant \hat{x}^p.$$

It is easy to see from the model that if there exists co-ordination between the theories, then the constraint $x^p \in X^p$ must not take into account constraints from other theories. If no co-ordination exists between the theories, the constraint $x^p \in X^p$ must consider the constraints of the other theories as far as possible.

2. Below we shall discuss the decomposition of a submodel (3) of one theory or decomposition within the framework of one theory (the theory index p is left out now). Here we begin with a decomposition by aspects [15]. We separate two models: 1) a branch model, and 2) a locational one. In the former we do not distinguish territorial location, and in the latter, alternative technologies. Thus, the dimensions of both models are reduced.

For the co-ordination of these models, consultation and limitation of aggregate activities [15] are heuristically applied. Consultation consists in considering the preliminary solutions of the locational model in the branch model, and vice versa. Limitation attempts at approximating the activity volumes in the branch model (over all technologies) to the respective activity volumes in the locational model (over all the studied territories).

Next we shall treat a branch model. Here it is advisable to decompose on the basis of groups of development indicators. This enables the experts to assess the values of the weighting factors belonging to the respective group to better advantage. As shown in the previous paragraph, mathematically it is suitable to describe the goal results as fictitious activities. So we assume the initial model to comprise fictitious activities, and only their intensities serve as arguments to the objective function.

Thus, a branch model is decomposed by goal results or development indicators. Ordinary activities are distributed so that a certain group of goal activities includes such ordinary activities that are more closely connected with this group of goal activities. Thus, economic activities are added to the indicators of economic development, educational activities to the indicators of the development of education, etc. The obtained submodels may be called models of sectoral socio-economic programmes, because every model includes certain related socio-economic goals and sectors directly engaged in achieving them.

For the co-ordination of the submodels, limitation of the results [15] is used. This is expedient for two reasons: firstly, an iteration of the feasible solution is guaranteed at every step, and secondly, an optimal solution can be found also in case of a linear problem. The principle of co-ordination is here as follows. The co-ordinating model gives the submodels the limits of the results, obtained by the allocation of the limits of the initial problem. The submodels find optimal solutions for the given limits and the respective dual solutions. The latter show the boundary efficiency of changing the limits, and on their basis the correction of the limits among the submodels is carried out.

Relying on the above-said, the co-ordinated forecast model of the programme c can be written in the following form:

$$\max_{x_i} u^c \sum_{i\in M_c} u^c_i x_i \Big\} \sum_{j\in N_c} g_j(x_j) + \sum_{i\in M_c} \hat{g}_i(x_i) \geqslant \hat{b}_c, \qquad x_j \in \hat{X}_j, \quad j \in N_c, \tag{4}$$

where $c\in D=\{c|c=1,\ldots,d\}$ is the index of the programme, x_i the intensity of a goal activity i, and $\hat{g}_i: E^1 \to E^m$ the respective fictitious result function. The sets M_c and N_c are obtained by the distribution of the sets N and M. The vector $\hat{b}_c$ is a co-ordinating result limit. The set $\hat{X}_j$ is an activity limit co-ordinated by means of the theories and the location model. The function g_j is a result function co-ordinated by consulting the location model. The weighting factor u_i^c expresses the importance of the initial indicator in the programme c, and u^c is the weighting factor of the composite indicator $\sum_{i\in M_c} u^c_i x_i$ from the standpoint of the higher level. Thus, comprised in the initial model $u_i = u^c \cdot u_i^c$. As already said, such a hierarchy of the weighting factors allows experts to assess their values to better advantage. Namely, the values of u_i^c, $i\in M_c$ are determined by the experts of the programme c alone. At that it is advisable that $u_i^c \geqslant 0$ and $\sum u_i = 1$, as this makes it easier for the higher level to determine u^c.

As to the co-ordination of the models of the programmes $c\in D$, the following ought to be said. We distinguish between two kinds of co-ordinatable results: results concerning many programmes or general ones, and results concerning few ($\leqslant$3—4) programmes or individual ones. In case of the former, it is advisable to organize co-ordination with the help of the respective centres (models), and thus vertical co-ordination is formed. In case of the latter results, co-ordination may be carried out directly between the models or horizontally.

Analogously to model (4), co-ordinated models of territories (towns, districts, etc.) can be written and their co-ordination system formed on the basis of the location model.

5. A schematic description of an example of a sectoral system of models

A schematic description of an example of a sectoral system of models is presented in Table 2. This comprises (in Column 1) 14 forecasting submodels and 1 co-ordinating model. The forecasting models are built so that they include related indicators of socio-economic development, and therefore they may be called also forecast models of socio-economic programmes.

The characterization of the activities used in the models can be found in Column 2, and that of the results in Columns 3 and 4. Column 5 characterizes the constraints imposed both on the results and the activities. The following columns describe the contents of the co-ordination data used. First, the data of vertical co-ordination are presented. Column 6 gives the data by which the co-ordinating model effects forecast models, and Column 7 presents the data that are directly exchanged among the forecast models.

Analogously to Table 2, a system of territorial models comprising models of towns and districts, can be formed.

The whole system works as follows. Firstly, the contents of the models are described, and their mathematical description is presented. Then the values of the initial data of the models are prognosticated. In case of objective data, descriptive forecasts (extrapolation forecasts and expert estimates) are used for this purpose. Estimates of subjective data are provided by experts [28—32]. At that there are two ways for employing experts. Firstly, experts form the estimates in the course of calculations (an adaptive method). Secondly, before the calculations are started, the so-called empirical experiment [33] may be arranged with the experts, and on its basis the values of the subjective data are assessed.

6. Conclusion

1. The treatment of a problem of a systems approach to normative forecasting of a region's socio-economic development by models of mathematical planning, especially models of stochastic planning is expedient. On the theoretical level it enables to more accurately clarify balance connections of the results of public and private activities and to better systematize the treatments, elucidate the role of objective and subjective data, etc.

On the applied (quantitative) level, the mathematical approach makes possible a systems (balanced) analysis of a considerable part of the problem. Because of the lack of objective measurable data, problems concerning, first of all, the allocation of socio-economic goods among population groups as well as the structure of private activities are not treated on the quantitative level. For the time being these problems must be studied with the help of less systematic and less quantified methods.

2. The dimensions of an applied (initial) model of a forecast problem of a region's socio-economic development having practical value are big, and it is expedient to solve the problem with the help of a system of

Table 2

A schematic description of an example of a sectoral system of the models (models of connected programmes) of a region's socio-economic normative forecasting

N	Name of Model	Activities	Results		Constraints	Co-ordination Data	
			Goal	Balancing		Vertical	Horizontal
0	1	2	3	4	5	6	7
1	Production	Branches of industry, agriculture, etc.	Efficiency, dynamics, etc.	Economic resources and environmental quality	On economic resources, some activities and environmental pollution	Limits of the final production of economic resources	Structure of specialists, natural resources, environmental conditions, etc.
2	Foreign trade	Branches of trade	„	„	On imports and exports	Limits of imports and exports	„
3	Consumption	Consumption by the population at home, public establishments, current and long-term	Consumption volume and quality	„	On kinds of products, volumes of services and also some activities	Limits of kinds of products and services	Types of housing units and passenger transportation
4	Housing	Types of dwelling houses and summer cottages and their maintenance	Living space and quality of housing	„	On economic resources, environmental conditions and also some types of dwellings	Limits of economic resources	Passenger transportation
5	Passenger transportation	Kinds of public and private transportation	Quality of transportation	„	Necessary volumes of transportation, economic resources, environmental conditions	„	Types of housing units and distribution of production
6	Education	Educational activities	People's educational level	„	On economic resources and some activities	„	Age and ethnic structure of the population, and need for specialists

0	1	2	3	4	5	6	7
7	Health care	Health care activities	Level of the population's health	Economic resources and environmental quality	On economic resources and some activities	Limits of economic resources	Age structure of the population
8	Culture	Cultural activities (science, fine arts, theatre, etc.)	Cultural level	"	"	"	"
9	Natural environment	Activities of environmental protection and recovery of natural resources	Quality of the natural environment	"	"	"	Environmental pollution by production and consumption
10	Social and political activity	Administration and activity of socio-political organizations	Quality of socio-political activity	"	"	"	Population structure by age, education and professions
11	Social security	Activities of the militia, army, fire guards, and the like	Level of security	"	"	"	"
12	Social insurance and maintenance	Activities of social insurance and maintenance	Level of social insurance	"	"	"	" and people's health condition
13	Population	Birth rate, death rate, migration, formation of families, etc.	Demographic development	Population number and structures (age, sex, and ethnic)	"	"	Social insurance, health condition, education, consumption, housing, etc.
14	Other socio-economic goals and activities	...	...	...	...	...	...
15	Co-ordination models	Allocation of economic resources	Socio-economic development	The rate of balance of economic resources	Limits of economic resources for the whole system	Marginal efficiencies of the limits of economic resources from the models	—

submodels. For deriving the submodels and their interconnections that are to be co-ordinated it is suitable to use principles of decomposition analysis.

It has become clear that the initial model might be decomposed according to different theories. Within the framework of one theory it is suitable to decompose first a sectoral model and a location model. It is expedient to decompose a sectoral model by socio-economic programmes.

For a co-ordination of the models of separate theories, limits of activities are suitable, while for a mutual co-ordination of a sectoral and a location model, consultation and activity limits (in a combined way) might be used. For a co-ordination of the models of programmes and regions it is appropriate to apply result limits. In so doing it is rational to organize a combined vertical (by means of a co-ordination model) and horizontal co-ordination. The former fits for resources of general use, the latter, for the results connecting a limited number of models.

3. Normative forecasting models must make extensive use of subjective information. This is caused by the fact that plan indicators are likewise forecast here. In the presented methods subjective information should be given in the shape of the so-called weighting factors, meant for forming composite indicators from simple ones. These factors can be interpreted as computational aids for forecasting an effective plan, or as essential indicators (socio-economic value estimates) for forecasting an optimal plan. In both cases, the values of the factors can be determined adaptively in the course of calculations.

To find compromise solutions of different theories, it is advisable to use subjective confidence coefficients.

4. A system of concrete models (programmes), concrete classifications of the indicators of the goals and the activities of socio-economic development must be worked out by the respective collective bodies of forecasters. Table 1 and 2 present an example of these classifications.

An application of a system of forecast models requires well coordinated long-term efforts of a large body of researchers.

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Ü. ENNUSTE

REGIOONI SOTSIAAL-MAJANDUSLIKU ARENGU NORMATIIVSE PROGNOOSIMISE DEKOMPOSITSIOONILISEST MUDELISÜSTEEMIST

Resümee

Rajooni sotsiaal-majandusliku arengu normatiivse prognoosimise ülesandeid on otstarbekas uurida optimaalse planeerimise mudelisüsteemide abil. Teoreetilisel tasandil võimaldab see süsteemsemalt ja rangemalt selgitada ühiskondlike ja isiklike tegevuste seoseid ning tulemusi. Rakenduslikul tasandil saab sellise käsitluse abil koostada tasakaalustatud prognoose suhteliselt detailselt.

Artiklis on esitatud regiooni sotsiaal-majandusliku normatiivse dekompositsioonilise mudelisüsteemi põhimõtteskeem, võtteid mudelisüsteemi moodustamiseks nn. tervikliku lähtemudeli alusel ning üksikmudelite töö koordineerimise mooduseid. Mudelisüsteemi põhimõtteskeemile on lisatud ka ühe regiooni sotsiaal-majandusliku arengu näitajate ja mudelisüsteemi klassifikatsioon.

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Ю. ЭННУСТЕ

О ДЕКОМПОЗИЦИОННОЙ СИСТЕМЕ МОДЕЛЕЙ НОРМАТИВНОГО ПРОГНОЗИРОВАНИЯ СОЦИАЛЬНО-ЭКОНОМИЧЕСКОГО РАЗВИТИЯ РЕГИОНА

Резюме

В статье утверждается, что проблемы прогнозирования социально-экономического развития региона целесообразно исследовать при помощи систем моделей оптимального программирования. На теоретическом уровне такой подход позволяет более системно и строго исследовать связи общественных и индивидуальных деятельностей и их результатов. На прикладном уровне эта трактовка дает возможность составлять более детальные сбалансированные прогнозы.

В статье представлена принципиальная схема декомпозиционной системы моделей нормативного социально-экономического прогнозирования региона. В ней показаны основные принципы выведения частных моделей из т. н. исходной целостной модели. Представлены также принципы координирования работы частных моделей. К принципиальной схеме системы моделей приложена классификация показателей социально-экономического развития региона и системы частных моделей.

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