

<https://doi.org/10.3176/hum.soc.sci.1970.4.06>

Ü. ENNUSTE

## ON THE PRINCIPLE OF THE ECONOMIC EFFECTIVENESS OF A PLANNING SYSTEM

A system planning economy or production can be regarded as an object of study by economic science. On the one hand, planning involves considerable costs. On the other, it may lead to a decrease of losses and expenditure, and give additional incomes. Here we are faced with an intricate problem of economic effectiveness of a planning system, its optimum characteristics, excessive or inadequate planning, etc.

In the following some problems of the optimization of the structures of planning systems are formulated, proceeding from the criterion of economic effectiveness. Such concepts as entropy of data and plan, informational costs and incomes and characteristic of planning system are used. Such problems as optimum informational state of data, optimum fullness of planning model, intricacy of relationships and others are discussed.

The treatment is of a qualitative character, and for illustration, extremely simplified examples are given. For quantitative application, further studies are required, especially in information economics and similitude theory of planning models. In this article, an attempt is made at strengthening intuition for a better choice of the characteristics of planning systems.

### 1. Concepts and principles

Let us define the most general necessary concepts and the principle of the effectiveness of a planning system.

#### 1.1. Entropy and information for a planning system

Set of data  $A = \{A_i\}$ ,  $i \in M = (1, \dots, m)$ , used in planning economic or production units in general, consists of random variables. Essentially these data are statistical or expectable values of the parameters of the planned system or unit. Only in special cases they are extremes of a random variable, i.e. determined variables.

A random variable is completely described by its probability distribution function  $P_i(a_i) = P(A_i < a_i)$  or density function  $p_i(a_i) = \frac{d}{da_i} P_i(a_i)$ , where  $a_i$  is some value of variable  $A_i$ .

In this article, for describing a random variable, also the concept of entropy is used [1]:

$$h_i(A_i) = - \int_{b_i}^{c_i} p_i(a_i) \log p_i(a_i) da_i, \quad (1)$$

where  $(b_i, c_i)$  is the interval of all possible values of random variable  $A_i$  (variation interval). The entropy of a determinate variable in expression (1) is zero, and that of an uncertain variable in general may approach infinity.

Assume that the entropy of a random variable  $A_i$  depends on the planner's informational state (volume of available information) with regard to the parameter  $i$ . Suppose that by gaining supplementary information it is possible for the planner to decrease entropy and this information is measured with the difference of entropies  $i_i = h_i^a(A_i) - h_i^p(A_i)$ , where  $h_i^a(A_i)$  and  $h_i^p(A_i)$  are entropies respectively before (a priori) and after (a posteriori) gaining a new piece of information.

Suppose that supplementary information is objective. By objectivity we understand the fact that with increasing information the entropy  $h_i(A_i)$  of parameter  $i$  for planner approaches the objective entropy  $h_i^0(A_i)$ , and the distribution function for planner  $P_i(a_i)$  approaches the objective one  $P_i^0(a_i)$ . In this case objectivity is a relative term alluding to causes of randomness which drop out of the area under study (entropy of environment [2]).

## 1.2. Entropy of plan

Let us define the planning system  $S$  as a system collecting data (information) and processing them to a plan, which we denote by  $X$ . The plan for a definite period comprises a set of indices  $X = (X_j)$ ,  $j \in N = (1, \dots, n)$ . The plan determines the values of controlled indices of an economic unit which, in a definite sense, are expected to be the best ones for the planned period and provide a basis for the choice of the best activity at a given moment (direct management, control).

Suppose that the planning system  $S$  is described by the following set of characteristics  $S = \{\mu, \eta, \xi, \varphi\}$ , where  $\mu$  characterizes the model (fullness of indices, intricacy of relationships between indices, number of intervals of the planned period, length of interval, etc.);  $\eta$  characterizes the informational state  $H = (h_i)$ ,  $i \in M$ , of data  $A$  used in the model;  $\xi$  characterizes the algorithm used for the solution of the model (accuracy of calculations, length of calculations, number of operations, etc.)  $\varphi$  indicates the starting point of plan, its length of elaboration, etc. Thus, in each characteristic we distinguish some subcharacteristics.

It can be said that initial data used by a real planning system  $S$  comprise entropy ( $\sum h_i(a_i) > 0$ ) that the models employed do not describe the planned objects in a homomorphous way, and that the accuracy of solution methods is limited. It follows from the above-said that the plan  $X$  is also a random variable, i. e. its coordinates  $X_j$ ,  $j \in N$ , are random variables. To simplify matters suppose that  $X_j$ ,  $j \in N$  are independent random variables. Let  $x_j$  be the realization of coordinate  $X_j$ , and  $p_j(x_j)$  density of its probability. Probability densities for the whole plan are described by the vector function  $p(x) = (p_j(x_j))$ ,  $j \in N$ . Thus we can speak of the entropy structure  $H(X) = (h_j(X_j))$ ,  $j \in N$  of plan  $X$  and the information structure  $i_x = h^a(X) - h^p(X)$ . The superscripts denote: "a" — before, and "p" — after specification.

Now it can be asserted that the total entropy of the plan  $h_x = \sum h_j(X_j)$  depends on the planning system  $S$ :

$$h_x = h_x(S), \quad (2)$$

where  $h_x(S)$  is an operator and  $S$  belongs to a set of possible planning systems  $S \in \bar{S}$ . The properties of operator  $h_x(S)$  and the set  $\bar{S}$  will be discussed later. Depending on the total entropy of the plan, we may also speak of a planning system with a greater or smaller entropy.

## 1.3. Incomes and costs of information, value of information and the optimum planning system

Planning means determination of future values of major variables of an economic unit so that timely preparations could be started to better utilize the conditions expected in the planned period. Unutilized possibilities [3] can be regarded as expectable losses. To illustrate the above, we bring the following example.

Example 1. Let the expectable needed capacity  $X$  be a random variable in the interval  $x \in [b, c]$  with even probability density  $p(x) = 1/(c-b)$  and mean value  $\bar{x} = (b+c)/2$ . Entropy of the planned index is

$$h(X) = -\frac{1}{c-b} \log \left( \frac{1}{c-b} \right) \int_b^c dx = \log 2g,$$

where  $g = (c-b)/2$  is called arm. It can be seen that entropy is increasing function of the arm  $g$ . Thus, the informational state of index  $X$  can be characterized by the arm  $g$ .

Suppose that for planned capacity is taken mean value  $\bar{x} = b+g$  of  $X$ . If the actually needed capacity (realization)  $x$  turns out to be smaller than we have constructed, i. e.  $x < \bar{x}$ , there is an overdraft on investments, a loss, which can be indicated by the expression  $k(\bar{x}-x)$ . If the needed capacity  $x$  is greater than the constructed one  $x > \bar{x}$ , there will be losses (replacement with a more expensive production). For the sake of simplicity we denote it also  $k(x-\bar{x})$ . Considering that density function equals  $p(x) = 1/2g$ , the mean value of loss can be calculated as follows:

$$K = \frac{k}{g} \int_0^g (g-x) dx = kg/2.$$

In the above example function shapes were simplified. Nevertheless, in principle it describes correctly the relationship between expectable loss (mean value of loss) and informational state of plan.

For a general case we can state that mean value of loss depends on informational state of plan or on planning system

$$K = K_H(h_z) = K_s(S). \quad (3)$$

On the other hand, informational state of plan and respective characteristic of planning system involve certain costs. Provided that planning systems are rational these costs increase with the improvement of informational state of plan and the decrease of entropy of planning system. A decrease in plan entropy makes it necessary to specify initial data more exactly or detail the model, improve the accuracy of computations, etc. More expenditure is required to conduct additional research and experiments, to specify initial data, to acquire a computer of a greater capacity, etc.

It can be asserted that a certain amount of resources or costs  $W$  corresponds to each planning system  $S$ :

$$W = S(S), \quad (4)$$

which are related to the determination of plan  $X$ .

Thus, planning costs depend on the characteristic  $S$  of the planning system. By means of these costs, attempts are made to gain additional incomes (as compensation for loss). It is easy to understand that the ratio of these costs and incomes should ensure maximum effectiveness in the utilization of the resources employed for planning and production.

To estimate effectiveness, the expectable incomes and concrete expenditure should be made economically comparable. First of all, we have to consider the differences between incomes and expenditure in time, problems involving risks and problems linked with the weight of the given plan in decision-making.

To eliminate differences in time, temporal discounting is used, for instance, incomes obtained in the planned period are discounted to the moment of planning (i. e. the time when planning costs become necessary). The incomes calculated represent mathematical expectations, the mean value of incomes. Depending on risk, these incomes can be given different weights in comparison with real planning costs. Finally, there is the possibility

that plan  $X$  elaborated by planning system  $S$  will be corrected when the final decision is taken, i.e. the plan will be given a certain weight. In other words, the final decision can be taken in co-operation (as a compromise) of several planning systems. Formally, the final decision can be looked upon as a weighted average of a number of plans, while weights are determined by the final decision-taker.

In sum, the expectable possible loss comparable with planning costs is

$$R=R(S), \quad (5)$$

where operator  $R$  permits the loss to be economically compared with planning costs.

Proceeding from the maximum economic effectiveness for the planning system, the optimum characteristic of the planning system can be found from the condition

$$\min_{S \in \bar{S}} (R(S) + W(S)). \quad (6)$$

Provided that  $S^0 \in \bar{S}$ , we find

$$\frac{dW(S^0)}{dS} = -\frac{dR(S^0)}{dS},$$

i.e. in case of the optimum planning system, marginal information costs are equal to marginal information incomes (decrease of loss).

It is practical to formulate the modification of the problem (6). Firstly, at fixed planning costs  $C$  it is advisable to determine the planning system  $S$  in such a way that the expectable loss of possibilities  $R$  is at a minimum.

$$\min R(S) \} W(S) = C. \quad (7)$$

After forming the Lagrange function

$$R(S) + \lambda(W(S) - C) \rightarrow \min,$$

we find that at an optimum solution

$$-\frac{dR(S^0)}{dS} = \lambda \frac{dW(S^0)}{dS},$$

where  $\lambda$  is the effectiveness of additional planning costs of the optimum planning system.

## 2. Effective characteristics of a planning system

It is difficult to set up complex hypotheses concerning operators  $R(S)$  and  $W(S)$  after all arguments  $\mu$ ,  $\eta$ ,  $\xi$  and  $\varphi$ . To simplify, we decompose them in the plane of isolated arguments. The values of remaining arguments are considered constant. Solutions of optimum problems formulated in such a way are effective points in the sense of mathematical optimization.

### 2.1. Optimum entropy of initial data

Suppose that the characteristics of a planning system are fixed, except the entropy of initial data. Let us determine the optimum entropy of initial data for this case.

Denote vector of initial data by  $A=(A_i)$ ,  $i \in M$ . To this vector corresponds vector of entropy  $H=(h_i)$ , where  $h_i$  is entropy of coordinate (parameter)  $i \in M$ . Assume that the entropy of plan is  $H_X=(h_j)$ ,  $j \in N$ , where  $h_j$  is the entropy of plan coordinate (index)  $j \in N$ , depending on the entropy of initial data  $H_X=H_{XA}(H)$ . Let possible loss due to plan entropy be  $r=r_X(H_X)$ . Thus, the possible loss can be expressed as a function of the entropy of initial data.

$$r=r_A(H). \quad (2.1.1)$$

Now consider the following hypothesis. Coordinates  $h_i$ ,  $i \in M$  of vector  $H$  can have values in the non-negative half-space,  $h_i \geq 0$ , only. An increase in entropy of coordinate  $i$  leads to increase of loss  $dr/dh_i > 0$ , and this increase has a growing tendency  $d^2r/dh_i^2 > 0$ .

Suppose that the dependence of a planning system's costs on the entropy of initial data is described by the function

$$\omega = \omega_A(H), \quad (2.1.2)$$

provided that with the growth of entropy costs decrease  $d\omega/dh_i < 0$  and this relation continues diminishing  $d^2\omega/dh_i^2 > 0$ .

With the help of functions  $r_A$  and  $\omega_A$ , a number of optimum problems can be formulated.

First of all, let us consider a case where information costs  $\omega$  are not limited. The optimum entropy of initial data is to be determined

$$u_A = r_A + \omega_A \rightarrow \min. \quad (2.1.3)$$

**Example 2** (to problem 2.1.3). Let the expectable capacity needed to turn out product  $l$  be  $X = AY$ , where  $Y$  is a given constant and  $A$  is the expectable input coefficient. The latter is a random variable with an even probability density in the interval  $(b, c)$ ;  $p(a) = 1/(c-b)$ . The entropy of parameter  $A$  is characterized by the arm  $g = (c-b)/2$ .

$Y$  and  $A$  are initial data. Because  $A$  is a random variable, the plan  $X$  is also a random variable in the interval  $(bY; cY)$  with an even probability density  $p(X) = 1/(c-b)Y = 1/2gY$ . Its arm is  $g_X = (c-b)Y/2 = gY$ .

Denote information costs by  $\omega = a/g$ , where  $a$  is given and information loss at realization  $x \in X = (bY; cY)$  is  $r = (\bar{x} - x)q$ , where  $q$  is given and  $\bar{x} = bY + g_X = (b+g)Y$ , i. e. the mean value of  $X$ .

The expectable loss is

$$R = 2 \int_0^{gY} (\bar{x} - x)q \frac{1}{2gY} dx = \frac{qgY}{2}.$$

Now we can formulate the problem as follows

$$u = \omega + r = \frac{a}{g} + \frac{qgY}{2} \rightarrow \min_g.$$

Thus

$$\frac{du}{dg} = -\frac{a}{g^2} + \frac{qY}{2} = 0,$$

and optimum value  $g = \sqrt{2a/qY}$ .

It can be seen that the greater the fine  $q$  for inaccuracy and the greater the volume  $Y$ , the more precisely initial data and variation interval should be determined. An increase in information cost  $a$  would require greater approximateness of initial data.

If the information costs  $\omega$  are bounded by above, we can construct a problem how the given information costs can be best distributed between initial data, in other words, how to find a state of entropy  $h_i$  best suited for application of the parameter  $i$  in planning. Thus, the solution represents the optimum entropy structure of initial data. The problem can be stated as

$$\min_H u_A = r_A + \omega_A \} \omega_A \leq \bar{\omega}_A. \quad (2.1.4)$$

Provided that  $\omega_A = \sum \omega_i(h_i)$ , the Lagrange function of the problem (2.1.3) is

$$u_A(H) - \lambda(\bar{\omega}_A - \sum \omega_i(h_i)) \rightarrow \min,$$

at optimum entropy structure

$$\frac{du_A}{dh_i} + \lambda \frac{dw_i}{dh_i} = 0,$$

where  $\lambda$  is the constant characterizing the effectiveness of additional information costs. If the total entropy state of data is bounded  $\sum h_i \geq h$ , we face a problem similar to (2.1.4), where information is to be distributed between parameters in such a way that the values of additional information units for all parameters are equal.

In constructing models of planning systems with limited resources, it is sometimes advisable to consider the upper boundaries for every group of resources separately. For instance, bounded by above are volume of human labour, computational capacity, etc. According to this formulation, the optimum entropy structure of initial data is determined.

**Example 3** (to Problem 2.1.4). Suppose the necessary production capacity is  $X = A_1 Y_1 + A_2 Y_2$ , where  $Y_1$  and  $Y_2$  are given constants, expectable coefficients  $A_1$  and  $A_2$  are random variables with even distribution densities and arms  $g_1$  and  $g_2$ . Thus,  $X$  is a random variable with arm  $g_x = g_1 Y_1 + g_2 Y_2$ . For the sake of simplicity, suppose that  $Y_1 = Y_2 = Y$ . Then  $g_x = (g_1 + g_2)Y$ . If  $\bar{x}$  — the mean value of  $X$  is selected as the capacity to be constructed, then information loss at realization  $x$  is  $r = (|\bar{x} - x|)Q$ , where  $Q$  is given. Expectable loss (see Example 1) is:  $R = 0,5Q(g_1 + g_2)Y$ . Information costs, respectively, are  $w = \alpha_1/g_1 + \alpha_2/g_2 \in \bar{w}$ , where  $\bar{w}$  is given. Now we have an optimum problem

$$\begin{aligned} \min u &= R + w \} w \leq \bar{w} \\ g_1, g_2 &\geq 0 \end{aligned}$$

The Lagrangian function is

$$L = u - \lambda(\bar{w} - w) \rightarrow \min.$$

Solution can be found from conditions  $\frac{dL}{dg_1} = \frac{dL}{dg_2} = \frac{dL}{d\lambda} = 0$ , where  $\lambda$  is the effectiveness of additional information costs (marginal effectiveness). If the amount of information gained by additional cost is known (measured with difference between arms), the additional effectiveness or value of information can be computed.

At this point, attention should be directed to a possibility of optimum utilization of information. In practice, the expectable necessary capacity  $X$  is a random variable with a known distribution density  $p(x)$ . Generally, the capacity to be constructed is selected as the mean value  $\bar{x}$  of  $X$ . But this solution is optimum only in a special case. To clarify the general case, let us define the loss as dependent of the choice of the capacity to be constructed  $v$ :

$$r = \begin{cases} r_1(v-x), & x \leq v, \\ r_2(x-v), & x > v. \end{cases}$$

The necessity to distinguish between function  $r_1$  and  $r_2$  is apparent.  $r_1$  describes loss at functioning below capacity, and  $r_2$  describes loss when there is a deficit of capacity. It is clear that these functions are essentially different. The expectable loss  $R$  is a function of  $v$ . Consequently, we have to find such a  $v$  that

$$R = \int_0^v r_1(v-x)p(x)dx + \int_v^c r_2(x-v)p(x)dx \rightarrow \min_v. \quad (2.1.5)$$

The difficulty of solving the problem (2.1.5) lies in its mathematical complexity, especially when the functions  $r_1$ ,  $r_2$  and  $p(x)$  are intricate.

For instance, in typical cases  $r_1 = (v-x)Q_1$  and  $r_2 = (x-v)Q_2$ . At normal distribution we face a rather difficult problem.

**Example 4** (to Problem 2.1.5). Suppose that the necessary expectable capacity is a random variable  $X$  with even probability density  $p(x) = 1/(c-b)$  in the interval  $[b, c]$ .

Denote the capacity to be constructed by  $v \in [b, c]$ . Suppose the loss according to realization  $x$  of  $X$  is

$$r = \begin{cases} r_1 = (v-x)Q_1, & x \leq v, \\ r_2 = (x-v)Q_2, & x > v. \end{cases}$$

Provided that  $b=0$ , expectable loss can be expressed as a function of  $v$

$$R = \int_0^v (v-x)Q_1 \frac{1}{c} dx + \int_v^c (x-v)Q_2 \frac{1}{c} dx = \frac{1}{2c} [Q_1 v^2 + Q_2 (c-v)^2].$$

The optimum  $v$  can be found from the condition

$$\frac{dR}{dv} = \frac{1}{c} [Q_1 v - Q_2 (c-v)] = 0.$$

Hence

$$v = \frac{Q_2 c}{Q_1 + Q_2}.$$

If  $Q_1=1$ ,  $Q_2=0,5$ ,  $c=1$ , then  $v=0,33$  and expectant loss  $R=0,16$ . If  $v=\bar{x}=0,5$ , then  $R=0,19$ .

Finally, such technique enables us to establish the set of number signs which can be used in planning calculations. The problem is as follows. The reduction of the number of signs, i.e. rounding off the numbers would cut time and effort, or in other words, expenditure associated with the calculations. Let us denote expenditure dependent of the number of signs  $n$ , by the function  $w(n)$ .

On the other hand, greater approximateness of planning indices causes higher expectable losses. Let us denote losses by  $r(n)$ . It will be understood that the best choice of the number of signs is

$$w(n) + r(n) \rightarrow \min_n. \quad (2.1.6)$$

Expenditure  $w$  depends on the character of operations and it is empirically possible to explore the shape and parameters of  $w(n)$ . To estimate the value of expectable losses assume that, if the planning indices contain a sufficient number of signs  $n_0$ , then, in terms of the number of signs (or the accuracy of scale) the expectable loss is practically zero  $r(n_0)=0$ . Switching over to a scale with a ten times greater unit (i.e. if in the decimal system the number of signs is reduced by one:  $n_1=n_0-1$ ), the expectable error would be:

$$E_{n_0-1} = 2 \int_0^5 (5-x) \frac{1}{10} dx = 2,5\Delta,$$

where  $\Delta$  is the length of the division of zero scale.

## 2.2 Dimensions and aggregation of a planning model. Complexity of relationships

The dimensions  $v$  of a planning model determine the necessary number of data and expenditure spent to obtain them (the informational state of data is given). On the other hand, greater dimensions mean a more detailed plan and reduction of expectable losses. It can be seen that this is an optimum problem: increase in dimensions would require much information and expenditure, reduction of dimensions would lead to a decrease in plan information and informational incomes.

In a planning problem, there often arises a need of aggregation, reduction of the problem's dimensions. A quality criterion of aggregation is that the results which are obtained by the use of a non-aggregated model and those that will be aggregated later

should possibly be in accordance (least-square method) with the calculations of the aggregated model [4, 5].

Proceeding from the concepts of informational incomes and costs, we come to a principally new criterion which enables us to estimate the quality of aggregation: informational loss at aggregation should be of minimum value. Indeed, deviation of some planning indices does not lead to essential losses (capacities can be employed in other branches, production can be replaced, advantageously exported or imported, etc.). However, increase in the approximateness of some indices may cause major informational losses. This line of reasoning gives us additional weights for the choice of aggregation. If the weights of all plan indices were equal, the new result would line up with the result obtained by the above-mentioned commonly used criterion.

Suppose the plan found with the help of a non-aggregate model is  $X=(X_j), j \in N$ , and its informational state is  $H_X=(h_j)$ . Expected informational loss associated with this plan is  $r=r(H_X)$ . Employ the aggregating operator  $G$  and obtain the model in aggregated form. The respective plan is  $\check{X}=(\check{X}_s), s \in \check{N}$ , with an entropy structure  $\check{H}_x=(h_s)$ . The respective expectable loss can be determined from the expression  $\check{r}(\check{H}_x)$ . Assume that  $\check{r}(\check{H}_x) = \sum_{s \in \check{N}} r_s(h_s)$ , where set  $\check{N}$  depends on the choice of aggregating operator  $G$ ,  $\check{N}=G(N)$ .

In sum, we have the problem

$$\min_{G, s \in G(N)} \sum r_s(h_s). \quad (2.2.1)$$

Quantitative analysis of this expression is rather difficult, but qualitative analysis allows to deduct some rules to improve aggregation. This, however, would be a study by itself.

A problem by itself is the formulation of every planning problem because plan indices represent aggregates. Which detailed indices existing in reality should be aggregated to one index is a question that should be solved on the principle of the least informational losses.

In working out the plan model, the question arises how to select the mathematical form of the balance relationships in the model, or the shape of production and consumption functions. A more simple form of these functions requires a lesser number of parameters. Thus, the informational costs are small. However, in this case entropy of the plan obviously is greater than that of more intricate functions. But the latter require that a greater number of parameters be determined.

Expression of plan entropy depending on the choice of relationships in planning model is an item of the theory of similitude. In the domain of economico-mathematical modelling, this theory has not yet been notably applied.

### 2.3. Choice of planning and re-planning period. Horizon of planning period

Considering planning in time, we can observe, as follows. Optimum planning of economics as a dynamic and stochastic process, according to R. Bellman's method of dynamic planning [6], is to be carried out continuously. With the passage of time, the system's (economic's) coordinates become more precise at the close of the time unit. New factors will be revealed that affect the parameters of the process and its course. With this in view, it would be possible to determine the optimum development plan of the system more precisely at the close of every time unit. It should be noted that in case of a stochastic process the system's optimum development plan is not intended for realization, but it provides a certain foundation to such an activity which ensures that the future expectable informational losses will be at a minimum (on the basis of the existing informational state).

Let us consider planning in discreet time, where  $\Omega=\{1, \dots, \omega\}$  is set of time units in the planning period,  $\tau$  — time (moment) of planning and  $\theta$  — time in future  $\theta > \tau$ ,  $\theta \in \Omega$ .



Now make the following assumptions about informational losses in the interval  $\tau$ . They depend on entropies of plans for subsequent intervals  $\tau+1, \dots, \theta, \dots, \omega$ , and temporal distance of these plans  $\kappa = \theta - \tau$ . The same assumptions are made about informational costs. Considering influence of these factors in isolation, we can distinguish three types of problems.

Firstly, let us consider a case where the interval to be planned and entropy of plan in this interval are fixed. Denote the plan for interval  $\theta$  by  $X_\theta = (X_{j\theta})$  and its entropy by  $H_{X_\theta} = (h_{j\theta})$ . Informational loss in interval  $\tau$  depends on the entropy of the plan for interval  $\theta$  as follows

$$r_{\tau\theta} = r_{\tau\theta}(H_{X_\theta}).$$

Informational costs  $w_{\tau\theta}$  in interval  $\tau$ , depending on entropy of plan  $X_\theta$ , can also be expressed as the function

$$w_{\tau\theta} = w_{\tau\theta}(H_{X_\theta}).$$

Consequently, we come to the problem

$$r_{\tau\theta} + w_{\tau\theta} \rightarrow \min. \quad (2.3.1)$$

$$H_{X_\theta}$$

The solution of this problem indicates suitable entropy structure of plans which in interval  $\tau$  are worked out for interval  $\theta$ . Under the assumption that at fixed  $h_{j\theta}$  the value of function  $w_{\tau\theta}$  infinitely increases due to the increase of  $\kappa = \theta - \tau$ , and value of function  $r_{\tau\theta}$  decreases at the growth of  $\kappa$ , it can be seen that for certain  $\bar{\kappa}$  planning of index  $j$  turns out to be inefficacious. Obviously,  $\bar{\kappa}$  will have greater values for indices with greater inertia which take more time to be realized. Index  $\kappa$  is called planning distance and maximum effective distance  $\bar{\kappa}$  is the horizon of a planning period.

In a similar manner, it is possible to consider the influence of planning distance  $\kappa$  on the fulness of plan  $X_\theta$ . The fulness of plan  $X_\theta$  is characterized by dimensions of vector  $X_\theta = (X_{j\theta})$ ,  $j \in N_\theta$ , where  $N_\theta$  indicates fulness of plan in interval  $\theta$ . It is intuitively clear that, with the increase of distance, the influence of detailed indices decreases and only aggregated indices retain their importance. Apparently, in case of fixed distance and aggregation, it is advisable to determine aggregated indices with smaller entropy because they have greater stability than the detailed ones.

Two closely interrelated questions arise: after what time interval should plans be corrected and at what speed should it be done. Correction of plans worked out in interval  $\tau$  for interval  $\theta$  which is carried out in the interval  $\eta$ , ( $\tau < \eta < \theta$ ) allows to cut informational losses for interval  $\eta$ , since with the passage of time  $\eta - \tau$  data can be specified more exactly, and informational incomes may top respective costs and also costs related to recalculation of plan.

If precise recalculation of plan takes up a considerable amount of time and such delay causes vital losses, an approximate correction of plan may be effected. Then the plan is specified approximately, in the first place, in the part where informational losses reach their maximum.

Analysis of effective correction problems reveals that those planning indices which are more aggregated and stabler in time are less frequently corrected. Detailed indices should be corrected more frequently. Most detailed indices are to be planned continuously or immediately before the realization of the respective plan.

In sum, if respective data are available, optimum aggregation of plan indices can be determined in consideration of the planning distance and the horizon of the planning period. Then the frequency of re-planning can be established. Detailed indices are re-planned more frequently than bigger aggregates.

### 3. Applications

For some time past, management and planning of national economy has been metaphorically termed "management industry". In connection with computerization, indeed, there will be more and more resemblance between this kind of social activity whose task is information processing and a technological branch of national economy. Support costs of this branch are ever growing, which urges the need to get them down to a reasonable level.

Like in every other industry, we should start from the study of "production costs", i.e. expenditure on information processing. It seems that at least in some aspects immediate practical results can be achieved. Most promising could be measurement of the informational state of planning data and respective costs, also estimation of the relationship between initial data and informational states of a plan, comparison of this relationship to informational losses.

There exist many ways to cut the entropy of initial data. Most relevant is an economico-technical survey. More detailed studies will contain less entropy, but involve higher costs. Experience gained so far already allows the latter to be prognosticated. Informational state of research depending on its fulness can also be predicted.

It appears that with more complex problems there arise mathematical difficulties in estimating the relationship between entropies of plan and initial data. A practical way out is the use of numerical methods which can be easily realized on a computer.

### REFERENCES

1. N. Wiener, *Küberneetika*. Tallinn, 1962.
2. R. Murphy, *Adaptive processes in economic systems*. New York, Acad. Press, 1965.
3. H. Raifa, R. Schlaifer, *Applied Statistical Decision Theory*. Boston. Harvard University, 1961.
4. Н. Дюмин, Ю. Архангельский, *Агрегирование в межотраслевом балансе. Экономика и математические методы*, 1966, т. 2, № 6, p. 841—847.
5. H. Theil, *Economics and Information Theory*. Amsterdam, North-Holland Publishing Company, 1967.
6. Р. Белман, С. Дрейфус, *Примерные задачи динамического программирования*. М., 1965.

*Academy of Sciences of the Estonian SSR,  
Institute of Economics*

Received  
July 4, 1969

U. ENNUSTE

### PLANEERIMISSÜSTEEMI MAJANDUSLIKU EFEKTIIVSUSE PÕHIMÕTTEST

#### *Resüme*

Artiklis käsitatakse majandust või tootmist planeerivat süsteemi majandusteaduse uurimisobjektina, sest ühelt poolt on täpsem planeerimine, eriti täpsemate algandmete tagamine, seotud oluliste kuludega, teiselt poolt võimaldab täpsem plaan vähendada mitmesuguseid võimalikke kaotusi ja seega anda täiendavat tulu. Siit kerkibki keerukas planeerimissüsteemi karakteristikute optimaalsete väärtuste määramise probleem.

Majandusliku efektiivsuse kriteeriumist lähtudes püstitatakse artiklis planeerimissüsteemi, öieti plaanikarakteristikute optimaalse määramise ülesandeid, kusjuures kasutatakse mõisteid *andmete ning plaani entroopia, mudeli optimaalne detailsus* jne.

Kogu käsitlus on kvalitatiivne ning illustreerimiseks esitatakse äärmiselt lihtsustatud näiteid. Artikkel aitab seega ainult «tugevdada intuitsiooni» planeerimissüsteemide karakteristikute praktilisel määramisel.

*Eesti NSV Teaduste Akadeemia  
Majanduse Instituut*

Saabus toimetuses  
4. VII 1969

Ю. ЭННУСТЕ

## О ПРИНЦИПЕ ЭКОНОМИЧЕСКОЙ ЭФФЕКТИВНОСТИ СИСТЕМЫ ПЛАНИРОВАНИЯ

Резюме

В статье рассматриваются вопросы определения экономически оптимальных показателей системы планирования. Применяются такие термины, как энтропия данных и плана, детальность плана и т. п.

Анализ поднятых в статье вопросов имеет качественный характер и предусмотрен только для формулировки проблем и «усиления интуиции» при практическом определении характеристик моделей планирования. Приведенные примеры даны лишь как иллюстрации. Для проведения практических расчетов необходимы дальнейшие исследования.

*Институт экономики  
Академии наук Эстонской ССР*

Поступила в редакцию  
4/VII 1969

## JUUBILARE \* ЮБИЛЕИ

### Akadeemik Joosep Saat 70-aastane

Eesti NSV Teaduste Akadeemia akadeemik Joosep Maksimi p. S a a t sündis 30. juulil 1900 Muhu saarel Kapi, praeguses Tupenurme külas ehitustöölise perekonnas.

1916. aastal, kui raske majanduslik olukord oli teda sundinud loobuma südame lähedase eriala õppimisest kunsttööstuskoolis, alustas J. Saat veel samal aastal oma töömeheteed Nõmmküla vallakooli õpetajana. Järgmisel aastal õppis J. Saat telegrafistide kursusel Kuressaares ning lõpetas need sama aasta sügisel. Sügavasti mõjutasid tema edasist elu revolutsioonilise 1917. aasta sündmused. Noore kooliõpetajana võttis ta osa kohaliku madruste klubi tööst, tutvus lähemalt toorkordse poliitiliste vooludega ning temast sai veendunud bolševik.

1917. aasta sügisel, kui Saksa väed tungisid Lääne-Eesti saartele, siirdus J. Saat mandrile ning asus tööle posti-telegraafiringi raudteeametnikuna. Suure Sotsialistliku Oktoobrirevolutsiooni päevil lülitus J. Saat aktiivselt revolutsioonilisse võitlusse, tegutsedes raudteelaste ametiühingus ning tehes kaastööd töölisajalehtedele.

Kodanliku diktatuuri aastail oli J. Saat sihiteadlik revolutsiooniline võitleja. 1921. aastal võeti ta Eestimaa Kommunistliku Partei liikmeks ning valiti partei Järvamaa komitee koosseisu. 1922. aastal asus ta partei ülesandel tööle Tallinna töölisajalehtede toimetusse.

1924. aasta jaanuaris J. Saat arreteeriti ja mõisteti sama aasta sügisel «149 protsessis» ühe kaebelusena eluks ajaks sunnitud tööle. Enam kui 14 vangla-aastat aga ei



murdnud mehist võitlejat. Nagu teisedki poliitvangid, kasutas J. Saat vähimaidki võimalusi teadmiste omandamiseks ja kirjutas artikleid poliitvangide käsikirjalisele häälekandjale «Punane Viisnurk».

Vabanenud vanglast üldise amnestia alusel 1938. aastal, töötas J. Saat ehitustöölisena ning arveametnikuna tööliskooperatii-