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RING THEORY

Morita contexts and unitary ideals of rings

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Abstract. In this paper we study Morita contexts between rings without identity. We prove that if two associative rings are connected by a Morita context with surjective mappings, then these rings have isomorphic quantales of unitary ideals. We also show that the quotient rings by ideals that correspond to each other under that isomorphism are connected by a Morita context with surjective mappings. In addition, we consider how annihilators and two-sided socles behave under that isomorphism.

Key words: ring theory, ring, ideal, bimodule, quantale, Morita context.

1. INTRODUCTION

Morita theory of rings has been a very fruitful area of research. Although originally only rings with identity were considered, starting from [1] and [2], also rings without identity have been studied. Morita equivalence is usually defined by requiring the equivalence of certain module categories (see, e.g. [6] and [7]), but a fundamental observation indicates that if the rings are "sufficiently good", then the equivalence of these modules categories is equivalent to the existence of a Morita context with certain properties. From the results of [7] it follows that two idempotent rings are Morita equivalent if and only if they are connected by a unitary surjective Morita context. In [11] we showed that these two conditions are also equivalent to the fact that the two rings have a joint enlargement. Morita contexts and enlargements are usually easier to use than the equivalence functors when we want to study properties of Morita equivalent rings.

Properties that are shared by all rings in the same Morita equivalence class are called Morita invariants. Studying such invariants has always been an important part of Morita theory. There are many properties of rings that are defined in terms of operations and the inclusion order of ideals. Such operations are multiplication, intersection (meet) and sum (join), and with these operations the set of ideals of a ring is equipped with the structure of quantale (Chapter 4 in [13] is an introduction to how quantales can be used in the ideal theory of rings). In several papers ([2, Proposition 3.3], [7, Proposition 3.5], [3, Theorem 3.3]) it has been shown that lattices of certain ideals of Morita equivalent rings (of some type) are isomorphic. In this paper, taking inspiration from the recent developments in semigroup theory [10], we study under which conditions on a Morita context the quantales of unitary ideals are isomorphic.

Section 2 presents our main definitons and some basic results. In Section 3 we show that if two rings are connected by a surjective (but not necessarily unitary) Morita context, then their quantales of unitary ideals

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are isomorphic. In addition, we study how annihilators of modules behave with respect to that isomorphism. Then we prove that if two idempotent rings are Morita equivalent, then also the corresponding quotient rings are Morita equivalent. Section 3 concludes with considering socles of Morita equivalent rings and in Section 4 we demonstrate that Morita contexts induce a certain semifunctor in a natural way.

2. PRELIMINARIES

Unless stated otherwise, R and S will stand for associative rings in this paper.

As usual, denote

$$NS' := \left\{ \sum_{k=1}^{k^*} n_k s_k \,\middle|\, k^* \in \mathbb{N},\ n_1,\dots,n_{k^*} \in N,\ s_1,\dots,s_{k^*} \in S' \right\},$$

where N is a right S-module and $S' \subseteq S$. If N is an (R,S)-bimodule, then $R'N$ and $R'NS'$ (where $R' \subseteq R$) are defined analoguously.

Definition 2.1. *A left R-module N is called **unitary** (see, e.g.* [2]) *if $RN = N$. Unitary right modules are defined analogously. An (R,S)-bimodule M is called **unitary** if $RM = M$ and $MS = M$.*

Lemma 2.2. *A bimodule ${}_RM_S$ is unitary if and only if $RMS = M$.*

Proof. Necessity. It is clear.

Sufficiency. Let $m \in M = RMS$. Then there exist $r_1,\dots,r_{k^*} \in R$, $s_1,\dots,s_{k^*} \in S$ and $m_1,\dots,m_{k^*} \in M$, such that $m = r_1m_1s_1 + \ldots + r_{k^*}m_{k^*}s_{k^*}$. Now

$$m = \sum_{k=1}^{k^*} r_k(m_ks_k) \in RM \quad \text{and} \quad m = \sum_{k=1}^{k^*} (r_km_k)s_k \in MS. \qquad \square$$

Definition 2.3. *A right (left) ideal $I \trianglelefteq R$ is called **unitary** if $IR = I$ ($RI = I$). An ideal $I \trianglelefteq R$ is called **unitary** if I is unitary both as a right and as a left ideal.*

Remark. Unitary ideals of a ring without identity are also studied in [3], but there they are called *lower closed ideals* (Def. 3.1).

We denote the set of all unitary ideals of a ring R by $\mathrm{UId}(R)$. According to Lemma 2.2, an ideal $I \trianglelefteq R$ is unitary if and only if $RIR = I$.

Definition 2.4 ([16]). *A ring R is called **s-unital** if for every $r \in R$ there exist $u, v \in R$ such that $r = ru = vr$.*

For example, every ring with local units (see [2]), including every von Neumann regular ring, is s-unital. We also need the following result (see [16]).

Proposition 2.5. *A ring R is s-unital if and only if for every finite subset $F \subseteq R$ there exist $u, v \in R$ such that $r = ru = vr$ for every $r \in F$.*

Definition 2.6. *An ideal $I \trianglelefteq R$ is **generated** by a subset $X \subseteq R$ if I is the smallest ideal that contains X. In that case we write $I = (X)_{\mathrm{g}}$. An ideal $I \trianglelefteq R$ is **finitely generated** if it is generated by a finite set $X \subseteq R$.*

One can give an explicit description of ideals generated by a nonempty subset $X \subseteq R$. Denote

$$\mathbb{Z}X := \left\{ \sum_{h=1}^{h^*} k_h x_h \,\middle|\, h^* \in \mathbb{N},\ k_1, \ldots, k_{h^*} \in \mathbb{Z},\ x_1, \ldots, x_{h^*} \in X \right\} \subseteq R.$$

According to [14] (page 5), the ideal $(X)_{\mathrm{g}}$ is

$$(X)_{\mathrm{g}} = \mathbb{Z}X + RX + XR + RXR. \tag{2.1}$$

Lemma 2.7. *If a unitary ideal $I \trianglelefteq R$ is generated by a set $X \subseteq R$*, then $I = RXR$.

Proof. Let $(X)_{\mathrm{g}} = I \in \mathrm{UId}(R)$. Then we have

$$I = RIR = R(\mathbb{Z}X + RX + XR + RXR)R = \mathbb{Z}RXR + RRXR + RXRR + RRXRR \subseteq RXR.$$

On the other hand, we see from the equality (2.1) that $RXR \subseteq I$. Therefore, we have $I = RXR$. □

Next we will prove that s-unital rings can be described as follows.

Proposition 2.8. *A ring R is s-unital if and only if all right ideals of R are unitary and all left ideals of R are unitary.*

Proof. Necessity. If I is a right ideal and $i \in I$, then $i = iu$ for some $u \in R$. Hence, $I = IR$. Analogously, $I = RI$ if I is a left ideal.

Sufficiency. Take an element $r \in R$. Since the right ideal $I = \mathbb{Z}r + rR$ is unitary, there exist $z_k \in \mathbb{Z}, r_k, u_k \in R$ such that

$$r = \sum_{k=1}^{k^*} (z_k r + r r_k) u_k = \sum_{k=1}^{k^*} (z_k r u_k + r r_k u_k) = \sum_{k=1}^{k^*} (r(z_k u_k) + r r_k u_k) = r \sum_{k=1}^{k^*} (z_k u_k + r_k u_k).$$

Similarly, $r = vr$ for some $v \in R$. □

Corollary 2.9. *All ideals of an s-unital ring are unitary.*

Definition 2.10 ([13]). *A complete lattice A is called a **quantale** if it is equipped with a binary algebraic operation $*\colon\ A \times A \longrightarrow A$, such that for every set I and for every $x, y_i \in A$, where $i \in I$, the following conditions hold*:

$$x * \left(\bigvee_{i \in I} y_i \right) = \bigvee_{i \in I} (x * y_i) \quad \text{and} \quad \left(\bigvee_{i \in I} y_i \right) * x = \bigvee_{i \in I} (y_i * x).$$

A quantale A is called **unital** if there exists an element $e \in A$ such that $a * e = e * a = a$ for every $a \in A$. The element e is called the **identity** element of the quantale A.

Let A and B be quantales. A mapping $f\colon A \longrightarrow B$ is called an **isomorphism of quantales** if it is an isomorphism of lattices and $f(a_1 * a_2) = f(a_1) * f(a_2)$ for every $a_1, a_2 \in A$. An **isomorphism of unital quantales** has to also preserve the identity element.

In the following proposition we will show that for any ring R the set of its unitary ideals possesses a natural quantale structure.

Proposition 2.11. *If R is a ring, then* $\mathrm{UId}(R)$ *is a quantale.*

Proof. The poset $(\mathrm{UId}(R), \subseteq)$ is a complete lattice where for every subset $U \subseteq \mathrm{UId}(R)$ we have

$$\bigvee U = \sum_{I \in U} I \quad \text{and} \quad \bigwedge U = \bigvee \left\{ V \in \mathrm{UId}(R) \,\middle|\, V \subseteq \bigcap_{I \in U} I \right\}.$$

By Proposition 3.2 in [3], any sum of unitary ideals is also a unitary ideal.

Define the operation $*\colon\ \mathrm{UId}(R) \times \mathrm{UId}(R) \longrightarrow \mathrm{UId}(R)$ as $(I_1, I_2) \mapsto I_1 I_2$. If $J \in \mathrm{UId}(R)$ and $U \subseteq \mathrm{UId}(R)$, then

$$J * \left(\bigvee_{I \in U} I \right) = J \left(\sum_{I \in U} I \right) = \sum_{I \in U} JI = \bigvee_{I \in U} (J * I).$$

The other compatibility condition in the definition of a quantale holds analogously. □

A ring R is called **idempotent** ([7]) if $RR = R$.

Proposition 2.12. *If R is an idempotent ring, then* $\mathrm{UId}(R)$ *is a unital quantale with identity element R.*

Proof. If R is an idempotent ring, then R is a unitary ideal of itself. It is also clear from the definition of a unitary ideal that for every $I \in \mathrm{UId}(R)$ we have $RI = IR = I$.

Meets are calculated here as follows: for any subset $U \subseteq \mathrm{UId}(R)$

$$\bigwedge U = R \left(\bigcap_{I \in U} I \right) R.$$

□

3. QUANTALES OF UNITARY IDEALS AND MORITA CONTEXTS

In this section we will study the quantales of unitary ideals of rings connected by a surjective Morita context. It is proved that these quantales are isomorphic. First, let us recall the definition of a Morita context.

Definition 3.1. *A six-tuple $(R, S, {}_RP_S, {}_SQ_R, \theta, \phi)$, where R and S are rings and ${}_RP_S$, ${}_SQ_R$ are bimodules, is called a **Morita context**, if*

$$\theta\colon\ {}_R(P \otimes_S Q)_R \longrightarrow {}_RR_R, \quad \phi\colon\ {}_S(Q \otimes_R P)_S \longrightarrow {}_SS_S$$

are bimodule homomorphisms such that

$$\theta(p \otimes q)p' = p\phi(q \otimes p'), \tag{3.1}$$

$$q\theta(p \otimes q') = \phi(q \otimes p)q' \tag{3.2}$$

for every $p, p' \in P$ and $q, q' \in Q$.

We say that a Morita context $(R, S, {}_RP_S, {}_SQ_R, \theta, \phi)$ is **unitary**, if the bimodules ${}_RP_S$ and ${}_SQ_R$ are unitary; and **surjective**, if the homomorphisms θ and ϕ are surjective. Unitary surjective Morita contexts connect only idempotent rings (see, e.g. [4]).

The following theorem is a ring theoretic analogue of Theorem 3.4 in [10].

Theorem 3.2. *If rings R and S are connected by a surjective Morita context $(R, S, {}_RP_S, {}_SQ_R, \theta, \phi)$, then their quantales of unitary ideals* $\mathrm{UId}(R)$ *and* $\mathrm{UId}(S)$ *are isomorphic. This isomorphism takes finitely generated ideals to finitely generated ideals. If the rings R and S are idempotent, then the previous isomorphism is a morphism of unital quantales.*

Proof. 1. Let $(R,S,{}_RP_S,{}_SQ_R,\theta,\phi)$ be a surjective Morita context. Note that, for every unitary ideal $J \in \mathrm{UId}(S)$, the set

$$\theta(PJ\otimes Q) = \left\{ \theta\left(\sum_{k=1}^{k^*} p_k j_k \otimes q_k\right) \middle| \forall k:\ p_k \in P,\ j_k \in J,\ q_k \in Q \right\} \subseteq R$$

is an ideal, because θ is an (R,R)-bimodule homomorphism. Additionally, we have

$$\begin{aligned}\theta(PJ\otimes Q) &= \theta(PSJS\otimes Q) = \theta(PSJ\otimes SQ) = \theta(P\,\mathrm{Im}(\phi)J\otimes \mathrm{Im}(\phi)Q)\\ &= \theta(\mathrm{Im}(\theta)PJ\otimes Q\,\mathrm{Im}(\theta)) = \theta(RPJ\otimes QR) = R\theta(PJ\otimes Q)R.\end{aligned}$$

Therefore, the ideal $\theta(PJ\otimes Q)$ is unitary. Analogously, we can show that, for every $I \in \mathrm{UId}(R)$, the set $\phi(QI\otimes P)$ is a unitary ideal of S. This allows us to define the mappings

$$\Theta\colon\ \mathrm{UId}(S) \longrightarrow \mathrm{UId}(R), \quad \Theta(J) := \theta(PJ\otimes Q), \tag{3.3}$$

$$\Phi\colon\ \mathrm{UId}(R) \longrightarrow \mathrm{UId}(S), \quad \Phi(I) := \phi(QI\otimes P). \tag{3.4}$$

Let $J_1, J_2 \in \mathrm{UId}(S)$ be such that $J_1 \subseteq J_2$. Then we have $\Theta(J_1) = \theta(PJ_1\otimes Q) \subseteq \theta(PJ_2\otimes Q) = \Theta(J_2)$, which means that the mapping Θ preserves the order. Analogously, the mapping Φ preserves the order. Note that if $J \in \mathrm{UId}(S)$, then

$$\Phi(\Theta(J)) = \phi(Q\theta(PJ\otimes Q)\otimes P) = \phi(\phi(Q\otimes P)JQ\otimes P) = \phi(Q\otimes P)J\phi(Q\otimes P) = SJS = S.$$

Analogously, $\Theta(\Phi(J)) = I$ holds for every $I \in \mathrm{UId}(R)$, which means that the mappings Φ and Θ are inverses of each other. Hence, the mappings Φ and Θ are actually isomorphisms of lattices.

If $J_1, J_2 \in \mathrm{UId}(S)$, then

$$\begin{aligned}\Theta(J_1)\Theta(J_2) &= \theta(PJ_1\otimes Q)\theta(PJ_2\otimes Q) = \theta(PJ_1\otimes Q\theta(PJ_2\otimes Q)) = \theta(PJ_1\otimes \phi(Q\otimes PJ_2)Q)\\ &= \theta(PJ_1\otimes\phi(Q\otimes P)J_2Q) = \theta(PJ_1\otimes SJ_2Q) = \theta(PJ_1\otimes J_2Q) = \theta(P(J_1J_2)\otimes Q) = \Theta(J_1J_2).\end{aligned}$$

Analogously, we can show that, for every $I_1, I_2 \in \mathrm{UId}(R)$, the equality $\Phi(I_1)\Phi(I_2) = \Phi(I_1I_2)$ holds. Hence, Θ and Φ are isomorphisms of quantales.

2. Let $J \in \mathrm{UId}(S)$ be a finitely generated ideal. Then there exists a finite set $X = \{x_1,\dots,x_n\} \subseteq J$ such that $J = SXS$ (see Lemma 2.7). Fix an index $k \in \{1,\dots,n\}$. Then the element x_k can be written as

$$x_k = \sum_{h=1}^{h^*} s_{kh}x_{kh}s'_{kh},$$

where $s_{k1}, s'_{k1}, \dots, s_{kh^*}, s'_{kh^*} \in S$ and $x_{k1},\dots,x_{kh^*} \in X$. Considering that the mapping ϕ is surjective, we can also express x_k as follows:

$$x_k = \sum_{t=1}^{t^*} \phi(q_t\otimes p_t)\xi_t\phi(q'_t\otimes p'_t),$$

where $q_1, q_1', \ldots, q_{t^*}, q_{t^*}' \in Q$, $p_1, p_1', \ldots, p_{t^*}, p_{t^*} \in P$ and $\xi_1, \ldots, \xi_{t^*} \in X$. Now, for every $p \in P$ and $q \in Q$, we have

$$\begin{aligned}
\theta\left(px_k \otimes q\right) &= \theta\left(p\left(\sum_{t=1}^{t^*} \phi(q_t \otimes p_t)\xi_t \phi(q_t' \otimes p_t')\right) \otimes q\right) \\
&= \sum_{t=1}^{t^*} \theta(p\phi(q_t \otimes p_t)\xi_t \phi(q_t' \otimes p_t') \otimes q) \\
&= \sum_{t=1}^{t^*} \theta(\theta(p \otimes q_t)p_t\xi_t \otimes \phi(q_t' \otimes p_t')q) \\
&= \sum_{t=1}^{t^*} \theta(\theta(p \otimes q_t)p_t\xi_t \otimes q_t'\theta(p_t' \otimes q)) \\
&= \sum_{t=1}^{t^*} \theta(p \otimes q_t)\theta(p_t\xi_t \otimes q_t')\theta(p_t' \otimes q) \in RYR,
\end{aligned}$$

where

$$Y = \{\theta(p_t\xi_t \otimes q_t') \mid t \in \{1, \ldots, t^*\}\} \subseteq R$$

is a finite set. Note that

$$\begin{aligned}
\Theta(J) = \theta(PJ \otimes Q) &= \left\{\theta\left(\sum_{\ell=1}^{\ell^*} p_\ell j_\ell \otimes q_\ell\right) \middle| \forall \ell\colon\ p_\ell \in P, q_\ell \in Q, j_\ell \in J\right\} \\
&= \left\{\theta\left(\sum_{\ell=1}^{\ell^*} p_\ell \left(\sum_{h=1}^{h^*} s_{h\ell}x_{h\ell}s_{h\ell}'\right) \otimes q_\ell\right) \middle| \forall \ell, h\colon\ p_\ell \in P; q_\ell \in Q; x_{h\ell} \in X; s_{h\ell}, s_{h\ell}' \in S\right\} \\
&= \left\{\theta\left(\sum_{\ell=1}^{\ell^*}\sum_{h=1}^{h^*} (p_\ell s_{h\ell})x_{h\ell} \otimes (s_{h\ell}'q_\ell)\right) \middle| \forall \ell, h\colon\ p_\ell \in P; q_\ell \in Q; x_{h\ell} \in X; s_{h\ell}, s_{h\ell}' \in S\right\} \\
&= \left\{\sum_{\ell=1}^{\ell^*}\sum_{h=1}^{h^*} \theta\left((p_\ell s_{h\ell})x_{h\ell} \otimes (s_{h\ell}'q_\ell)\right) \middle| \forall \ell, h\colon\ p_\ell \in P; q_\ell \in Q; x_{h\ell} \in X; s_{h\ell}, s_{h\ell}' \in S\right\} \subseteq RYR.
\end{aligned}$$

On the other hand, $Y \subseteq \Theta(J)$. Since $\Theta(J)$ is an ideal of R which contains Y,

$$(Y)_{\mathrm{g}} \subseteq \Theta(J) \subseteq RYR \subseteq (Y)_{\mathrm{g}},$$

which implies that $\Theta(J) = (Y)_{\mathrm{g}}$. Hence, $\Theta(J)$ is a finitely generated ideal.

3. Let the rings R and S be idempotent. Then, by Proposition 2.12, the quantales $\mathrm{UId}(R)$ and $\mathrm{UId}(S)$ are unital quantales with identity elements R and S, respectively. Since lattice isomorphisms preserve the largest elements, $\Theta(S) = R$ and $\Phi(R) = S$. □

Remark. In Proposition 3.5 in the article [7], it has been shown that if idempotent rings R and S are connected by a unitary surjective Morita context, then the lattices $\mathrm{UId}(R)$ and $\mathrm{UId}(S)$ are isomorphic. Additionally, we have proved that they are isomorphic as quantales and that these isomorphisms behave well with respect to finitely generated ideals. We have also shown that assuming the idempotence of rings and the unitariness of bimodules in the Morita context is not necessary.

Theorem 3.2 implies that the isomorphisms Θ and Φ preserve all the properties of unitary ideals that are defined by using multiplication of ideals, inclusion relation, joins or meets. For example, if I is a semiprime element in the quantale $\mathrm{UId}(R)$ ([13, Def. 3.2.5]), then $\Phi(I)$ is semiprime in $\mathrm{UId}(S)$. An analogous statement holds for prime elements ([13, Def. 3.2.8]). In [15], the radical of a complete lattice is defined as the meet of all coatoms. Thus, Φ takes the radical of the lattice $\mathrm{UId}(R)$ to the radical of $\mathrm{UId}(S)$.

Here is another application of Theorem 3.2. Let $\mathrm{Mat}_n(R)$ denote the full matrix ring of $(n\times n)$-matrices over a ring R.

Corollary 3.3. *If R is an idempotent ring and n a natural number, then* $\mathrm{UId}(R)$ *and* $\mathrm{UId}(\mathrm{Mat}_n(R))$ *are isomorphic quantales.*

Proof. By Corollary 3.7 in [11], R is Morita equivalent to the full matrix ring $\mathrm{Mat}_n(R)$. The last ring is idempotent by Proposition 2.6 and Proposition 2.2 in [11]. As mentioned in the introduction, Morita equivalence in this case means that the rings R and $\mathrm{Mat}_n(R)$ are connected by a unitary surjective Morita context. Now the claim follows from Theorem 3.2. □

Corollary 3.4. *If R is an s-unital ring and n a natural number, then* $\mathrm{Id}(R)$ *and* $\mathrm{Id}(\mathrm{Mat}_n(R))$ *are isomorphic quantales.*

Proof. If R is s-unital, then by using Proposition 2.5, one can show that also $\mathrm{Mat}_n(R)$ is an s-unital ring. The claim follows from Corollary 3.3 and Corollary 2.9. □

Recall that the **annihilator** of an R-module M_R is defined as:

$$\mathrm{ann}(M_R) := \{r\in R \mid \forall m\in M\colon\ mr=0\} = \{r\in R \mid Mr=0\}.$$

It is easy to see that, for any R-module M_R, its annihilator $\mathrm{ann}(M_R)$ is an ideal of R. An R-module M_R is called **faithful** if $\mathrm{ann}(M_R)=0$.

Now we will prove a result which generalizes Proposition 18.47 in [12].

Proposition 3.5. *Let R and S be s-unital rings. If R and S are connected by a surjective Morita context $(R,S,{}_RP_S,{}_SQ_R,\theta,\phi)$, then there exists an isomorphism $\Phi\colon\ \mathrm{Id}(R)\longrightarrow \mathrm{Id}(S)$. Moreover, for every R-module M_R,*
- $\Phi(\mathrm{ann}(M_R)) = \mathrm{ann}(M\otimes P_R)$;
- M_R is faithful if and only if the module $M\otimes P$ is faithful.

Proof. If R and S are s-unital rings, then, by Corollary 2.9, $\mathrm{Id}(R)=\mathrm{UId}(R)$ and $\mathrm{Id}(S)=\mathrm{UId}(S)$. Due to Theorem 3.2, we now have $\mathrm{Id}(R)\cong\mathrm{Id}(S)$, where the isomorphism Φ is defined as in (3.4). Note that

$$\begin{aligned}(M\otimes P)\Phi(\mathrm{ann}(M)) &= (M\otimes P)\phi(Q\,\mathrm{ann}(M)\otimes P) = M\otimes\theta(P\otimes Q)\,\mathrm{ann}(M)P\\ &= M\otimes R\,\mathrm{ann}(M)P = MR\,\mathrm{ann}(M)\otimes P\subseteq M\,\mathrm{ann}(M)\otimes P = 0\otimes P=0.\end{aligned}$$

Therefore, we have $\Phi(\mathrm{ann}(M))\subseteq\mathrm{ann}(M\otimes P)$. Analogously, we can show that $\Theta(\mathrm{ann}(M\otimes P))\subseteq\mathrm{ann}(M\otimes P\otimes Q)$, where $\Theta\colon\ \mathrm{Id}(S)\longrightarrow\mathrm{Id}(R)$ is defined as in (3.3).

Now, let $r\in\mathrm{ann}(M\otimes P\otimes Q)\subseteq R$. Since R is s-unital, there exists $u\in R$ such that $r=ru$ and, due to surjectivity of θ, there exist elements $p_1,\dots,p_{k^*}\in P$ and $q_1,\dots,q_{k^*}\in Q$ such that $u=\sum_{k=1}^{k^*}\theta(p_k\otimes q_k)$. Note that, for any $m\in M$, we have

$$\begin{aligned}mr = mur = \mu_M(m\otimes u)r &= \sum_{k=1}^{k^*}\mu_M(m\otimes\theta(p_k\otimes q_k))r = \sum_{k=1}^{k^*}\mu_M((\mathrm{id}_M\otimes\theta)(m\otimes p_k\otimes q_k))r\\ &= \sum_{k=1}^{k^*}\mu_M((\mathrm{id}_M\otimes\theta)((m\otimes p_k\otimes q_k)r)) = \sum_{k=1}^{k^*}\mu_M((\mathrm{id}_M\otimes\theta)(0))=0,\end{aligned}$$

where $\mu_M\colon M\otimes R \longrightarrow M$, $\mu_M(m\otimes r) = mr$ is a homomorphism of right R-modules. Hence, $r\in \mathrm{ann}(M_R)$. Now we have proved the inclusions

$$\Theta(\mathrm{ann}(M\otimes P)) \subseteq \mathrm{ann}(M\otimes P\otimes Q) \subseteq \mathrm{ann}(M).$$

Applying the lattice isomorphism Φ to the previous sequence of inclusions, we obtain

$$\mathrm{ann}(M\otimes P) = \Phi(\Theta(\mathrm{ann}(M\otimes P))) \subseteq \Phi(\mathrm{ann}(M)).$$

In conclusion, we have shown that $\Phi(\mathrm{ann}(M)) = \mathrm{ann}(M\otimes P)$.

If M_R is faithful, then $0 = \mathrm{ann}(M) = \Theta(\mathrm{ann}(M\otimes P))$, which implies that $\mathrm{ann}(M\otimes P) = 0$ because Θ is an isomorphism. □

If ${}_RM_S$ is a bimodule, then by $\mathrm{USub}(M)$ we denote the set of all unitary sub-bimodules of M.

Proposition 3.6. *If ${}_RM_S$ is a bimodule, then* $\mathrm{USub}(M)$ *is a complete lattice. If R and S are idempotent rings, then this lattice is modular.*

Proof. It is easy to see that the sum of any set of unitary sub-bimodules of M is unitary. Hence, $\mathrm{USub}(M)$ is a complete lattice.

Assume that R,S are idempotent and let us prove that the lattice is modular. Let $A,B,C\in \mathrm{USub}(M)$ such that $A\subseteq C$. Then $(A+B)\cap C = A+B\cap C$ because the lattice of all submodules of M is modular. Hence,

$$(A\vee B)\wedge C = R((A+B)\cap C)S = R(A+(B\cap C))S = RAS + R(B\cap C)S = A + R(B\cap C)S = A\vee(B\wedge C). \ \square$$

It follows that $\mathrm{UId}(R)$ is a modular lattice when R is an idempotent ring.

In [3] Buys and Kyuno showed that the lattices $\mathrm{UId}(R)$ and $\mathrm{UId}(S)$ are also isomorphic to the lattices $\mathrm{USub}(P)$ and $\mathrm{USub}(Q)$, where P and Q are taken from the Morita context.

Theorem 3.7 ([3, Theorem 3.3]). *Let the rings R and S be connected by a surjective Morita context $(R,S,{}_RP_S,{}_SQ_R,\theta,\phi)$, then the following lattices are isomorphic:*

1. $\mathrm{UId}(R)$,
2. $\mathrm{UId}(S)$,
3. $\mathrm{USub}({}_RP_S)$,
4. $\mathrm{USub}({}_SQ_R)$.

The isomorphisms in the previous theorem are obtained by using the following mappings:

$$\begin{aligned} \Psi\colon &\quad \mathrm{UId}(R)\longrightarrow \mathrm{USub}(P), \quad \Psi(I) := IP,\\ \Omega\colon &\quad \mathrm{USub}(P)\longrightarrow \mathrm{UId}(R), \quad \Omega(A) := \theta(A\otimes Q);\\ \Psi'\colon &\quad \mathrm{UId}(R)\longrightarrow \mathrm{USub}(Q), \quad \Psi'(I) := QI,\\ \Omega'\colon &\quad \mathrm{USub}(Q)\longrightarrow \mathrm{UId}(R), \quad \Omega'(B) := \theta(P\otimes B). \end{aligned}$$

Remark. Among other things, Theorem 3.7 implies that if R and S are s-unital rings, then

$$R \text{ is uniform } \iff S \text{ is uniform } \iff {}_RP_S \text{ is uniform } \iff {}_SQ_R \text{ is uniform},$$

where **uniformity** means that the intersection of every two non-zero ideals (sub-bimodules) is non-zero. An analogous claim holds for the dual notion – hollowness.

The following theorem is a generalization of Corollary 18.49 in [12]. It will imply that if R and S are Morita equivalent idempotent rings, then every quotient ring of R is Morita equivalent to a certain quotient ring of S.

Theorem 3.8. *Let $\Gamma = (R, S, {}_RP_S, {}_SQ_R, \theta, \phi)$ be a Morita context. Then, for every ideal $I \in \mathrm{Id}(R)$, the quotient rings R/I and $S/\Phi(I)$ are connected by a Morita context $\Gamma_I = (R/I, S/\Phi(I), P/\Psi(I), Q/\Psi'(I), \zeta, \eta)$, where*

$$\begin{aligned} \Phi\colon \quad & \mathrm{Id}(R) \longrightarrow \mathrm{Id}(S), \quad \Phi(I) := \phi(QI \otimes P), \\ \Psi\colon \quad & \mathrm{Id}(R) \longrightarrow \mathrm{Sub}(P), \quad \Psi(I) := IP, \\ \Psi'\colon \quad & \mathrm{Id}(R) \longrightarrow \mathrm{Sub}(Q), \quad \Psi'(I) := QI. \end{aligned}$$

Moreover,
- if Γ is surjective, then Γ_I is surjective;
- if Γ is unitary, then Γ_I is unitary.

Proof. Let $I \in \mathrm{UId}(R)$. First, we must show that the abelian group $P/\Psi(I)$ is an $(R/I, S/\Phi(I))$-bimodule. Consider the mappings

$$R/I \times P/\Psi(I) \longrightarrow P/\Psi(I), \quad ([r],[p]) \mapsto [rp], \tag{3.5}$$
$$P/\Psi(I) \times S/\Phi(I) \longrightarrow P/\Psi(I), \quad ([p],[s]) \mapsto [ps]. \tag{3.6}$$

Let $p_1, p_2 \in P$ and $s_1, s_2 \in S$ be such that $[p_1]_{\Psi(I)} = [p_2]_{\Psi(I)}$ and $[s_1]_{\Phi(I)} = [s_2]_{\Phi(I)}$. Then we have $p_1 - p_2 \in \Psi(I) = IP$ and $s_1 - s_2 \in \Phi(I) = \phi(QI \otimes P)$. Note that

$$\begin{aligned} & p_1s_1 - p_2s_1 = (p_1 - p_2)s_1 \in IPS \subseteq IP, \\ & p_2s_1 - p_2s_2 = p_2(s_1 - s_2) \in P\phi(QI \otimes P) = \theta(P \otimes Q)IP \subseteq RIP \subseteq IP, \end{aligned}$$

which implies that

$$[p_1s_1]_{\Psi(I)} = [p_2s_1]_{\Psi(I)} = [p_2s_2]_{\Psi(I)}.$$

Therefore, the mapping (3.6) is well defined. Analogously, the mapping (3.5) is well defined. Now it is easy to see that with the mappings (3.5) and (3.6) $P/\Psi(I)$ is a bimodule.

Analogously, the abelian group $Q/\Psi'(I)$ is an $(S/\Phi(I), R/I)$-bimodule.

Define the mappings ζ and η as follows:

$$\begin{aligned} \zeta\colon \ P/\Psi(I) \otimes Q/\Psi'(I) \longrightarrow R/I, \quad & \sum_{t=1}^{t^*} [p_t] \otimes [q_t] \mapsto \sum_{t=1}^{t^*} [\theta(p_t \otimes q_t)], \\ \eta\colon \ Q/\Psi'(I) \otimes P/\Psi(I) \longrightarrow S/\Phi(I), \quad & \sum_{t=1}^{t^*} [q_t] \otimes [p_t] \mapsto \sum_{t=1}^{t^*} [\phi(q_t \otimes p_t)]. \end{aligned}$$

To show that these mappings are well defined, we consider the following two mappings:

$$\begin{aligned} \hat{\zeta}\colon \ P/\Psi(I) \times Q/\Psi'(I) \longrightarrow R/I, \quad & ([p],[q]) \mapsto [\theta(p \otimes q)], \\ \hat{\eta}\colon \ Q/\Psi'(I) \times P/\Psi(I) \longrightarrow S/\Phi(I), \quad & ([q],[p]) \mapsto [\phi(q \otimes p)]. \end{aligned}$$

Let $p_1, p_2 \in P$ and $q_1, q_2 \in Q$ be such that $[p_1] = [p_2]$ and $[q_1] = [q_2]$. Then $p_1 - p_2 \in \Psi(I) = IP$ and $q_1 - q_2 \in \Psi'(I) = QI$, therefore there exist elements $\lambda_1, \ldots, \lambda_{k^*} \in P$, $\kappa_1, \ldots, \kappa_{h^*} \in Q$ and $\iota_1, \ldots, \iota_{k^*}, \iota'_1, \ldots, \iota'_{h^*} \in I$ such that $p_1 - p_2 = \iota_1\lambda_1 + \ldots + \iota_{k^*}\lambda_{k^*}$ and $q_1 - q_2 = \kappa_1\iota'_1 + \ldots + \kappa_{h^*}\iota'_{h^*}$. Now

$$\hat{\zeta}([p_1],[q_1]) - \hat{\zeta}([p_2],[q_1]) = [\theta((p_1 - p_2) \otimes q_1)]_I = \left[\sum_{k=1}^{k^*} \iota_k\theta(\lambda_k \otimes q_1)\right]_I = [0]_I,$$

$$\hat{\zeta}([p_2],[q_1]) - \hat{\zeta}([p_2],[q_2]) = [\theta(p_2 \otimes (q_1 - q_2))]_I = \left[\sum_{h=1}^{h^*} \theta(p_2 \otimes \kappa_h)\iota'_h\right]_I = [0]_I.$$

Therefore, we have

$$\hat{\zeta}([p_1],[q_1]) = \hat{\zeta}([p_2],[q_1]) = \hat{\zeta}([p_2],[q_2]),$$

which shows that the mapping $\hat{\zeta}$ is well defined. Since $\hat{\zeta}$ is also $S/\Phi(I)$-balanced, then, due to the definition of tensor product, the mapping ζ is also well defined. Analogously, the mappings $\hat{\eta}$ and η are well defined. Also, ζ and η are bimodule homomorphisms because θ and ϕ are bimodule homomorphisms.

Now, for every $p, p' \in P$ and $q, q' \in Q$, we have

$$\zeta([p]\otimes[q])[p'] = [\theta(p\otimes q)][p'] = [\theta(p\otimes q)p'] = [p\phi(q\otimes p')] = [p]\eta([q]\otimes[p']),$$
$$[q']\zeta([p]\otimes[q]) = [q'][\theta(p\otimes q)] = [q'\theta(p\otimes q)] = [\phi(q'\otimes p)q] = \eta([q']\otimes[p])[q].$$

In conclusion, we have shown that $(R/I, S/\Phi(I), P/\Psi(I), Q/\Psi'(I), \zeta, \eta)$ is a Morita context.

If θ and ϕ are surjective, then also ζ and η are surjective. If P and Q are unitary, then their quotient bimodules are unitary, too. □

In [3, Def. 4.1], the **two-sided socle** of a ring R was defined as

$$\mathrm{Soc}(R) := \sum\{I \mid I \text{ is a minimal ideal of } R\}.$$

Minimal ideals of R are precisely the atoms of the lattice $\mathrm{Id}(R)$.

Definition 3.9. *We define the **unitary two-sided socle** of a ring R as*

$$\begin{aligned}\mathrm{USoc}(R) &:= \sum\{I \mid I \in \mathrm{UId}(R), I = 0 \textit{ or } I \textit{ is an atom of the lattice } \mathrm{UId}(R)\}\\ &= \bigvee\{I \mid I \in \mathrm{UId}(R), I = 0 \textit{ or } I \textit{ is an atom of the lattice } \mathrm{UId}(R)\},\end{aligned}$$

where the join is calculated in the lattice $\mathrm{UId}(R)$ (see also [15], Section 2).

If R and S are connected by a surjective Morita context, then, by Theorem 3.2 we have a lattice isomorphism $\Theta\colon \mathrm{UId}(S) \longrightarrow \mathrm{UId}(R)$, and it follows that

$$\Theta(\mathrm{USoc}(S)) = \mathrm{USoc}(R).$$

If the ring R (and analogously S) satisfies the condition

$$\forall r \in R: \quad (RrR = 0 \implies r = 0), \tag{3.7}$$

then every minimal ideal of R is unitary ([3, Prop. 3.5]). Hence, $\mathrm{USoc}(R) = \mathrm{Soc}(R)$ and we may write

$$\Theta(\mathrm{Soc}(S)) = \mathrm{Soc}(R).$$

Definition 3.10 ([3, Def. 4.5]. *A ring R is called **completely reducible** if* $\mathrm{Soc}(R) = R$.

The fact that the ring R is completely reducible means that R is the join of all atoms in the lattice $\mathrm{Id}(R)$.

Propositionn 3.11. *Let two idempotent rings R and S be connected by a surjective Morita context. Then R is completely reducible if and only if S is completely reducible.*

Proof. Assume that S is completely reducible. Note that if R is idempotent, then it satisfies (3.7). Hence, $\mathrm{Soc}(R) = \mathrm{USoc}(R)$ and analogously $\mathrm{Soc}(S) = \mathrm{USoc}(S)$. Due to Theorem 3.2, we have a lattice isomorphism $\Theta\colon \mathrm{UId}(S) \longrightarrow \mathrm{UId}(R)$. Now

$$\mathrm{Soc}(R) = \Theta(\mathrm{Soc}(S)) = \Theta(S) = R,$$

yielding that R is completely reducible. The other direction is similar. □

4. MORITA CONTEXTS INDUCE A SEMIFUNCTOR

Consider the following category, which we denote by Ltc:

- Objects are lattices.
- Morphisms $L \longrightarrow L'$ are pairs (f,g) of order-preserving mappings $f : L \longrightarrow L'$ and $g : L' \longrightarrow L$ such that $(gf)(a) \le a$ for every $a \in L$ and $(fg)(b) \le b$ for every $b \in L'$.
- The composite of morphisms $(f,g) : L \longrightarrow L'$ and $(f',g') : L' \longrightarrow L''$ is defined as

$$(f',g')(f,g) := (f'f, gg') : L \longrightarrow L''.$$

This category is somewhat similar to the category of lattices and Galois connections (see page 21 in [8]), although the pairs (f,g) do not form a Galois connection.

Next, we wish to construct a semicategory by using Morita contexts. We say that two Morita contexts $\Gamma = (R,S,{}_RP_S,{}_SQ_R,\theta,\phi)$ and $\Delta = (R,S,{}_RP'_S,{}_SQ'_R,\theta',\phi')$ are **isomorphic** if there exist biact isomorphisms $h : {}_RP_S \longrightarrow {}_RP'_S$ and $k : {}_SQ_R \longrightarrow {}_SQ'_R$ such that the diagrams

$$\begin{tikzcd} P\otimes Q \arrow[dd, "h\otimes k"'] \arrow[dr, "\theta"] & \\ & R \\ P'\otimes Q' \arrow[ur, "\theta'"'] & \end{tikzcd} \qquad \begin{tikzcd} Q\otimes P \arrow[dd, "k\otimes h"'] \arrow[dr, "\phi"] & \\ & S \\ Q'\otimes P' \arrow[ur, "\phi'"'] & \end{tikzcd}$$

commute (cf. [5], page 5). We write $\Gamma \cong \Delta$. It is easy to see that the relation $\cong$ is an equivalence relation on the class of all Morita contexts connecting R and S. We denote the equivalence class of Γ by $[\Gamma]$.

Now consider the semicategory Rng_{MC} with the following ingredients:

- Objects are rings.
- Morphisms $R \longrightarrow S$ are equivalence classes of Morita contexts between R and S.
- Let R,S,T be rings and $[\Gamma] : R \longrightarrow S$, $[\Gamma'] : S \longrightarrow T$ be morphisms, where $\Gamma = (R,S,{}_RP_S,{}_SQ_R,\theta,\phi)$ and $\Gamma' = (S,T,{}_SP'_T,{}_TQ'_S,\theta',\phi')$. The composite of $[\Gamma]$ and $[\Gamma']$ is defined as the equivalence class of the Morita context

$$\Gamma' \circ \Gamma = (R,T,{}_RP \otimes_S P'_T, {}_TQ' \otimes_S Q_R, \overline{\theta}, \overline{\phi}) : R \longrightarrow T,$$

where

$$\overline{\theta} : (P \otimes_S P') \otimes_T (Q' \otimes_S Q) \longrightarrow R, \qquad p \otimes p' \otimes q' \otimes q \mapsto \theta(p\theta'(p' \otimes q') \otimes q),$$
$$\overline{\phi} : (Q' \otimes_S Q) \otimes_R (P \otimes_S P') \longrightarrow T, \qquad q' \otimes q \otimes p \otimes p' \mapsto \phi'(q'\phi(q \otimes p) \otimes p').$$

A straightforward verification shows that this composition of morphisms is well defined and associative.

Also, it can be seen that Ltc is a category with involution where $(f,g)^\dagger = (g,f)$ and Rng_{MC} is a semicategory with involution, where

$$[(R,S,{}_RP_S,{}_SQ_R,\theta,\phi)]^\dagger = [(S,R,{}_SQ_R,{}_RP_S,\phi,\theta)].$$

Proposition 4.1. *The assignment*

$$\begin{tikzcd} R \arrow[r, mapsto] \arrow[d, "{[\Gamma]}"'] & \mathrm{Id}(R) \arrow[d, "{(\Phi_\Gamma,\Theta_\Gamma)}"] \\ S \arrow[r, mapsto] & \mathrm{Id}(S) \end{tikzcd}$$

defines an involution preserving semifunctor $\mathrm{Id} : \mathsf{Rng}_{MC} \longrightarrow \mathsf{Ltc}$.

Proof. First we need to check that the mapping $[\Gamma] \mapsto (\Phi_\Gamma, \Theta_\Gamma)$ is well defined. Suppose that $\Gamma \cong \Delta$, where Γ and Δ are as indicated above. Then, for every $I \in \mathrm{Id}(R)$, $q \in Q$, $i \in I$ and $p \in P$,

$$\phi(qi \otimes p) = \phi'(k(qi) \otimes h(p)) = \phi'(k(q)i \otimes h(p)) \in \phi'(Q'I \otimes P'),$$

so $\phi(QI \otimes P) \subseteq \phi'(Q'I \otimes P')$. Analogously, $\phi'(Q'I \otimes P') \subseteq \phi(QI \otimes P)$, and thus

$$\Phi_\Gamma(I) = \phi(QI \otimes P) = \phi'(Q'I \otimes P') = \Phi_\Delta(I).$$

A similar argument demonstrates that $\Theta_\Gamma = \Theta_\Delta$. Therefore, the mapping $[\Gamma] \mapsto (\Phi_\Gamma, \Theta_\Gamma)$ is indeed well defined.

Let now Γ and Γ' be as indicated above. Then $\mathrm{Id}([\Gamma' \circ \Gamma]) = (\Phi, \Theta)$, where

$$\Phi(I) = \overline{\phi}((Q' \otimes Q)I \otimes (P \otimes P')) = \left\{ \sum_{k=1}^{k^*} \phi'(q'_k \phi(q_k i_k \otimes p_k) \otimes p'_k) \;\middle|\; q'_k \in Q', q_k \in Q, i_k \in I, p_k \in P, p'_k \in P' \right\}$$

for every $I \in \mathrm{Id}(R)$, and Θ is defined analogously. On the other hand,

$$\mathrm{Id}([\Gamma']) \circ \mathrm{Id}([\Gamma]) = (\Phi_{\Gamma'}, \Theta_{\Gamma'}) \circ (\Phi_\Gamma, \Theta_\Gamma) = (\Phi_{\Gamma'}\Phi_\Gamma, \Theta_\Gamma\Theta_{\Gamma'}),$$

where

$$(\Phi_{\Gamma'}\Phi_\Gamma)(I) = \Phi_{\Gamma'}(\phi(QI \otimes P)) = \phi'(Q'\phi(QI \otimes P) \otimes P') = \Phi(I).$$

We see that $\Phi_{\Gamma'}\Phi_\Gamma = \Phi$ and, similarly, $\Theta_\Gamma\Theta_{\Gamma'} = \Theta$. Thus, $\mathrm{Id}([\Gamma' \circ \Gamma]) = \mathrm{Id}([\Gamma']) \circ \mathrm{Id}([\Gamma])$. Also,

$$\mathrm{Id}([\Gamma]^\dagger) = (\Theta_\Gamma, \Phi_\Gamma) = (\Phi_\Gamma, \Theta_\Gamma)^\dagger = \mathrm{Id}([\Gamma])^\dagger. \qquad \square$$

Remark. Note that if we do not consider sub-bimodules as in Theorem 3.7, but only one-sided submodules, then a Morita context $(R, S, {}_RP_S, {}_SQ_R, \theta, \phi)$ induces in a natural way certain contravariant Galois connections between the lattices of left ideals of S, right ideals of S, submodules of ${}_RP$ and submodules of Q_R (see [9]).

5. CONCLUSIONS

We have proved that if two associative rings are connected by a surjective Morita context, then the quantales of unitary ideals of those rings are isomorphic and finitely generated ideals correspond to finitely generated ideals under that isomorphism. This implies that all the properties of rings that are defined by using unitary ideals and quantale operations are Morita invariants. We have also shown that quotient rings of Morita equivalent idempotent rings by ideals that correspond to each other under isomorphism are Morita equivalent. In Section 4 we have demonstrated that Morita contexts induce in a natural way a certain semifunctor between semicategories.

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Ringide Morita kontekstid ja unitaarsed ideaalid

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On uuritud ühikuta ringide Morita kontekste. On tõestatud, et kui kaks assotsiatiivset ringi on seotud Morita kontekstiga, mille kujutused on sürjektiivsed, siis nende ringide unitaarsete ideaalide kvantaalid on isomorfsed. Samuti on näidatud, et faktoringid selliste ideaalide järgi, mis on üksteisega vastavuses selle isomorfismi korral, on ka seotud Morita kontekstiga, mille kujutused on sürjektiivsed. Lisaks on vaadeldud, kuidas annullaatorid ja soklid käituvad selle isomorfismi suhtes.