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ELASTIC MANIPULATORS, VIBRATION CONTROL

RESEARCH ARTICLE

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Vibration control of a rotating exponentially axially functionally graded flexible beam

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ABSTRACT

Elastic manipulators are commonly used in industrial applications. Therefore, understanding their dynamics is one of the most important engineering challenges. Vibration is a crucial phenomenon due to the elastic nature of the manipulators. The accurate positioning and trajectory tracking of elastic manipulators is achieved through vibration control. In this study, boundary control of an axially exponentially graded flexible manipulator with exponential convergence was investigated. The manipulator was modeled using Euler–Bernoulli beam theory. Boundary control inputs were applied at the boundaries of the manipulator. A proportional-derivative boundary controller was designed, and exponential convergence was achieved. A Lyapunov function was designed for the stability of the system. Equations were solved using the finite difference discretization method. Angle tracking and boundary control inputs were obtained.

1. Introduction

Elastic manipulators are mechanical systems that are frequently used in modern industry. Especially over the last 50 years, with the increasing use of robotics in continuous production systems, the control and dynamic behavior of mechanical elements have gained importance in engineering applications. In this context, controlling the dynamic behavior of rotating elastic manipulators is one of the subjects examined in detail. There are many studies in the literature on the vibration control of rotating manipulators. In the literature, manipulators are examined as flexible, rigid, and flexible-rigid structures (Dwivedy and Eberhard 2006; Kiang et al. 2015; Liu and He 2018; Lee and Alandoli 2020; Subedi et al. 2020). It is known in the control theory that modeling manipulators as a rigid body is insufficient in terms of sensitivity. In recent years, incorporating the elasticity of manipulators has been the preferred approach. Euler–Bernoulli beam theory is mostly used in the modeling of elastic manipulators (Wen et al. 2011; Zhang and Liu 2013; Jiang et al. 2015; Jiang et al. 2017; Liu and Liu 2017; Liu et al. 2017). Manipulator arms can be linked as single, double, or multiple. In the literature, it is observed that many methods are used in flexible manipulator control, including model-dependent and model-free methods. There are some studies where boundary control methods are preferred because inputs are applied only at the boundaries.

Exponential convergence (Jiang et al. 2015), LaSalle analysis (Jiang et al. 2015), state constraints (Jiang et al. 2017), input constraints (Liu et al. 2017) and guaranteed temporal performance as boundary control methods (Liu and Liu 2017) were used. Previous studies have shown that axially functionally graded elastic manipulators have not been examined using the boundary control method. The main contribution of this study is that, to the best of the authors' knowledge, it is the first to examine the controls of axially functionally graded elastic manipulators in the literature. The use of these manipulators has the potential to provide weight and energy savings both in theory and in practice.

In this study, the controls of elastic manipulators with axially graded material properties were examined using the boundary control method. The beam was modeled as an Euler–Bernoulli beam. First, the equations of motion and the boundary conditions for a beam with a mass added at its end point were obtained. Then, control and stability conditions were established using the exponential convergence method.

2. Analysis

Elastic manipulators are widely used in modern industry. In general, they are moved by applying a torque connected to a motor from one end, and they reach a certain position by rotating a predetermined amount. Vibrations occur in the elastic arm due to inertial forces during the initial movement and deceleration at the reached position. Properly reducing and eliminating these vibrations is one of the most important robotic control challenges today. Many methods are used to control elastic manipulator vibrations. In this study, the boundary control method was used. In this method, the vibration control of the elastic manipulator was examined by applying a moment determined by the boundary control method from the fixed end of the manipulator and a force from the free end. In this study, the elastic manipulator was modeled as a rotating clamped elastic beam.

Now, consider a rotating elastic manipulator with length L (Fig.1). Here, XOY and xOy correspond to the global inertia coordinate system and the body-fixed coordinate system attached to the manipulator, respectively. F is the control force provided by the actuator, $\tau(t)$ is the control torque generated by the motor at the shoulder, and $\theta(t)$ is the angular position of the motor. A point mass payload m is attached at the free end of the manipulator.

It was assumed that the material properties (Young modulus E and density ρ) were changing along the length of the beam. The equations of motion and the boundary conditions of the manipulator were obtained using Hamilton's principle (Meirovitch 2001) as follows:

$$\rho(x)\ddot{w}(x,t) = -E(x)Iw_{xxxx} - 2E_x(x)Iw_{xxx} - E_{xx}(x)Iw_{xx}, \tag{1}$$

$$\tau = I_h\ddot{w}_x(x) - E(x)Iw_{xx}(0), \tag{2}$$

$$F = m\ddot{w}_x(L) - E(x)Iw_{xxx}(L) - E_xIw_{xx}, \tag{3}$$

$$w(0) = 0,\ w_x(0) = \theta,\ w_{xx}(L) = 0, \tag{4}$$

where w is the total displacement of the beam, the sub-index denotes the derivative with respect to x and the dot is the time derivative. I_h is the hub inertia. Also, the offset of the robot arm $w(x,t)$ can be written as:

$$w(x,t) = x\theta(t) + y(x,t),\ \ddot{w}(x,t) = x\ddot{\theta} + \ddot{y}(x),\ \ddot{w}(L,t) = L\ddot{\theta} + \ddot{y}(L,t), \tag{5}$$

where y is the elastic deformation of the manipulator. The following equations can be obtained using Eqs (4) and (5): $w_{xx}(x) = y_{xx}(x)$, $\ddot{w}_x(0) = \ddot{\theta}$, $w_{xx}(0) = y_{xx}(0)$, $w_{xx}(L,t) = y_{xx}(L,t)$, $w_{xxx}(L,t) = y_{xxx}(L,t)$. The control goal is:

$$\theta(t) \to \theta_d(t),\ \dot{\theta}(t) \to \dot{\theta}_d(t),\ y(x,t) \to 0,\ \dot{y}(x,t) \to 0. \tag{6}$$

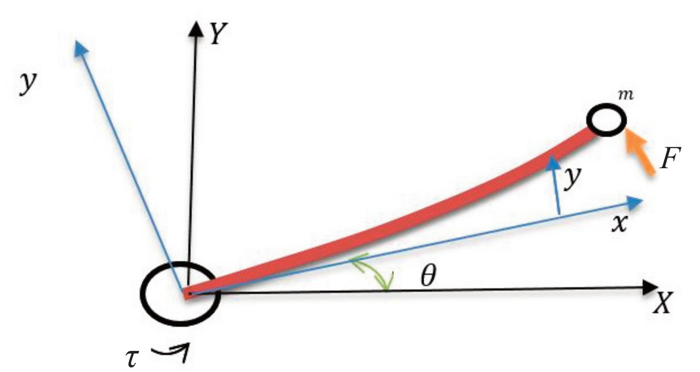

Fig. 1. Rotating elastic manipulator.

In order to achieve the control goal, the following control torque and control force were used:

$$\tau = -K_p e - K_d \dot{e}, \tag{7}$$

$$F = -k u_a + m \dot{w}_{xxx}(L), \tag{8}$$

where K_p (proportional constant), K_d (derivative constant), k are constants, and u_a is an auxiliary variable. The tracking error is defined as $e = \theta - \theta_d$. For stability analysis, the following Lyapunov function was assumed:

$$V(t) = \frac{1}{2}\int_0^L \rho\, \dot{w}^2\, dx + \frac{1}{2}\int_0^L EI y_{xx}^2\, dx + \frac{1}{2} I_h \dot{e}^2 + \frac{1}{2} k_p e^2 + \frac{1}{2} m u_a^2 + \alpha\rho \int_0^L x \dot{w}(x) z e_x(x) dx + \alpha I_h \dot{e}, \tag{9}$$

where E_1 is the sum of the kinetic and potential energy of the manipulator (the first two terms in Eq. (9)), E_2 is the angle tracking error (the third–fifth terms in Eq. (9)), and E_a is the auxiliary function (the last two terms in Eq. (9)).

The positive definiteness of the function $V(t)$ was shown; however, it is not given here due to limited space. Using the time derivative of the Lyapunov function, the following differential equations were obtained:

$$\dot{V}(t) \leq \lambda(E_1 + E_2) \leq \lambda_o \frac{V(t)}{\alpha_3} = \lambda V(t). \tag{10}$$

The solution of Eq. (10) leads to

$$V(t) \leq V(0) e^{-\lambda t}. \tag{11}$$

Since $V(0)$ is limited, $\dot{V}(t)$ converges to 0 exponentially. The solution of the equation of motion was obtained using the finite difference method, and the angular position and deflection equations were obtained. The domain of the beam was divided into ten parts and the time interval was divided into 10 000 steps.

3. Results

In this section, the numerical results for a rotating manipulator are given. The used parameters are $\theta_d = 0.7$ (desired angle in radians), $K_p = 50$, $K_d = 30$, $k = 20$, $m = 0.1$, $L = 1$, $I_h = 1$, $\rho_R = 1$, $\rho_L = 1.5$. The elasticity modulus and the density variations are assumed as

$$E(x) = E_L e^{\beta_1 x}, \ \ \rho(x) = \rho_L e^{\beta_2 x}, \ \ \beta_1 = \frac{1}{L}\ln\left(\frac{E_R}{E_L}\right), \ \ \beta_2 = \frac{1}{L}\ln\left(\frac{\rho_R}{\rho_L}\right), \tag{12}$$

where the sub-indices L and R are the material properties at the left and right ends, respectively, and β_i are the constants. The angular position, angular speed, deflection and deflection rate of the manipulator without control are shown in Fig. 2. At t =1 s, a 100 Nm torque is applied. The vibration of the manipulator can be observed.

The deflection and deflection rate of the manipulator are presented in Fig. 3 for various E_L and E_R values. It is observed that the deflection of the beam is higher near the free end (x = 1) for the first two seconds. After two seconds, control is applied to the beam, and the vibrations cease. The present results are in good agreement with the results by Liu and He (2018).

Figure 4 shows the deformation at L (free end of the beam). It is observed that the deflection $y(L, t)$ is suppressed to zero within three seconds. The highest deflection is obtained for the uniform beam.

The input torque and end force variations are given in Fig. 5. The time interval was chosen as 0.3 seconds in order to observe the effect of the material properties. The initial torque values reach 20 Nm, then decrease to the 0–2 Nm range. The control torque for the E_L = 6, E_R = 3 beam is the highest for the examined materials. The control force initially shows a sharp variation between positive and negative values, then settles into the 0–2 Nm range. The beam with E_R = 6 has the highest control force. This is due to high stiffness at the free end, and high force is required in order to decrease vibrations. The use of axially graded beam may decrease the required control torque, and this leads to energy saving in applications.

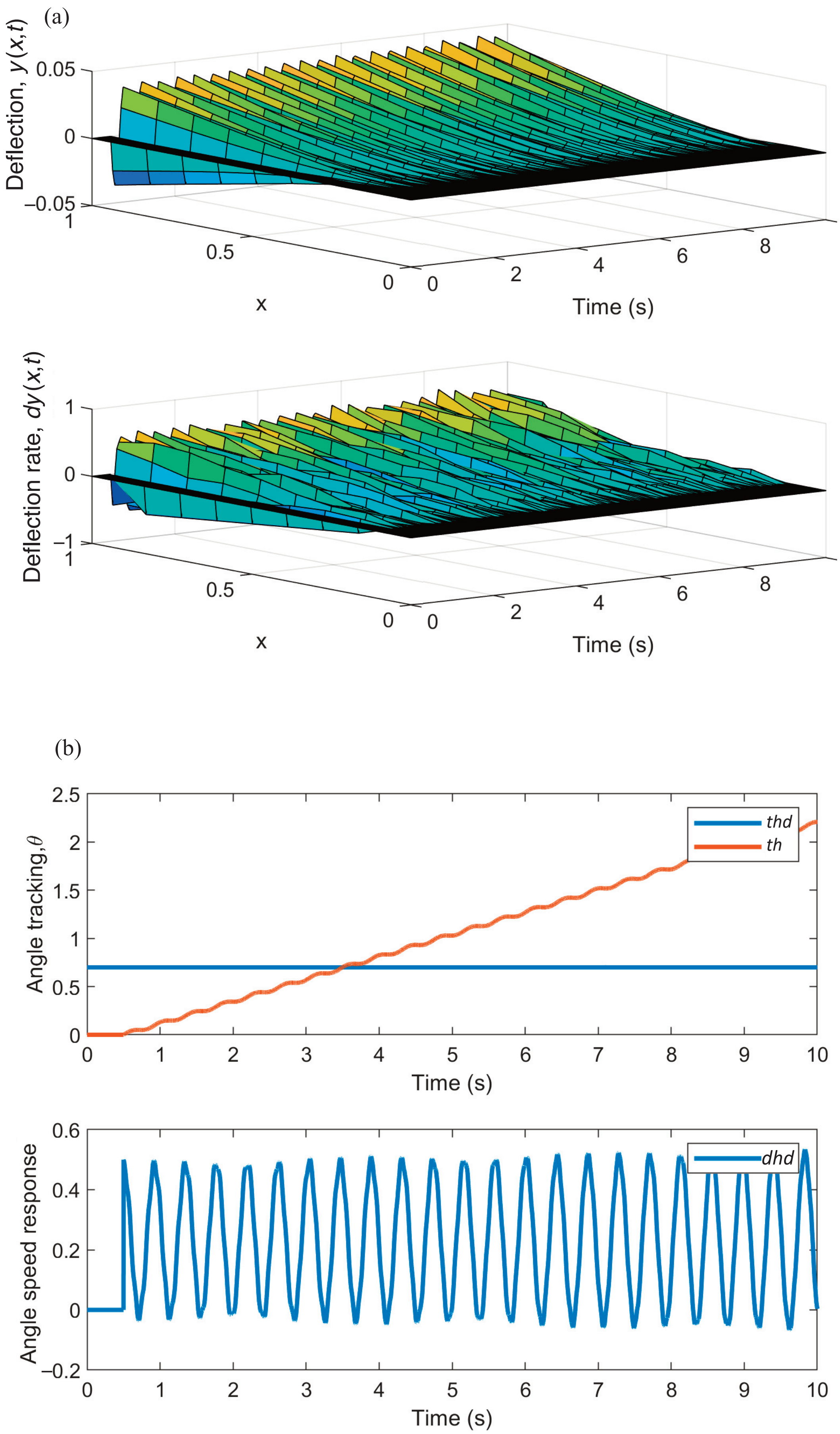

Fig. 2. Deflection and deflection rate (a) and angular tracking, angular speed response (b) of the beam (E_L = 3, E_R = 6).

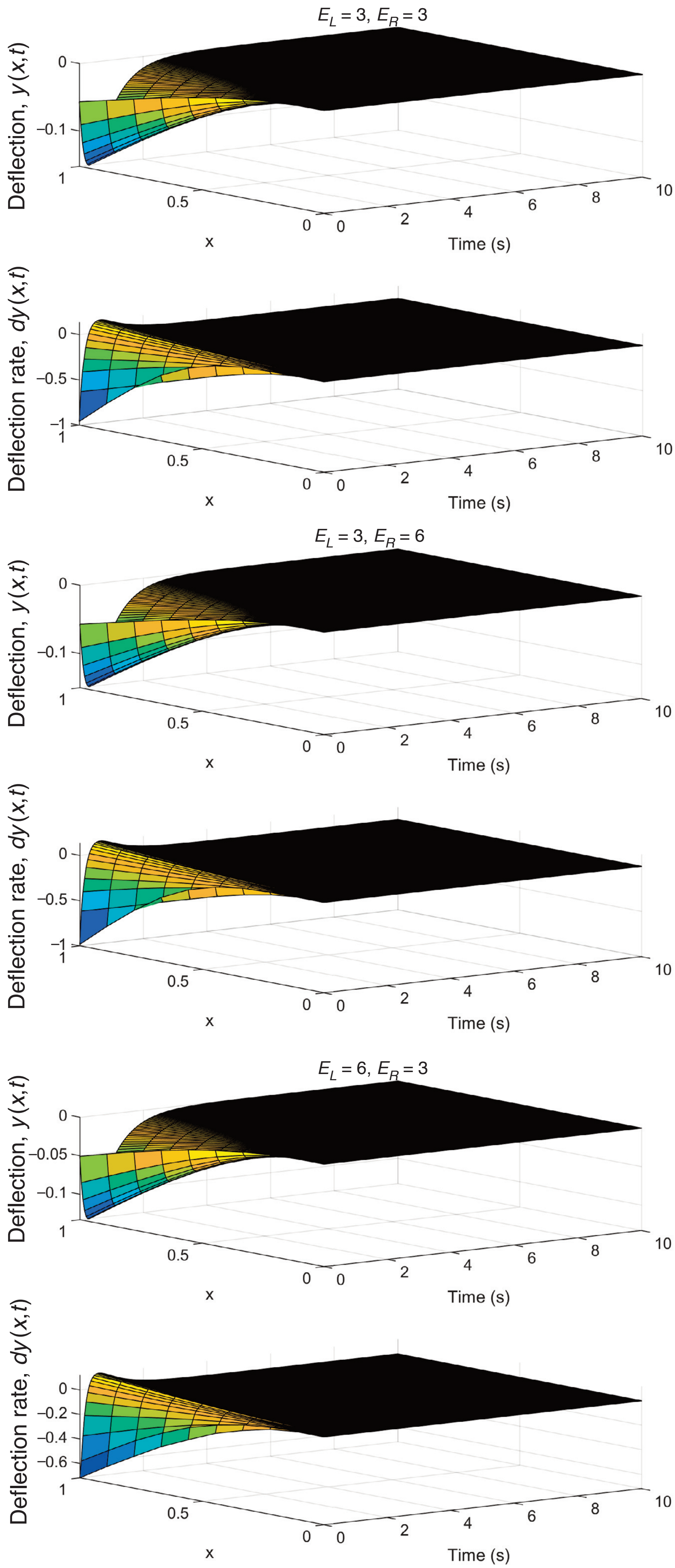

Fig. 3. Deflection and deflection rate of the beam.

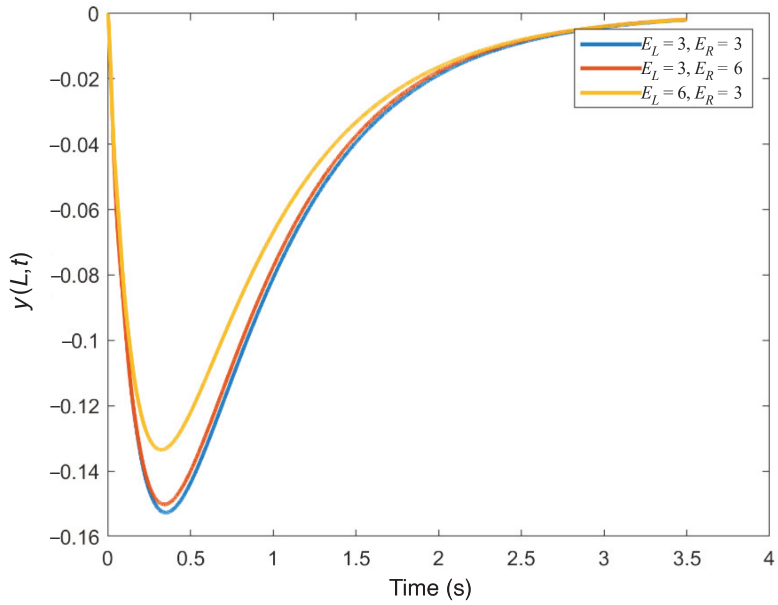

Fig. 4. End point displacement of the manipulator.

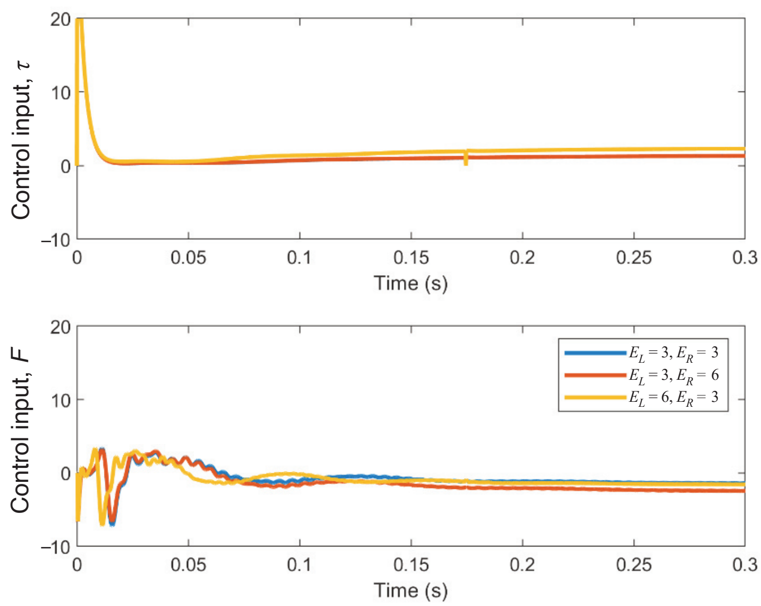

Fig. 5. Boundary control input torque and force.

4. Conclusion

The boundary control of an exponentially axially graded flexible manipulator with exponential convergence was investigated. The beam was modeled using Euler–Bernoulli beam theory. Force and torque boundary control inputs were applied at the boundaries of the manipulator. A proportional-derivative boundary controller was designed and the exponential convergence method was applied. A Lyapunov function was designed for the stability of the system. The equations of motion were solved using the finite difference method. It was found that the deflection of the axially graded beam can be smaller than that of a homogeneous beam, which may provide new design opportunities. Using the present formulation, the control of an axially graded elastic manipulator can be achieved. This study can be extended to other beam theories and multilink manipulators.

Data availability statement

All data are available in the article.

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Telje suunas eksponentsiaalselt gradueeritud pöörleva tala vibratsiooni juhtimine

Feza Eralp Aydogdu ja Bahar Uymaz

Elastseid manipulaatoreid kasutatakse sageli tööstuslikes rakendustes. Seetõttu on nende dünaamika mõistmine inseneriteaduse üks võtmeküsimusi. Vibratsioon on oluline nähtus, mis tuleneb manipulaatorite painduvusest ehk elastsusest. Elastsete manipulaatorite positsiooni ja trajektoori täpne jälgimine saavutatakse vibratsioonijuhtimise abil. Töös uuriti telje suunas eksponentsiaalselt gradueeritud painduva manipulaatori piirijuhtimist. Manipulaator modelleeriti, rakendades Euleri-Bernoulli talade teooriat. Manipulaatori piiridele lisati piirikontrolli sisendid. Koostati eksponentsiaalse koonduvusega proportsionaaltuletise (vastab vea muutumise kiirusele) piirijuhtimisseade. Süsteemi stabiilsuse tagamiseks kasutati Lapunovi funktsiooni. Diferentsiaalvõrrandite süsteem lahendati lõplike vahede meetodi abil. Tulemusena saadi nurga jälgimine ja piirijuhtimise sisendite väärtused.