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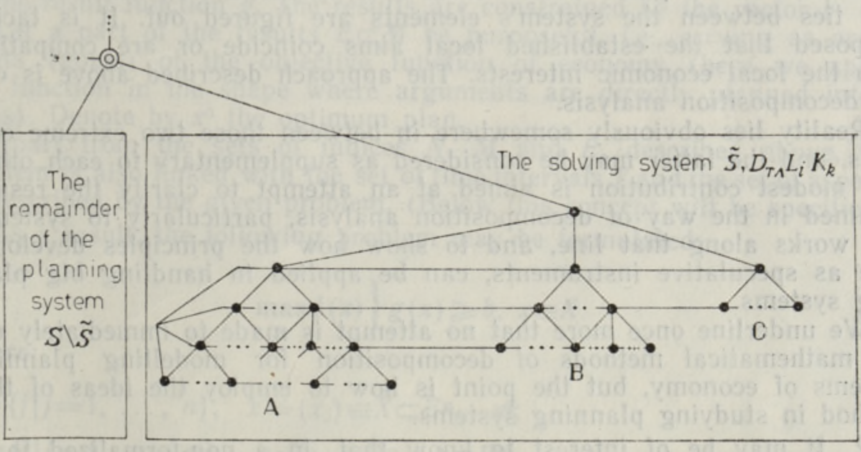
Ü. ENNUSTE

PRINCIPLES OF DECOMPOSITION OF OPTIMUM PLANNING

1. At the Twenty-Fifth Congress of the CPSU, the importance of adaption of economic planning and management systems was pointed out, particularly in connection with new possibilities of computing techniques and mathematical economics. The said systems belong to a class of the so-called big management systems. Their pattern can be best represented by a graph (See Fig.), where the bones denote information flows and the knots — elements of the system, for instance, planning bodies, computers or models. So in the graph in our Figure we can distinguish the subsystems of planning and forecast periods with various horizons, of different levels and pattern of ties.



Graph of the planning systems (S), solving system ($\tilde{S}$), an example. Bones denote information flows and knots denote elements (planning bodies, computers, models, etc.). A — the solving system of the tactical planning S_t, $D_N \vee D_N \vee D_N \vee S_{ih} L_i$; B — the solving system of the strategic planning S_s, $D_{\varepsilon} \wedge D_N \vee S_k L_k L_i$; C — the solving system of the forecasting S_f, $D_A \wedge L_k S_k$.

What, then, are the modern concrete theoretical conceptions for analysis and synthesis of such systems, for their design and adequacy assessment? Unfortunately, one has to agree with the main assertion of the superb book "Anti-Equilibrium" by J. Kornai that solid, constructive, systematic theoretical fundamentals are lacking, especially in the field of formal logical theories. There are sufficient theoretical foundations to work out aggregate plans for national economy, for branches of economy, regions and enterprises (production associations), but the question of a co-ordination of those plans remains open. Or another example. There exist foundations for drawing up long- and short-term plans, but the question of how to couple them is obscure.

All these theoretical bottle-necks did not show up sharply when planning was based upon experience and intuition. However, automation and introduction of mathematics call for stricter precepts which cannot be replaced by inductive approach.

It will suffice to state that investigation into the theoretical fundamentals of big systems effecting the planning of Socialist economy is undoubtfully topical, in particular along the lines of formal logic. It should be added, of course, that the actual processes of planning are highly complicated. Their formalized description requires simplifications whose influence makes itself felt on the results. Therefore, these processes should be interpreted very carefully. However, great variety in the treatment is justified here by the object's complexity, i. e., both formalized and non-formalized lines of investigation deserve attention.

In Soviet mathematical economics, big planning systems are studied along two lines. The game theoretical approach assumes the system's elements to be preassigned. The question is what general planning problem is solved by such a system. It should be noted that a wide involvement of local interests lends that approach a certain vitality.

The second line proceeds from the Marxist-Leninist thesis that a Socialist state plans national economy in its entity toward global goals. Here a respective integral hypothetical problem of optimum planning is posed, and the game which solves it is specified, i. e., the local aims and ties between the system's elements are figured out. It is tacitly supposed that the established local aims coincide or are compatible with the local economic interests. The approach described above is called decomposition analysis.

Reality lies obviously somewhere in between those two extreme concepts, and the latter may be considered as supplementary to each other. Our modest contribution is aimed at an attempt to clarify the results obtained in the way of decomposition analysis, particularly to systematize works along that line, and to show how the principles developed here as speculative instruments, can be applied in handling big planning systems.

We underline once more that no attempt is made to immediately use the mathematical methods of decomposition for modelling planning systems of economy, but the point is how to employ the ideas of that method in studying planning systems.

2. It may be of interest to know that, in a non-formalized form, already Aristotle meditated upon the problem of management decomposition, or, more precisely, on how to deploy troops for their best command. In our Socialist country the questions of decomposition of politico-economical management were thoroughly considered by V. I. Lenin. He formulated the principle of democratic centralism, which also is the foundation to the decomposition analysis of the optimum planning of Socialist economy.

The first major breakthrough in solving the mathematical problem of optimum planning was achieved by G. Danzig and P. Wolfe in 1960. Another important work was published two years later by J. Kornai and T. Liptak. Below we shall discuss their and other authors' results in greater detail.

For economic interpretations of decomposition methods the most renowned authors are O. Lange, K. Arrow, L. Hurwicz, V. Kantorovich,

V. Novozhilov, V. Nemchinov, J. Kornai, A. Aganbegyan, K. Bagrinovsky, N. Fedorenko, B. Volkonsky, and others.

In summary, we can say that now several methods for the decomposition of optimum planning, and their interpretations, have been devised. However, there are no general system principles allowing to encompass all the literature, to systematize the problems solved and to show the open ones. Such principles could help us to attain a better understanding of the economico-mathematical fundamentals of the problems related to planning and perfecting those systems. As noted previously, in this paper an attempt is made to describe one of the possible system approaches to decomposition analysis and to draw respective conclusions. We shall start with a description of the theoretical problem of optimum integral planning of economy. Then we shall discuss its solving by means of systems consisting of simpler subproblems, as well as their construction principles and properties. Attention will be called to the need of making allowances for stochastics. Finally, we shall consider the problems of synthesis or composition of big planning systems.

3. We proceed from the hypothetical problem of optimum planning of Socialist economy on the whole, assuming that the number of the activities planned is n, their set being denoted by N. Suppose that the planned intensity or planned index of activity j is x_j. Thus, the full plan is expressed by $x=(x_j)$, $j\in N$. On the plan x, direct constraints are imposed by the set $x\in X$.

Further let us suppose that the plan is linked with the set of results M, and the respective relation between plan and results is described by the result function g. The results are constrained by the vector b.

Let a part of the results $E\subset M$ be purposeful, i.e., serving as arguments (goals) of the objective function of economy (here we apply this function in the shape where arguments are directly planned intensities). Denote by x^0 the optimum plan.

Apart from the sets of indices N, M and E, described above, the problem is also linked with the set of time intervals T and the set of aspects A, described by the given problem. (Below this concept will be specified).

As a result, the following problem may be formulated:

$$\max_x f(x) \Big\} g(x) \geqslant b,\ x\in X. \tag{1}$$

Where:

$N=\{j\,|\,j=1, \ldots, n\}$, $x=(x_j)\in X\subset E^n$, x^0;

$M=\{i\,|\,i=1, \ldots, m\}$, $g:E^n \to E^m$, $b\in E^m$;

$E=\{c\,|\,c=1, \ldots, e\}\subset M$, $F:E^e \to E^1$, $F\equiv f:E^n \to E^1$;

$T=\{t\,|\,t=1, \ldots, v\}$;

$A=\{a_1, \ldots, a_s\}$.

4. The iterative system of subproblems constructed for solving the initial problem will be described in this way:

$$\begin{cases} p_r(a^k_r,\ u^k_r,\ v^k_r), \\ u^{k+1}_r = C^k_r\,(v^k_0, \ldots, v^k_z), \\ r\in R \end{cases} \tag{2}$$

where the subproblem is signified by $p_r(\ldots)$, $R=\{r|r=1, \ldots, z\}$; the iteration by $k=1, 2, \ldots$; a_r — part of parameters a of problem (1) which are contained in the subproblem r. They are determined by the decomposition principle laid down by matrix $D^k=(D^k_r)$. Thus, $a^k_r = D^k_r a$; u_r — co-ordination parameters determined by the co-ordination principle and here described with the help of operator $C^k=(C^k_r)$; $v^k_r \in E^{z_r}$, $N_r=\{j_r|j_r=1_r, \ldots, z_r\}$, represent the solution of subproblem r at the step k. The full solution of the whole system is $v^k=(v^k_r)$, a certain composition $G(v^k)$ of which gives the assessment x^k of the initial problem's solution. If x^k converges upon the optimum solution x^0 of the initial problem, then the system of subproblems is equivalent to it. v^k_0 is a co-ordination parameter determined exogenously.

In the system of subproblems, distinction can be made between the planning and co-ordinating subproblems. Solutions of planning problems contain components from the initial problem's plan, but co-ordinating problems do not involve them. Thus, the set of planning subproblems may be presented in the form of $P=\{r|r\in R, N_r \cap N \neq \varnothing\}$, and that of co-ordinating subproblems in the shape of $O=R \setminus P$.

Moreover, we call the system disjunctive, $\vee$, if every plan index of the initial problem is contained in only one subproblem, $N_r \cap N_q = \varnothing$, $r, q \in P$, $r \neq q$, and conjunctive or overlapping, $\wedge$, if one plan index is present in several subproblems, $N_r \cap N_q \neq \varnothing$, $r, q \in P$, $r \neq q$.

5. It is apparent that for constructing the described system of subproblems, both decomposition and co-ordination principles are needed.

The principles of decomposition produce those indices of the initial problem, upon whose basis the latter is dismembered. Setting up the initial problem, we described five sets of indices, and on the base of these we define five principles of decomposition: 1) by time or temporal, D_T, 2) by activities and units, D_N, 3) by goals, D_E, 4) by results or constraints, D_M, 5) by aspects, D_A.

Applying decomposition by time, D_T, planning subproblems consider various intervals of the planning period of the initial problem, or the subproblems have various time horizons within the planning period.

In the case of decomposition by activities and units (unit is a group of activities), D_N, planning subproblems contain activities or groups of activities of the initial problem.

As concerns decomposition by goals, D_E, the arguments of the initial problem's objective function are dismembered into objective functions of subproblems. Arguments of the initial problem may be both results and intensities of the activities.

If decomposing is done by results, D_M, the constraints for the initial problem are distributed between planning subproblems.

The principle of decomposition by aspects, D_A, in turn, divides into the principles of decomposition by problems, D_P, and by properties, D_O. The first of them cuts up the content of the initial problem between subproblems. The second principle allows to distribute the initial problem's formal properties between subproblems.

The described principles may be adopted to the initial problem in combination, $D_C(T, N, \ldots, A)$, but the sequence of their application is essential. They may be applied repeatedly.

All the principles of decomposition can be used in two types: 1) disjunctively, $\vee$, and 2) conjunctively, $\wedge$. It is noteworthy that until now conjunctive decomposition has been given little attention, although it is of great theoretical and practical interest.

Thus, a classification of the principles of decomposition may be presented:

$$K_D = \{D_T, D_N, D_E, D_M, D_A = \{D_P, D_O\}, \quad D_{C(T, N, \ldots, A)}\}; \vee; \wedge. \tag{3}$$

On the principles of co-ordination, parts of the initial problem are integrated into a system, the solution of which produces the solution of the initial problem. The principles of co-ordination are divided depending on how they affect solutions of the planning subproblems. Suppose that in the most general form a planning subproblem is a problem of optimum planning. It has two main components: the objective function and the constraint system.

If the solution of a subproblem is affected 1) by means of the objective function, the procedure is called stimulation, S, and if 2) by means of the constraint system, then we speak of limitation, L. These are the two main principles of co-ordination.

Within stimulation, several principles are applied: 1) the price principles, S_h, 2) the penalty principles, S_t, and 3) the principles of stimulating consultation, S_k.

Within the price principles we speak of: 1) prices of results, S_{ih}, and 2) prices of activities, S_{jh}. Prices of results, y, by their mathematical content are Lagrangian multipliers, y, of the initial problem, or solutions of the dual problem. By their economical content they represent prices of products, services, resources, information, waste matter, etc., or tax rates. Prices of activities, e, in terms of mathematics are solutions of the problem, conjugate to the initial one. By their economic content they represent tax rates for intensity units of the activities planned.

In the case of penalty principles, S_t, the solution of the initial problem is affected with the help of penalties that depend on deviations. Here two possibilities exist: the penalty is imposed 1) directly, on the basis of a deviation from plan indices, which is called the "activity penalty" or conformation, S_{jt}, and 2) on the basis of deviations from results corresponding to the plan indices, which is called the "results penalty", S_{it}.

By stimulating consultation, S_k, we mean adjustment of component parameters of the initial problem's objective function, which are contained in subproblems.

The main principle of limitation, L, is divided into the following principles: 1) limitation of results, L_i, 2) limitation of activities, L_j, and 3) limiting consultation, L_k.

Limitation of results, L_i, exerts an influence upon the solutions of planning subproblems with the help of respective constraints on results. In essence, this means imposing some constraints on production, consumption, utilization of resources, waste matter, etc., of subproblems.

Limitation of activities, L_j, is imposed directly upon the intensities of planning activities. A special case of that principle is dictation, where activity intensities of a planning subproblem are determined by a co-ordinating element.

Limiting consultation, L_k, means adjusting parameters of constraint or result functions of subproblems.

The principles of co-ordination may be applied in combination, $C_C(S, L)$. From the economic viewpoint, of particular interest are obviously combinations of result prices, result limits and activity prices: (S_{ih}, L_i) and (S_{ih}, L_i, S_{jh}).

Intuitively it may be asserted that combined co-ordination permits to solve more intricate problems, increasing the speed of the solutions' convergence. However, the co-ordination and the subproblems get more complicated.

In summary, a classification of co-ordination principles K_C and description of principles may be presented as follows:

$$K_C=\{S=\{S_{ih}, S_{jh}, S_{it}, S_{jt}, S_k\},\quad L=\{L_i, L_j, L_k\},\ C_C(S,L)\}, \tag{4}$$

where *

$$\left.\begin{array}{ll} \left.\begin{array}{ll} f_r(x_r)+\tilde{y}g_r(x_r) & \to S_{ih} \\ f_r(x_r)+\tilde{e}x_r & \to S_{jh} \end{array}\right\} S_h & \\ \left.\begin{array}{ll} f_r(x_r)-q_r(g_r(x_r)-\tilde{z}_r) & \to S_{it} \\ f_r(x_r)-r_r(x_r-\tilde{x}_r) & \to S_{jt} \end{array}\right\} S_t & \\ \tilde{f}_r(x_r) \qquad\qquad \to S_k & \end{array}\right\} S$$

$$\left.\begin{array}{ll} g_r(x_r)\geqslant\tilde{b}_r & \to L_i \\ x_r\in\tilde{X}_r,\quad (x_r=\tilde{x}_r) & \to L_j \\ \tilde{g}_r(x_r)\geqslant b_r & \to L_k \end{array}\right\} L$$

$$\left.\begin{array}{ll} f_r(x_r)+\tilde{y}g_r(x_r)\}\, g_r(x_r)\geqslant\tilde{b}_r,\ x_r\in X_r & \to (S_{ih}, L_i) \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ f_r(x_r)+\tilde{y}g_r(x_r)-q_r(g_r(x_r)-\tilde{z}_r),\ x_r\in X_r & \to (S_{ih}, S_{it}) \end{array}\right\} C_C(S,L).$$

6. The principles of co-ordination, decomposition and the initial problem, taken together, determine the system of subproblems or the scheme of the decomposition method. To classify those schemes, the classification K may now be used, i. e., the product of the decomposition and co-ordination principles: $K=K_D\times K_C$. For describing the initial problem, we introduce the notations LP — linear planning, KP — convex planning, D — determined, $\tilde{S}$ — stochastic, K — compact, S — separable.

This can be illustrated by the following table:

Classification of the systems of subproblems

$D_{N\vee}S_{ih}L_j$	$LPDS$	Danzig-Wolfe	1960
$D_{N\vee}L_i$	$LPDS$	Kornai-Liptak	1962
$D_{N\vee}S_{ih}$	$KPDS$	Everett	1963
$D_{M\wedge}S_{jt}$	$KPDS$	Lions-Temam	1966
$D_{T\vee}S_{ih}$	$KPDS$	Hwang-Fan	1967
$D_{N\vee}L_i$	$KPDS$	Bagrinovsky	1968
$D_{N\vee}L_i$	$KPDS$	Geoffrion	1970
$D_{N\vee}S_{jh}L_i$	$KPDS$	Martinez-Soler	1972

* Here, for clarity, the co-ordinated elements are covered with tildes.

$D_{T\vee}L_i$	*KPDS*	Dementyev	1972
$D_{A\wedge}S_kL_k$	*LPDS*	Aganbegyan-Bagrinovsky	1972
$D_{N\vee}S_{ih}L_{it}$	*LPDS*	Polywak-Tretyakov	1974
$D_{N\vee}S_{ih}L_i$	*KPDS*		
"	*KPSK*		
$D_{A\wedge}S_kL_r$	*KPDK*		
"	*KPSK*		
$D_{N\wedge}S_{ih}L_i$	*KPDS*		
....	....		

The known methods and some unsolved problems are shown in the Table.

7. One of the principles of decomposition analysis is the modelling principle. It uses systems of subproblems or decomposition methods as models of the so-called solving systems.

Best suited for modelling of those systems are the more general methods which possess combined links of various directions, as well as hierarchical and inverse hierarchical structures.

In the modelling of solving systems the notions of "number of levels" and "direction of links" are employed. We assume that the co-ordinating problem is of a higher level than the planning one. The direction of links may be horizontal or vertical. Systems with horizontal links have only one level. Multi-level systems may be hierarchical, pyramidal or general. The general systems contain several levels, at each of which horizontal links may exist. Vertical links may be both hierarchical and inverse-hierarchical. In the latter case we say that the system is polycentrical (one element is co-ordinated by several elements of a higher level).

Of course, different methods of decomposition describe different structures of solving systems. These methods may often be modified in such a way that they will describe new structures. Besides, each method of decomposition possesses several properties, first of all, convergence upon the solution of and prerequisites for the initial problem. Essential also are speed of convergence, monotony of convergence (in terms of both objective functions and plans), admissibility of the approximate plans, location of data, etc. According to these properties, a certain method is more or less suited for modelling particular planning systems.

For instance, co-ordination by means of prices does not secure permanent admissibility of plans (the soft method). If this condition must be met, then, e. g., limitation of results (the rigid method) may be recommended.

8. Making allowance for incomplete information exerts considerable influence upon the methods of decomposition, as well as on their economic interpretations. In this connection we shall make a special remark about the stochastic problems.

Parameters of the planning problem of economy are random variables. The more remote the time moment they belong to, the greater their randomness. This circumstance is one of the factors determining the length of the period which is taken into consideration when plans are worked out (the plan period).

However, generally it is not advisable to fix the values of controllable indices for the duration of this period, i. e., to draw up a definite plan (by plan here such values of controllable indices are meant, whose observance is connected with a certain agreement). By and large, a definite plan should be only for the beginning of the plan period and only for such a space of time that is necessary for ensuring a regular management of economy. The principle of stochastic planning, consisting of several stages, in terms of mathematics, may be put down like this:

$$\max_{x_I} \underset{\zeta_I}{E} [\varphi_I(x_I, \zeta_I) + \max_{x_{II}} \underset{\zeta_{II}}{E} \varphi_{II}(x_{II}, \zeta_{II}, x_I, \zeta_I)],$$
$$x^0_I = \text{fix}, \quad x^0_{II} = x^0_{II}(\zeta_I). \tag{5}$$

Generally this principle is used so that random values of controllable indices, $x^0_{II}(\zeta_I)$, are given implicitly. However, obviously in the case of macroeconomical objects, it is expedient to give them explicitly. Such a planning method is called the definite-probable one.

The probable controllable indices may be 1) linked with some agreement, 2) not linked with any agreement. Respectively we speak of a probable plan, or of a plan forecast.

By means of definite-probable planning it is possible to more adequately describe the relationship between planning and forecast, as well as the flexibility of plans, building of capacity reserves, etc. The plan is not always a determined index, it may also be a random quantity. In principle, the results related to a plan are always random values. They may be declared plans (i. e., linked with an agreement about their fulfilling), or considered as forecasts, depending on the system of planning.

One of the simple ways of drawing up problems of definite probable planning is their description with the help of mean values and dispersions (variances) only. Those problems are called problems using mean values and dispersions. Their solutions produce mean values $E[\ldots]$, and dispersions $D[\ldots]$ of both plan indices and respective indices of results. The latter make it possible to easily find confidence intervals of all indices that are suited for practical use.

Here dispersions of the planned indices of activities show the flexibility of a plan, which is not valid for definite plans.

It appears that consideration of stochastic elements markedly complicates the system of planning, and particularly the system of co-ordination, due to the higher complexity of the initial problem. Determined approximation of a stochastic problem in the shape of a problem using mean values and dispersions is more multidimensional and compact than an analogous determined initial problem. Dimensions grow when dispersions are taken into account, and, as a result, the compactness of a problem increases, too.

When problems using mean values and dispersions are decomposed, both mean values and dispersions of the indices contained in the subproblems, are subjected to co-ordination.

For instance, applying the principle of result price S_{ih}, we obtain the prices of mean values (y) and dispersions (η). In a problem of a unit r both prices should be taken into consideration.

Within the period of a definite plan the intensities of activities are fixed, except the use of stocks. Thus, stocks are here the only activities reducing risk. The influence of stocks in terms of risk reduction is modelled in such a manner that they have negative co-variation with the results of other activities.

Local co-ordination of the volume of stocks and degree of risk of other activities is effected through co-ordination of the values of the respective dispersions. When co-ordination is done by prices, the latter are called prices of risk. On the basis of those prices, activities that cause dispersion of results, pay the centre fines for risk. Activities that reduce dispersion of results (negative co-variation with the results of previous activities), on the basis of those prices, get income to build stocks.

Within the period of a probable plan and plan forecast, dispersion is congenial with the values of plan indices, too. Dispersion of a planned activity reveals the possible flexibility of that activity, and, simultaneously, the ability to reduce the scattering of results. However, flexibility calls for standby capacities which means expenses. Local adjustment of the respective indices is effected through co-ordination. For this purpose the risk prices, risk limits or their combinations may be used.

9. Systems of economic planning cannot be built by technological methods, but their perfecting, at least within the framework of qualitative analysis, may be considered as an optimization process. A perfected system has to possess certain properties, according to which it should be superior to the original one.

The most essential properties to be reckoned with are quality of plans, speed of plan-making, complexity of planning process and its cost, adjustment of approximate plans, monotony of convergence process, location of data, etc.

Properties of a planning system, s, are worked upon during alternative selection of its components. The major classes of the alternative components are solving systems $\bar{S}$, teaching systems L, check-up and reward systems R, and systems of technological equipment W.

By means of decomposition analysis, suitable solving systems can be built. Within the latter, still other components may be distinguished, such as information systems I, systems of subproblems P and systems of solving methods F. The class of possible alternatives may be expressed as follows: $S=P\cup I\cup F\cup L\cup R\cup W$. This class should contain the system synthesized: $s\in S$. The selection of a suitable system can be handled as an optimization problem:

$$\operatorname*{extr}_{s} v(s) \left.\right\} p\wedge s \rightarrow (k_o,\ k_b,\ k_m,\ \ldots),\quad s\in S \tag{6}$$

where $v(s)$ — critical index of a system; vector $(k_o, k_b, k_m, \ldots)$ describes the required characteristics of a system; k_o — optimization; k_b — permanent balancing; k_m — monotony of convergance, etc.; p — initial planning problem.

The given problem (6) is not yet solved. Therefore, the synthesis of systems should be begun by solving narrower problems.

Generally it is advisable to start with the decomposition by time, for instance, dismembering the system into systems of strategic and tactical planning. The first of them, in turn, may be cut up into systems of forecasting, long- and medium-term planning, etc.

The system of strategic planning can be best analyzed along the principles of decomposition by aims and aspects. For a co-ordination of the subsystems so obtained, horizontal links are essential.

The problems of tactical planning are of comparatively large dimensions. Here it is expedient to apply principles of decomposition by units and kinds of activities. For a co-ordination of those systems, both horizontal and vertical links are important.

Analysis of decomposition models in the aspect of organization reveals that regional centres should co-ordinate production/consumption of the respective regional resources and products. Branch centres should co-ordinate production/consumption of products and resources of a more global character. When plans are drawn up at the level of regions and branches, the latter should co-ordinate plans of their common enterprises. Co-ordination is also possible through setting up a respective higher level.

Making allowances for flexibility and randomness of results complicates co-ordination, in particular, the questions of rewarding the planners. For instance, here arises the question of informing, as well as that of the reliability of the stochastic planning indices. To achieve higher reliability, the principle of rewarding should be applied in such a way that a reward is dependent both on the accuracy of presenting the result planned and its realization according to plan. This principle of rewarding can be easily realized by the so-called co-ordination through intervals, where the plans of results are presented by intervals. To illustrate, we shall give an example of a reward function:

$$r_{ij}(z_{ij}, \overline{\vartheta}_{ij}) = y_i z_{ij} - (\vartheta^{\urcorner}_{ij} - {}_{\llcorner}\vartheta_{ij})\, u_i - \begin{cases} p_i, & z_{ij} \in \overline{\vartheta}_{ij} \\ 0, & z_{ij} \in \overline{\vartheta}_{ij} \end{cases}, \qquad (7)$$

here z_{ij} — realization of result i in unit j;

$\overline{\vartheta}_{ij}$ — plan interval of that result with the end points $\vartheta^{\urcorner}_{ij}$ and ${}_{\llcorner}\vartheta_{ij}$.

Secondly, there arises the question of rewarding local risk. To solve it, the co-ordination system should be provided with an insurance system, by means of which local risk may be co-ordinated with global expedient risk. A function of guaranteed rewarding may look like this:

$$r_j(t_j, t^0_j) = t_j + \Psi(t^0_j - t_j) + \pi(t^0_j) \qquad (8)$$

where t_j — realization of revenue of unit j; t^0_j — planned level. It can be easily seen that the modified reward function alters the unit's attitude to economic risk.

10. Being equipped with the concept of decomposition analysis, let us return to our graph (See Fig.). Now we are able to somewhat more profoundly analyze and more precisely synthesize the system of optimum planning of economy.

For instance, it may be said that the given system is at first conjunctively cut up by time and co-ordinated by result limits. Proceeding from the respective method, something may be stated about the properties of that system. Probably, for a person not familiar with the conjunctive decomposition by time, the procedure of building such a system seems to be wrapt in mystery, at least from the mathematical point of view. Further it may be asserted that the system of long-term forecasting is conjunctively dismembered by aspects and co-ordinated by activity fines. Suppose that the system of stategic planning is at first cut up by units and conjunctively (regions and branches of industry), as well as by kinds of activities, and is co-ordinated by combined consultation and limitation of results, etc.

Apparently, the method of decomposition analysis described above allows to specify the structure and the economico-mathematical foundations of a system of optimum planning of economy, and to make available new instruments for analysis and synthesis of such systems. However, lots of questions remain open or arise again. A great deal of work on the development of mathematical methods of decomposition, building and experimenting of model systems still lies ahead.

Academy of Sciences of the Estonian SSR,
Institute of Economics

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Ü. ENNUSTE

OPTIMAALSE DEKOMPOSITSIOONPLANEERIMISE PRINTSIIPE

Artiklis käsitletakse sotsialistliku majanduse teoreetilise optimaalse planeerimise ülesande lahendamist lihtsamatest osaülesannetest koosnevate süsteemide abil. Selgitatakse nimetatud süsteemide moodustamise ja koordineerimise põhimõtteid. Erimärkus tehakse stohhastika arvestamise kohta. Lõpuks vaadeldakse suurte planeerimissüsteemide sünteesi probleeme.

Eesti NSV Teaduste Akadeemia
Majanduse Instituut

Toimetusse saabunud
10. XI 1975

Ю. ЭННУСТЕ

ПРИНЦИПЫ ДЕКОМПОЗИЦИОННОГО ОПТИМАЛЬНОГО ПЛАНИРОВАНИЯ

Резюме

В статье рассмотрено решение теоретической задачи оптимального планирования социалистической экономики с помощью систем, состоящих из более простых задач. Сделана попытка внести ясность в принципы построения и координации таких систем. Обращается внимание также на необходимость учета стохастики. Наконец, рассмотрены проблемы синтеза крупных систем планирования.

Институт экономики
Академии наук Эстонской ССР

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