A comparative analysis of Fuzzy AHP and Fuzzy VIKOR methods for prioritization of the risk criteria of an autonomous vehicle system

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Abstract. In the current study, two widely used multi-criteria decision-making methods, the Fuzzy analytic hierarchy process (AHP) and the Fuzzy VIKOR method, have been implemented to prioritize the criteria of a multi-criteria decision-making problem. Herein, the case study is an autonomous vehicle, the TalTech iseAuto AV shuttle, developed at TalTech University. The criteria of the present problem are evaluated by experts, and after forming the pairwise matrices, these matrices are aggregated by the max-min method with the arithmetic mean. Subsequently, in the case of Fuzzy AHP, by calculating the weights and normalizing them, the relative importance of each criterion is obtained, which leads to the ranking of the criteria. Moreover, in the case of the Fuzzy VIKOR method, the aggregated pairwise matrix is weighted and normalized. The ranking obtained from both methods is presented and compared. The advantages and disadvantages of the multi-criteria decision-making methods Fuzzy AHP and VIKOR, featured for risk analysis of the autonomous vehicle systems, are discussed.

Keywords: multi-criteria decision-making problem, Fuzzy analytic hierarchy process (AHP), Fuzzy VIKOR method, prioritization of criteria, autonomous vehicle (AV).

INTRODUCTION

The importance of studying multi-criteria decision-making (MCDM) problems cannot be underestimated in our increasingly complex world of decision-making. In the context of decision processes, such problems arise due to simultaneous consideration of multiple, often conflicting criteria. For a systematic evaluation of options, MCDM methods provide an approach where the diverse qualitative and numerical aspects are considered. Their relevance extends to a wide variety of fields, from business and engineering to environmental management and health care. These criteria are often not of equal importance, and the performance of alternatives is quite different. Formal methods are necessary to provide for a structured decision-making process. A number of techniques have been introduced for handling multiple criteria, for instance, evolutionary optimization [1–4], the analytic hierarchy process (AHP), the technique for order of preference by similarity to ideal solution (TOPSIS), and the vlsekriterijumska optimizacija i kompromisno resenje (multi-criteria optimization and compromise solution – VIKOR) method [5–9]. In order to ascertain the efficacy of two well-known MCDM techniques in terms of ranking these important criteria in the context of autonomous vehicle systems, this paper compares the AHP and the VIKOR methods.

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Fuzzy sets, first introduced by Zadeh [10], have been widely used in conjunction with Fuzzy AHP, which was introduced by Saaty [11], to assess complicated multi-criteria situations. For example, Kaganski et al. [12] assigned priority indexes to metrics using Fuzzy AHP based on the SMARTER (simple multi-attribute rating technique exploiting ranks) criterion. In addition, to improve the management of product data or lifecycle management, Paavel et al. [13] used Fuzzy AHP. However, VIKOR, which was introduced in 1998 by Opricovic [14], is notable for its capacity to provide solutions that strike a compromise when faced with multi-criteria decision-making challenges. Since then, many studies have implemented this method in many areas, such as the energy sector [15], health [16], education [17,18], manufacturing, and industry [19,20]. The purpose of this comparison study is to shed light on whether these methods are appropriate for handling the difficulties connected to determining how criteria should be prioritized in the context of autonomous vehicle systems. Over the years, many investigations have been conducted to compare and review the VIKOR method with other methods [21–24].

The autonomous vehicle known as the AV shuttle, which was developed by Astrov et al. [25], Sell and Petritsenko [26], and Rassõlkin et al. [27], serves as the case study for this investigation. Pikner et al. [28,29] provide a detailed introduction to the criteria and risks of such a vehicle. In particular, [29] examines the risk analysis of this model by combining Fuzzy AHP with TOPSIS.

In the current study, the Fuzzy AHP and VIKOR methods are configured/tuned for risk evaluation of the autonomous vehicle systems. In the first step, the criteria are introduced and linguistically evaluated by experts, which leads to the formation of pairwise matrices. These matrices are aggregated by the max-min method with the arithmetic mean, and after obtaining the final pairwise matrix, two different paths are followed according to the two multi-criteria decision-making methods.

In the case of the Fuzzy AHP method, by implementing the arithmetic mean as the aggregation method, the fuzzy weights are calculated and, after normalization, can be sorted to obtain the ranking of criteria.

In the case of the Fuzzy VIKOR method, a linear normalization is used to stabilize the problem, and then the utility measures are calculated. Consequently, the VIKOR scores are calculated and sorted to obtain the rank of each criterion.

**FORMULATION DEVELOPMENT FOR THE MCDM PROBLEM**

As mentioned before, the current study is an investigation in continuation of a previous work by Pikner et al. [29]; in that work the criteria for the mobile robot were introduced, which will be considered here. There are seven criteria for the current problem: mission computer and AI performance (C1), cybersecurity (C2), malfunction of an AV mechanical component (C3), sensor system (C4), communication link (C5), weather factors (C6), and low-level cyber-physical system performance (C7).

After establishing the criteria involved in the system, the criteria are linguistically evaluated by experts, and the relative importance of the criteria in terms of linguistics variables from the least important to the most important is equated to the numerical values from 1 to 9, crisp AHP scale. Then, the scales are translated into triangular fuzzy numbers (TFN) [12]. Thus, each expert evaluates the importance of the criteria against each other and forms a pairwise comparison matrix linguistically, which then is transformed according to the numerical values.

The experts can indicate that a criterion is equally preferred, or equally to moderately preferred, or moderately preferred, or moderately to strongly preferred, or strongly preferred, or strongly to very strongly preferred, or very strongly preferred, or very strongly to extremely preferred, or extremely preferred to another criterion.

Since each expert could have a unique opinion regarding the importance of the criteria, these pairwise matrices are going to be different, and in order to form a final pairwise matrix, an appropriate aggregation method is utilized. The upper and lower bounds of the aggregated TFN, \( h \) and \( l \), are chosen by the max-min method, and the middle-value \( m \) is calculated by the arithmetic mean. The aggregated fuzzy number \( X_{ij} = (l_{ij}, m_{ij}, h_{ij}) \), the relative importance of criterion \( X_i \) over criterion \( X_j \), can be calculated by the max-min method with the arithmetic mean in Eq. (1):

\[
X_{ij} = (l_{ij}, m_{ij}, h_{ij})
\]
\[ h_{ij} = \max_{t=1,2,...,q} \left( h_{ij}^{(t)} \right), \]
\[ m_{ij} = \frac{1}{q} \sum_{t=1}^{q} m_{ij}^{(t)}, \]
\[ l_{ij} = \min_{t=1,2,...,q} \left( l_{ij}^{(t)} \right), \]

\[ (1) \]

where \( q \) is the number of experts. First, the final pairwise comparison matrix of criteria is determined. Next, two various methods for solving the MCDM problem are implemented to rank the criteria. Below, the Fuzzy AHP and the Fuzzy VIKOR methods are explored. Figure 1 shows the process of both methods in one glance. As can be observed, both methods’ processes are the same until the calculation of the aggregated pairwise matrix, after which the differentiation of these methods begins.

**The analytic hierarchy process (AHP)**

AHP, as described before, is a systematic and structured way of making decisions by breaking down the problem into pairwise comparisons and then aggregating the results [5]. After obtaining the final pairwise matrix in Eq. (2), the weights are calculated by utilizing the arithmetic mean as the aggregation method, as shown in Eq. (3).

\[
C = \begin{bmatrix}
  x_{11} & x_{12} & \cdots & x_{1n} \\
  x_{21} & x_{22} & \cdots & x_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{m1} & x_{m2} & \cdots & x_{mn}
\end{bmatrix}, \quad i = 1,2,...,m; j = 1,2,...,n. \tag{2}
\]
To conclude, the centroid method is used to defuzzify the fuzzy weight, and the crisp weight is calculated as such:

$$w_i = \frac{x_i}{\sum_{j=1}^{n} x_j}$$

which is normalized as such:

$$x_i = \frac{1}{n} (x_{i1} \oplus x_{i2} \oplus \cdots \oplus x_{in}).$$

The VIKOR method

As previously stated, this method determines the compromise solution and the best solution from a set of alternatives. The compromise solution will be presented by comparing the degree of closeness to the ideal alternative, and each alternative can be evaluated by each criterion function [21]. A systematic approach of the Fuzzy VIKOR method for multi-criteria in a fuzziness environment is given in this section, based on [6,8,16].

At first, the fuzzy pairwise matrix in Eq. (2) needs to get normalized in order to ensure that all criteria are standard and comparable to each other. Here, linear normalization is utilized for stabilisation, which is provided by $R_{ij}$ as follows:

$$R_{ij} = \left(\frac{x_{ij} - x_{ij}^{-}}{x_{ij}^{-} - x_{ij}^{+}}\right),$$

where $R_{ij}$ is the normalized score for the alternative $i$ of the $j$th criteria, $x_{ij}$ is the value of the $j$th criteria for the alternative $i$, and the best value of the $j$th criteria is $x_{ij}^{+}$ and the worst value $x_{ij}^{-}$.

The utility measures will be calculated at this stage:

$$S_{i} = \sum_{j=1}^{n} w_{j} R_{ij},$$

$$R_{i} = \max_{j}[w_{j} R_{ij}],$$

where $w_{j}$ is the weight of the $j$th criteria, and $S_{i}$ and $R_{i}$ are the alternative distance values to positive and negative ideal solutions, respectively. Hence, the VIKOR score $Q_{i}$ is obtained by

$$Q_{i} = \nu \left(\frac{S_{i} - S^{-}}{S^{+} - S^{-}}\right) + (1 - \nu) \left(\frac{R_{i} - R^{-}}{R^{+} - R^{-}}\right),$$

where

$$S^{+} = \max_{i} S_{i}, \quad S^{-} = \min_{i} S_{i},$$

$$R^{+} = \max_{i} R_{i}, \quad R^{-} = \min_{i} R_{i}.$$
Also, the parameter $v$ is the strategy of the maximum utility, which is a value between 0 to 1. In this paper, this parameter is chosen to be 0.5.

Finally, the defuzzification is done for parameters $S$, $R$, and $Q$, and ranking is done based on the obtained value for $Q$. The smaller the VIKOR score value, the better the alternative solution.

NUMERICAL RESULTS AND DISCUSSION

In this section, the final results of the ranking of the criteria are presented. For the system with seven criteria mentioned above, six experts give their evaluations of the criteria, which are then transformed into a pairwise comparison matrix [29]. Then the linguistic values are transferred to TFN according to [28]. The aggregated evaluation matrix is obtained by the max-min method with the arithmetic mean. First, to study the results of the Fuzzy AHP method, the fuzzy weights and crisp weights are calculated, and the criteria are ranked from the most important to the least important. The results are validated with results from the literature (Table 1).

As can be observed, the results show a good agreement with each other. For this case study, the final values of $S$, $R$, and $Q$ are presented in Table 2. In the Fuzzy VIKOR method, the ranking of the VIKOR score is done so that the smallest value refers to the most important criteria and the largest value to the least important criteria.

It is worth mentioning that these results are for $v = 0.5$, and, as mentioned before, this value is chosen according to the literature. In the case of higher values, the value of $Q$ will tend toward a majority agreement and, consequently, when it has lower values, $Q$ will tend toward a majority negative attitude [24].

As can be seen in Table 2, the parameter values, both fuzzy and crisp, can get extremely close, and extra care is needed to ensure the accuracy of final values and ranking.

Table 1. Weights and ranking of criteria based on Fuzzy AHP and the results from [29]

| Criteria | Fuzzy AHP | | | | |
|----------|-----------|-----------|-----------|-----------|
|          | Aggregated fuzzy comparison value | Crisp weights | Final rank | Aggregated fuzzy comparison value | Crisp weights | Final rank |
| C1       | (0.37, 1.03, 2.64) | 0.08940 | 7 | (0.51, 0.60, 0.71) | 0.076 | 6 |
| C2       | (0.39, 1.99, 5.14) | 0.14225 | 4 | (0.86, 1.07, 1.34) | 0.137 | 4 |
| C3       | (0.42, 2.21, 5.29) | 0.15139 | 3 | (0.98, 1.19, 1.46) | 0.151 | 3 |
| C4       | (0.94, 3.07, 6.57) | 0.23719 | 1 | (1.83, 2.17, 2.50) | 0.268 | 1 |
| C5       | (0.28, 0.99, 3.86) | 0.09187 | 6 | (0.44, 0.52, 0.65) | 0.067 | 7 |
| C6       | (0.50, 0.99, 2.29) | 0.09726 | 5 | (0.56, 0.64, 0.73) | 0.079 | 5 |
| C7       | (0.56, 2.74, 6.43) | 0.19065 | 2 | (1.49, 1.81, 2.09) | 0.223 | 2 |

Table 2. The results of the Fuzzy VIKOR method and final ranking

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Fuzzy value</th>
<th>Fuzzy value</th>
<th>Fuzzy value</th>
<th>Crisp value</th>
<th>Final rank</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$S$</td>
<td>$R$</td>
<td>$Q$</td>
<td>$Q$</td>
<td></td>
</tr>
<tr>
<td>C1</td>
<td>(0.80, 0.77, 0.62)</td>
<td>(0.36, 0.26, 0.23)</td>
<td>(0.76, 0.79, 0.78)</td>
<td>0.77739</td>
<td>6</td>
</tr>
<tr>
<td>C2</td>
<td>(0.78, 0.75, 0.60)</td>
<td>(0.35, 0.26, 0.23)</td>
<td>(0.71, 0.72, 0.76)</td>
<td>0.73010</td>
<td>4</td>
</tr>
<tr>
<td>C3</td>
<td>(0.70, 0.78, 0.36)</td>
<td>(0.34, 0.26, 0.23)</td>
<td>(0.34, 0.81, 0.50)</td>
<td>0.55151</td>
<td>3</td>
</tr>
<tr>
<td>C4</td>
<td>(0.77, 0.65, 0.47)</td>
<td>(0.34, 0.26, 0.20)</td>
<td>(0.55, 0.04, 0.12)</td>
<td>0.23855</td>
<td>1</td>
</tr>
<tr>
<td>C5</td>
<td>(0.92, 0.83, 0.64)</td>
<td>(0.36, 0.26, 0.23)</td>
<td>(1.00, 0.90, 0.79)</td>
<td>0.90162</td>
<td>7</td>
</tr>
<tr>
<td>C6</td>
<td>(0.75, 0.88, 0.83)</td>
<td>(0.36, 0.26, 0.23)</td>
<td>(0.64, 0.65, 1.00)</td>
<td>0.76163</td>
<td>5</td>
</tr>
<tr>
<td>C7</td>
<td>(0.68, 0.63, 0.77)</td>
<td>(0.31, 0.26, 0.22)</td>
<td>(0.00, 0.50, 0.91)</td>
<td>0.46919</td>
<td>2</td>
</tr>
</tbody>
</table>
Finally, the rankings obtained from both methods, Fuzzy AHP and Fuzzy VIKOR, are compared, and they show a good agreement (Table 3). However, it should be kept in mind that each of these methods has its advantages; for instance, the Fuzzy AHP method is straightforward and does not require an additional tool for criteria prioritization. Likewise, Fuzzy VIKOR provides a ranking procedure for positive attributes and negative attributes and ensures the balance between them.

**CONCLUSIONS**

The current investigation, in continuation of previous work, develops a formulation to solve a multi-criteria decision-making problem. The current case study is an AV shuttle system that has seven criteria; these criteria are linguistically evaluated by six experts. After forming the pairwise matrices, the max-min method with the arithmetic mean has been utilized to aggregate them to form a singular pairwise matrix, comparing the importance of the criteria to each other. In the case of Fuzzy AHP, based on the aggregated matrix, the fuzzy weights are calculated and defuzzified to obtain crisp weights. The ranking of the criteria is performed based on crisp weights. In the case of Fuzzy VIKOR, the linearization has been carried out, and the values of the utility measures and, subsequently, the VIKOR scores are calculated. The results of the Fuzzy AHP method are in accordance with the results of the previous study, and the results for both methods are compared, which exhibit a good agreement. The Fuzzy AHP method is beneficial in the case of a certain degree of inconsistency during pairwise comparisons and provides a robust way to transform decision-makers’ judgments into numerical results [9]. However, in the Fuzzy VIKOR method, compromise in conflict resolution may occur [21].

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**Fuzzy AHP ja Fuzzy VIKOR meetodite võrdlev analüüs autonoomse sõiduki süsteemiriskide prioriseerimisel**

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