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Numerical modeling of fragment flight dynamics

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ABSTRACT

To study the flight dynamics of fragments, the following input data are considered: initial coordinates, velocities, air density, the fragment's exposed area, drag coefficient, and mass. The parameters for the fragmentation process are determined through experimental studies and finite element analysis of the natural fragmentation of a high-explosive projectile. The simulation of the natural fragmentation of an explosive projectile shell leverages the finite element method, and stochastic failure theory is applied using the Ansys Autodyn software. The point mass trajectory model is employed to predict the trajectory of a fragment moving under the impact of drag and gravitational force. In the current study, the main focus was on the development of the methods and tools for implementing trajectory models with varying drag coefficients for different flow speeds. Different approaches for determining drag coefficient are discussed. The nonlinear trajectory model was converted to a linear system of differential equations by employing quasi-linearization. The linear system of differential equations was solved using the Haar wavelet method. The fragment trajectory model with improved accuracy can be considered as the final result of the study.

1. Introduction

The study of fragment flight dynamics offers a means to evaluate the hazards posed by rapidly moving fragments dispersed into the surroundings. The associated risks are contingent on the fragment density per unit volume and the kinetic energy of the fragment under investigation at the specific location. The initial fragmentation parameters are dictated by the object causing the fragmentation and the nature of its formation. Fragmenting objects can range from fuel tanks and explosive devices to vehicle parts, with the formation typically resulting from explosions, collisions, or fractures.

The study of fragments resulting from explosions covers simulations, experiments, and statistical models. Collecting the data required for fragment flight from experiments, such as mass, dimensions, velocities, accelerations, and direction vectors, is both resource- and labor-intensive [1–3]. On the other hand, statistical models, while useful in certain scenarios based on specific experiments, may not always be suitable for a given case [4].

Various numerical methods have been introduced for the analysis of the fragments of metallic objects, including the probabilistic mass method by Djelosevic and Tepic [5], the arbitrary Lagrangian–Eulerian approach by Ahmed et al. [2], and the stochastic failure theory adapted by Ugrčić [6]. The results from these simulations can serve as initial data for the point mass trajectory model, which is described by a nonlinear system of ordinary differential equations (ODE). To solve the nonlinear ODE system, Kljuno and Catovic [7] as well as Szmelter and Lee [8] employed the Runge–Kutta method, while Djelosevic and Tepic [5] utilized the Taylor series-based method.

The drag coefficient plays a crucial role in determining the trajectory of fragments due to its direct effect on the aerodynamic resistance experienced by a fragment moving through the air. Experimental studies, finite element simulations, and numerical simulations have demonstrated that the drag coefficient value is directly related to flying object variables, such as shape, size, and roughness [4,9–16]. Fragment shape plays a crucial role in determining the drag coefficient due to its impact on flow separation and turbulence generation around the fragment [7]. Irregularly shaped fragments, typical of high-explosive projectile breakups, exhibit complex aerodynamic behavior compared to standard geometric shapes, such as spheres or cylinders [8]. Recent studies, including those by Hu et al. [17] and Seltner et al. [18], have refined empirical models for drag coefficient estimation under varying flow conditions, expanding on foundational work by Mott [11] and Grady and Kipp [12].

In the current study, the finite element model developed for the simulation of natural fragmentation of a high-explosive projectile is combined with the Haar wavelet approach for the solution of the nonlinear trajectory model.

2. Methodology

To analyze the flight dynamics of fragments, first, it is necessary to establish certain parameters. These include the initial coordinates, velocities relative to these coordinates, air density, the fragment's exposed area, drag coefficient, and mass. The parameters for the fragmentation process are determined through an experimental study and finite element analysis of the natural fragmentation of a high-explosive projectile.

The simulation of the natural fragmentation of an explosive projectile shell leverages the finite element method and stochastic failure theory and is carried out using the Ansys Autodyn software. The arbitrary Lagrange–Eulerian approach, in conjunction with the Johnson–Cook strength and fracture method, is employed to simulate fragmentation and the subsequent dispersion of fragments into the surrounding air. Numerical analysis is performed to ascertain the fragment's initial position, velocity, mass, and volume. The simulation's coordinate system is derived from the computer-aided design model and is transformed to align with the situation's coordinate system. Figure 1 provides a visualization of the simulation's geometry, the coordinate system, and the explosion-induced scattering of fragments.

In the simulation, a projectile with a casing that weighed 38 kg and had a diameter of 155 mm was investigated. The simulation spanned a duration of 0.43 ms, during which roughly 2500 fragments were formed. These fragments had masses varying from 0.3×10^{-5} kg to 0.5 kg, velocities in the range of 250 m/s to 1400 m/s, and volumes from 3 mm^3 up to 6.6×10^4 mm^3. The specific parameters for chosen fragments can be found in Table 1. The fragments with high kinetic energy were selected to ensure sufficient flight path.

The effects of drag and gravitational forces dictate the movement of fragments through the air. This movement can be modeled by employing the point mass trajectory model based on Rayleigh's drag equation [4,13,19].

$$x'' = -\frac{A\rho C_D}{2m} V \cdot x', \; y'' = -\frac{A\rho C_D}{2m} V \cdot y', \; z'' = -\frac{A\rho C_D}{2m} V \cdot z' - g, \tag{1}$$

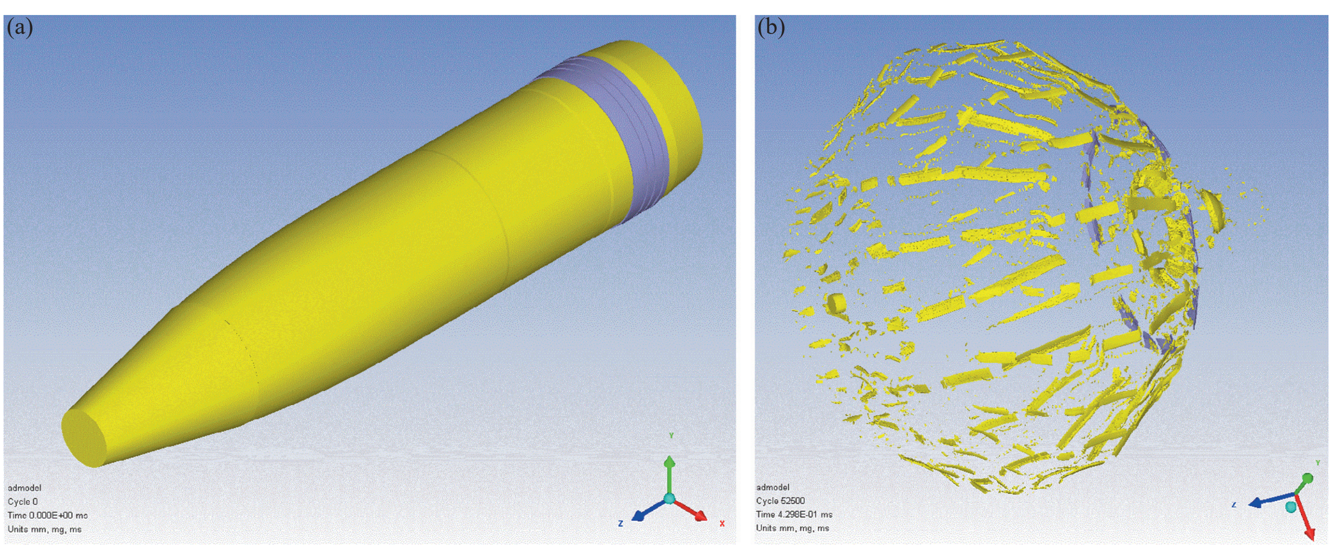

Fig. 1. (a) Projectile shell and (b) shell fragmentation.

Table 1. The initial position and velocities of selected fragments; unit system (m, kg, s)

Fragment No.	x_0	y_0	z_0	x_0'	y_0'	z_0'	Mass	Volume
1	–0.175	–0.384	1.093	–101.9	–994.1	444.9	6.88E-04	8.95E-06
2	–0.671	0.434	0.849	–337.4	1000.9	346.0	5.36E-04	6.87E-06
3	–0.621	0.415	1.121	–470.2	995.8	879.7	6.13E-04	7.85E-06
4	–0.622	–0.415	1.120	–471.4	–997.2	879.1	6.13E-04	7.85E-06
5	–0.455	0.569	0.840	–78.9	1353.4	203.5	5.06E-04	6.48E-06
6	–0.531	0.533	0.794	–156.4	1246.47	149.1	4.77E-04	6.11E-06
7	–0.176	0.351	1.134	–156.1	916.1	518.7	3.89E-04	4.98E-06
8	–0.576	–0.102	1.295	–548.1	–249.5	1210.6	1.40E-03	1.79E-05

where V is the velocity of the fragment and equates to $\sqrt{x'^2 + y'^2 + z'^2}$ in which x', y' and z' are velocities in each direction. The apostrophe ($'$) is defined as the derivative of a function with respect to time. Also, ρ is the air density, g is the gravitational acceleration, and C_D is the drag coefficient, which can be presented as

$$C_D = \frac{2F_D}{A\rho V^2}, \tag{2}$$

where F_D is the drag force, which acts on fragments due to their relative motion against the surrounding air. For accurate modeling, C_D must be determined based on fragment shape, speed, flow regime, and orientation relative to the airflow. However, in the case of irregularly shaped fragments, determining an accurate function to represent C_D could be quite challenging; thus, estimating the drag coefficient value range could be the next best option in determining the distance that a fragment will travel at any moment.

Irregular fragments from explosive events often fall into shape categories such as parallelepiped, box, wedge, and mountain ridge geometries, each with varying C_D profiles based on experimental data. Each shape category has a unique drag coefficient range, which varies based on how the fragment interacts with airflow [4]. Fragments with a larger frontal area perpendicular to the flow direction generally experience higher drag coefficients.

After assigning a value for the drag coefficient, the nonlinear system of differential equations presented in Eq. (1) is transformed into a linear one by applying the quasilinearization technique. In this study, the Taylor series expansion has been utilized for this purpose [20]. The linear system of differential equations is solved using the Haar wavelet method (HWM) [21–23], and the influence of the variation C_D on fragment trajectory is analyzed. Future studies are planned to refine the modeling and design of the fragment by utilizing powerful AI-based methods and tools [24–29].

3. Results

This section investigates the effect of the drag coefficient on the fragment trajectory. For this study, the value of the air density is chosen to be 1.20 kg/m^3 and the gravitational acceleration 9.81 kg/s^2. For two fragments, fragment No. 1 and No. 6, with the initial values presented in Table 1, the effect of various drag coefficient values is presented in Table 2. According to the initial values, the fragments start traveling through the air at supersonic speeds, and the drag coefficient values are chosen based on [4] in the range of 0.6 to 2.07.

As can be observed, the drag coefficient can significantly change the location and speed of the fragment after 2.5 seconds. As expected, by increasing the drag coefficient value, the fragments slow down faster and remain closer to the initial point. This assessment could be a crucial aspect of the safety analysis. This matter is presented visually in Fig. 2.

It should be mentioned that the fragments start within a supersonic speed range and fall into the subsonic range. Hence, it would be more accurate to consider the drag coefficient as a function that is dependent on the Mach number and fragment size. However, introducing the drag coefficient as a function will create more complexity to the problem due to the irregularity of the fragment's shape and roughness.

Table 2. Position and velocities of two fragments with various drag coefficient values at t = 2.5 s

	C_D	x	y	z	x'	y'	z'
Fragment No. 1	0.6	–82.7056	–806.6848	342.1496	–14.5318	–142.1023	49.2556
	1.27	–52.5861	–513.6224	211.9643	–7.4630	–73.2175	19.1801
	1.45	–48.2381	–471.3466	193.1371	–6.6050	–64.8564	15.5232
	1.72	–43.0582	–420.9962	170.6929	–5.6356	–55.4098	11.3882
	2.07	–37.9570	–371.4284	148.5714	–4.7369	–46.6525	7.5510
Fragment No. 6	0.6	–108.2442	863.1932	85.4217	–17.3385	139.2097	2.91141
	1.27	–67.0990	537.4383	47.5523	–8.6303	69.9509	–4.7025
	1.45	–61.2927	491.5597	42.1939	–7.5927	61.6974	–5.6079
	1.72	–54.4169	437.2727	35.8440	–6.4252	52.4097	–6.6243
	2.07	–47.6887	384.2037	29.6261	–5.3474	43.8335	–7.5581

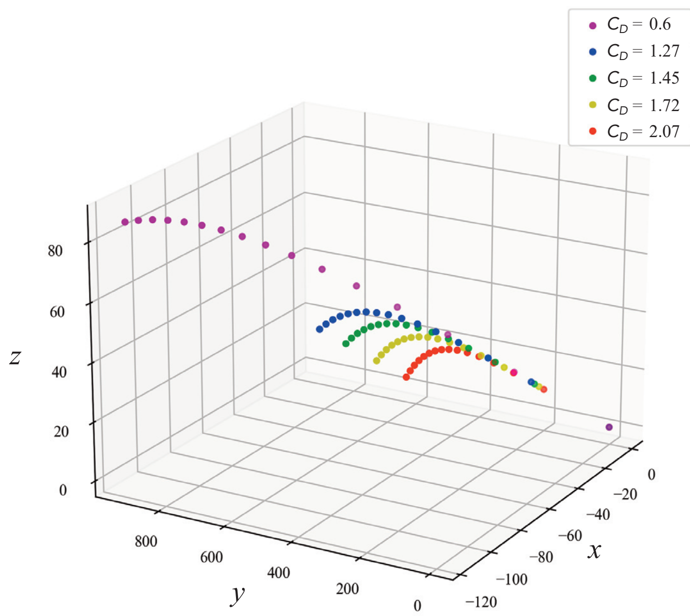

Fig. 2. Trajectory model of fragment No. 6 for different drag coefficient values.

4. Conclusion

The numerical modeling of fragment flight dynamics is performed, covering the whole cycle from experimental study and finite element analysis of the natural fragmentation of a high-explosive projectile to the development of a nonlinear trajectory model, quasilinearization, and the solution of a system of linear differential equations using the Haar wavelet method. The results obtained for different drag coefficient values are visualized in Fig. 2.

The planned future studies include the development and implementation of artificial neural networks and Haar wavelet-based models for describing the behavior of the drag coefficient.

Data availability statement

All research data are contained within the article and can be shared upon request from the authors.

Acknowledgments

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References

1. Angel, J. Methodology for dynamic characterization of fragmenting warheads. Presented at *Guns and Missiles Symposium, New Orleans, LA, 22 April 2008*.
2. Ahmed, K., Malik, A. Q., Hussain, A., Ahmad, I. R. and Ahmad, I. Blast and fragmentation studies of a scaled down artillery shell-simulation and experimental approaches. *Int. J. Multiphys.*, 2021, **15**(1), 49–72.
3. Arnold, W. and Rottenkolber, E. Fragment mass distribution of metal cased explosive charges. *Int. J. Impact Eng.*, 2008, **35**(12), 1393–1398. https://doi.org/10.1016/j.ijimpeng.2008.07.049
4. Zecevic, B., Terzic, J., Catovic, A. and Serdarevic-Kadic, S. Characterization of distribution parameters of fragment mass and number for conventional projectiles. In *New Trends in Research of Energetic Materials, Czech Republic, 2011.*
5. Djelosevic, M. and Tepic, G. Probabilistic simulation model of fragmentation risk. *J. Loss Prev. Process Ind.*, 2019, **60**, 53–75. https://doi.org/10.1016/j.jlp.2019.04.003
6. Ugrčić, M. and Ivanišević, M. Characterization of the natural fragmentation of explosive ordnance using the numerical techniques based on the FEM. *Sci. Tech. Rev.*, 2015, **65**(4), 16–27.
7. Kljuno, E. and Catovic, A. Trajectory estimation model for a solid body with an irregular shape undergoing extremely high aerodynamic forces. *Period. Eng. Nat. Sci.*, 2021, **9**(2), 545–568.
8. Szmelter, J. and Lee, C. K. Prediction of fragment distribution and trajectories of exploding shells. *J. Battlef. Technol.*, 2007, **10**(3), 1–6.
9. Zhang, F., He, Y., Xie, W., Wei, N., Li, J., Wang, S. et al. Drag coefficients for elongated/flattened irregular particles based on particle-resolved direct numerical simulation. *Powder Technol.*, 2023, **418**, 118290. https://doi.org/10.1016/j.powtec.2023.118290
10. McCleskey, F. Drag coefficients for irregular fragments. *Naval Surface Warfare Center*, 1988, **87**.
11. Mott, N. F. Fragmentation of shell cases. *Proc. R. Soc. Lond. A*, 1947, **189**, 300–308. https://doi.org/10.1098/rspa.1947.0042
12. Grady, D. E. and Kipp, M. E. Mechanisms of dynamic fragmentation: factors governing fragment size. *Mech. Mater.*, 1985, **4**(3–4), 311–320. https://doi.org/10.1016/0167-6636(85)90028-6
13. Moxnes, J. F., Frøyland, Ø., Øye, I. J., Brate, T. I., Friis, E., Ødegårdstuen, G. et al. Projected area and drag coefficient of high velocity irregular fragments that rotate or tumble. *Def. Technol.*, 2017, **13**(4), 269–280. https://doi.org/10.1016/j.dt.2017.03.008
14. Catovic, A., Zečević, B., Kadic, S. S. and Terzic, J. Numerical simulations for prediction of aerodynamic drag on high velocity fragments from naturally fragmenting high explosive warheads. In *New Trends in Research of Energetic Materials, Pardubice, Czech Republic, 18–20 April 2012.*
15. Li, Y., Yu, Q., Gao, S. and Flemming, B. W. Settling velocity and drag coefficient of platy shell fragments. *Sedimentology*, 2020, **67**(4), 2095–2110. https://doi.org/10.1111/sed.12696
16. Gubinelli, G. and Cozzani, V. Assessment of missile hazards: evaluation of the fragment number and drag factors. *J. Hazard. Mater.*, 2009, **161**(1), 439–449. https://doi.org/10.1016/j.jhazmat.2008.03.116
17. Hu, J., Chen, H., Yu, Y., Xue, X., Feng, Z. and Chen, X. Experimental study on motion law of the fragment at hypersonic speed. *Processes*, 2023, **11**(4), 1078. https://doi.org/10.3390/pr11041078
18. Seltner, P. M., Willems, S., Gülhan, A., Stern, E. C., Brock, J. M. and Aftosmis, M. J. Aerodynamics of inclined cylindrical bodies free-flying in a hypersonic flowfield. *Exp. Fluids*, 2021, **62**(9), 182. https://doi.org/10.1007/s00348-021-03269-6
19. Kivistik, L., Eerme, M., Majak, J. and Kirs, M. Numerical analysis of dynamics of flight of the fragments. *AIP Conf. Proc.*, 2024, **2989**(1), 030016. https://doi.org/10.1063/5.0189331
20. Kivistik, L., Mehrparvar, M., Eerme, M. and Majak, J. Dynamics of flight of the fragments with higher order Haar wavelet method. *Proc. Estonian Acad. Sci.*, 2024, **73**(2), 108–115. https://doi.org/10.3176/proc.2024.2.02
21. Arda, M., Majak, J. and Mehrparvar, M. Longitudinal wave propagation in axially graded Raylegh–Bishop nanorods. *Mech. Compos. Mater.*, 2024, **59**(6), 1109–1128. https://doi.org/10.1007/s11029-023-10160-4
22. Ratas, M., Majak, J. and Salupere, A. Solving nonlinear boundary value problems using the higher order Haar wavelet method. *Mathematics*, 2021, **9**(21), 2809. https://doi.org/10.3390/math9212809
23. Mehrparvar, M., Majak, J., Karjust, K. and Arda, M. Free vibration analysis of tapered Timoshenko beam with higher order Haar wavelet method. *Proc. Estonian Acad. Sci.*, 2022, **71**(1), 77–83. https://doi.org/10.3176/proc.2022.1.07
24. Abouzaid, K., Bassir, D., Guessasma, S. and Yue, H. Modelling the process of fused deposition modelling and the effect of temperature on the mechanical, roughness, and porosity properties of resulting composite products. *Mech. Compos. Mater.*, 2021, **56**(6), 805–816. https://doi.org/10.1007/s11029-021-09925-6
25. Lee, J. K. and Lee, B. K. Coupled flexural-torsional free vibration of an axially functionally graded circular curved beam. *Mech. Compos. Mater.*, 2022, **57**(6), 833–846. https://doi.org/10.1007/s11029-022-10003-8
26. Teng, S., Chen, G., Yan, Z., Cheng, L. and Bassir, D. Vibration-based structural damage detection using 1-D convolutional neural network and transfer learning. *Struct. Health Monit.*, 2023, **22**(4), 2888–2909. https://doi.org/10.1177/14759217221137931
27. Bassir, D., Lodge, H., Chang, H., Majak, J. and Chen, G. Application of artificial intelligence and machine learning for BIM: review. *Int. J. Simul. Multidisci. Des. Optim.*, 2023, **14**, 5. https://doi.org/10.1051/smdo/2023005
28. Xiao, Y., Li, Z., Liu, Z., Zang, M. and Zhu, Y. Effect of material design and weak link setting on the energy absorption of composite thin-walled beams under transverse loading. *Mech. Compos. Mater.*, 2021, **57**(3), 401–414. https://doi.org/10.1007/s11029-021-09963-0
29. Yan, Z., Jin, Z., Teng, S., Chen, G. and Bassir, D. Measurement of bridge vibration by UAVs combined with CNN and KLT optical-flow method. *Appl. Sci.*, 2022, **12**(10), 5181. https://doi.org/10.3390/app12105181

Fragmentide lennudünaamika numbriline modelleerimine

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Töös uuritakse fragmentide lennudünaamikat alates fragmentatsiooniprotsessi modelleerimisest kuni lennutrajektoori määramiseni. Fragmentatsiooniprotsessi parameetrid on määratud eksperimentaalselt. Protsessi simuleerimiseks on kasutatud lõplike elementide meetodit rakendustarkvaras Ansys Autodyn. Fragmentatsiooni ja sellele järgneva fragmentide ümbritsevasse õhku hajumise modelleerimiseks on rakendatud Lagrange–Euleri meetodit koos Johnson–Cooki tugevus- ja purunemismudelitega. Fragmendi trajektoori modelleerimisel on kasutatud punktmassi trajektoorimudelit. Mittelineaarne trajektoorimudel on teisendatud lineaarseks diferentsiaalvõrrandite süsteemiks kvasi-lineariseerimise abil. Lineaarse diferentsiaalvõrrandite süsteemi lahendamiseks on rakendatud Haari lainikute meetodit. Uuring keskendub meetodite ja tööriistade arendamisele trajektoorimudelite rakendamiseks, kus takistustegur varieerub sõltuvalt voolukiirusest. Analüüsitakse erinevaid lähenemisviise takistusteguri määramiseks. Lõpptulemusena esitatakse täpsem fragmendi trajektoorimudel.