

On a class of Lorentzian para-Sasakian manifolds

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Abstract. We classify Lorentzian para-Sasakian manifolds which satisfy $P \cdot C = 0$, $Z \cdot C = L_C Q(g, C)$, $P \cdot Z - Z \cdot P = 0$, and $P \cdot Z + Z \cdot P = 0$, where P is the v -Weyl projective tensor, Z is the concircular tensor, and C is the Weyl conformal curvature tensor.

Key words: contact metric manifold, Lorentzian para-Sasakian manifold, Sasakian manifold, v -Weyl projective tensor, concircular tensor.

1. INTRODUCTION

Matsumoto [1] introduced the notion of Lorentzian para-Sasakian (LP -Sasakian for short) manifold. Mihai and Rosca defined the same notion independently in [2]. This type of manifold is also discussed in [3,4].

Let M be an n -dimensional Riemannian manifold of class C^∞ . A v -projective symmetry is a projectable vector field X with the property in which every diffeomorphism φ of its one-parametric group is a projective map between leaves. In the theory of the projective transformations of connections the Weyl projective tensor plays an important role.

Recently, the authors of [5] studied the contact metric manifold M^n satisfying the curvature conditions $Z(\xi, X) \cdot R = 0$ and $R(\xi, X) \cdot Z = 0$, where Z is the *concircular tensor* of M^n defined by

$$Z(X, Y)W = R(X, Y)W - \frac{\tau}{n(n-1)}R_0(X, Y)W, \quad (1)$$

where

$$R_0(X, Y)W = g(Y, W)X - g(X, W)Y,$$

R and τ are the *Riemannian–Christoffel curvature tensor* and the *scalar curvature* of M^n , respectively. They observed immediately from the form of the concircular curvature tensor that Riemannian manifolds with a vanishing concircular curvature tensor are of constant curvature. Thus one can think of the concircular curvature tensor as a measure of the failure of a Riemannian manifold to be of constant curvature.

In the theory of the projective transformations of connections the Weyl projective tensor plays an important role. The *v–Weyl projective tensor* P in a Riemannian manifold (M^n, g) is defined by [6]

$$P(X, Y)W = R(X, Y)W - \frac{1}{n-1}R_1(X, Y)W, \quad (2)$$

where

$$R_1(X, Y)W = S(Y, W)X - S(X, W)Y,$$

with S being the *Ricci tensor* of M .

In the present study we give a classification of the *LP-Sasakian manifold* M^n satisfying the curvature conditions $P \cdot C = 0$, $Z \cdot C = L_C Q(g, C)$, $P \cdot Z - Z \cdot P = 0$, and $P \cdot Z + Z \cdot P = 0$, where Z is the concircular tensor defined by (1), P is the *v–Weyl projective tensor*, and C is the *Weyl conformal curvature tensor* of M^n .

2. PRELIMINARIES

A differentiable manifold of dimension n is called an *LP-Sasakian manifold* [1,2] if it admits a $(1, 1)$ -tensor field ϕ , a contravariant vector field ξ , a covariant vector field η , and a Lorentzian metric g which satisfy

$$\eta(\xi) = -1, \quad (3)$$

$$\phi^2 = I + \eta \otimes \xi, \quad (4)$$

$$g(\phi X, \phi Y) = g(X, Y) + \eta(X)\eta(Y), \quad (5)$$

$$g(X, \xi) = \eta(X), \quad \nabla_X \xi = \phi X, \quad (6)$$

$$\Phi(X, Y) = g(X, \phi Y) = g(\phi X, Y) = \Phi(Y, X), \quad (7)$$

$$(\nabla_X \Phi)(Y, W) = g(Y, (\nabla_X \Phi)W) = (\nabla_X \Phi)(W, Y), \quad (8)$$

where ∇ is the covariant differentiation with respect to g . The Lorentzian metric g makes a timelike unit vector field, that is, $g(\xi, \xi) = -1$. The manifold M^n equipped with a Lorentzian almost paracontact structure (ϕ, ξ, η, g) is said to be a *Lorentzian almost paracontact manifold* (see [1,3]).

If we replace in (3) and (4) ξ by $-\xi$, then the triple (ϕ, ξ, η) is an almost paracontact structure on M^n defined by Sato [7]. The Lorentzian metric given by (6) stands analogous to the almost paracontact Riemannian metric for any almost paracontact manifold (see [7,8]).

A Lorentzian almost paracontact manifold M^n equipped with the structure (ϕ, ξ, η, g) is called a *Lorentzian paracontact manifold* (see [1]) if

$$\Phi(X, Y) = \frac{1}{2} ((\nabla_X \eta)Y + (\nabla_Y \eta)X).$$

A Lorentzian almost paracontact manifold M^n , equipped with the structure (ϕ, ξ, η, g) , is called an *LP-Sasakian manifold* (see [1]) if

$$(\nabla_X \phi)Y = g(\phi X, \phi Y)\xi + \eta(Y)\phi^2 X.$$

In an *LP-Sasakian manifold* the 1-form η is closed. In [1] it is also proved that if an n -dimensional Lorentzian manifold (M^n, g) admits a timelike unit vector field ξ such that the 1-form η associated to ξ is closed and satisfies

$$(\nabla_X \nabla_Y \eta)W = g(X, Y)\eta(W) + g(X, W)\eta(Y) + 2\eta(X)\eta(Y)\eta(W),$$

then M^n admits an *LP-Sasakian structure*.

Further, on such an *LP-Sasakian manifold* M^n with the structure (ϕ, ξ, η, g) the following relations hold:

$$g(R(X, Y)W, \xi) = \eta(R(X, Y)W) = g(Y, W)\eta(X) - g(X, W)\eta(Y), \quad (9)$$

$$R(\xi, X)Y = g(X, Y)\xi - \eta(Y)X, \quad (10)$$

$$R(X, Y)\xi = \eta(Y)X - \eta(X)Y, \quad (11)$$

$$R(\xi, X)\xi = X + \eta(X)\xi, \quad (12)$$

$$S(X, \xi) = (n - 1)\eta(X), \quad (13)$$

$$S(\phi X, \phi Y) = S(X, Y) + (n - 1)\eta(X)\eta(Y) \quad (14)$$

for any vector fields X, Y (see [1,2]), where S is the Ricci curvature and Q is the Ricci operator given by $S(X, Y) = g(QX, Y)$.

An *LP-Sasakian manifold* M^n is said to be η -Einstein if its Ricci tensor S is of the form

$$S(X, Y) = ag(X, Y) + b\eta(X)\eta(Y) \quad (15)$$

for any vector fields X, Y , where a, b are functions on M^n (see [9,10]).

Next we define endomorphisms $R(X, Y)$ and $X \wedge_A Y$ of $\chi(M)$ by

$$R(X, Y)W = \nabla_X \nabla_Y W - \nabla_Y \nabla_X W - \nabla_{[X, Y]}W, \quad (16)$$

$$(X \wedge_A Y)W = A(Y, W)X - A(X, W)Y, \quad (17)$$

respectively, where $X, Y, W \in \chi(M)$ and A is the symmetric $(0, 2)$ -tensor.

For a $(0, k)$ -tensor field T , $k \geq 1$, on (M, g) we define $P \cdot T$, $Z \cdot T$, and $Q(g, T)$ by

$$\begin{aligned} (P(X, Y) \cdot T)(X_1, \dots, X_k) &= -T(P(X, Y)X_1, X_2, \dots, X_k) \\ &\quad -T(X_1, P(X, Y)X_2, \dots, X_k) \\ &\quad - \dots -T(X_1, X_2, \dots, P(X, Y)X_k), \end{aligned} \quad (18)$$

$$\begin{aligned} (Z(X, Y) \cdot T)(X_1, \dots, X_k) &= -T(Z(X, Y)X_1, X_2, \dots, X_k) \\ &\quad -T(X_1, Z(X, Y)X_2, \dots, X_k) \\ &\quad - \dots -T(X_1, X_2, \dots, Z(X, Y)X_k), \end{aligned} \quad (19)$$

$$\begin{aligned} Q(g, T)(X_1, \dots, X_k; X, Y) &= -T((X \wedge Y)X_1, X_2, \dots, X_k) \\ &\quad -T(X_1, (X \wedge Y)X_2, \dots, X_k) \\ &\quad - \dots -T(X_1, X_2, \dots, (X \wedge Y)X_k), \end{aligned} \quad (20)$$

respectively [11].

By definition the Weyl conformal curvature tensor C is given by

$$\begin{aligned} C(X, Y)Z &= R(X, Y)Z - \frac{1}{n-2} \left[\begin{array}{l} g(Y, Z)QX - g(X, Z)QY \\ +S(Y, Z)X - S(X, Z)Y \end{array} \right] \\ &\quad + \frac{\tau}{(n-1)(n-2)} [g(Y, Z)X - g(X, Z)Y], \end{aligned} \quad (21)$$

where Q denotes the Ricci operator, i.e., $S(X, Y) = g(QX, Y)$ and τ is scalar curvature [9]. The Weyl conformal curvature tensor C is defined by $C(X, Y, Z, W) = g(C(X, Y)Z, W)$. If $C = 0$, $n \geq 4$, then M is called conformally flat.

3. MAIN RESULTS

In the present section we consider the LP -Sasakian manifold M^n satisfying the curvature conditions $P \cdot C = 0$, $Z \cdot C = L_C Q(g, C)$, $P \cdot Z - Z \cdot P = 0$, and $P \cdot Z + Z \cdot P = 0$.

First we give the following proposition.

Proposition 1. *Let M be an n -dimensional ($n > 3$) LP -Sasakian manifold. If the condition $P \cdot C = 0$ holds on M , then*

$$\begin{aligned} S^2(X, U) &= \left[\frac{\tau}{n-1} - (n-1)^2 - 1 \right] S(X, U) \\ &\quad + (n-1)[\tau - (n-1)]g(X, U) \\ &\quad + n[\tau - n(n-1)]\eta(X)\eta(U) \end{aligned}$$

is satisfied on M , where $S^2(X, U) = S(QX, U)$.

Proof. Assume that M is an n -dimensional, $n > 3$, LP -Sasakian manifold satisfying the condition $P \cdot C = 0$. From (18) we have

$$\begin{aligned} (P(V, X) \cdot C)(Y, U)W &= P(V, X)C(Y, U)W \\ &\quad - C(P(V, X)Y, U)W - C(Y, P(V, X)U)W \\ &\quad - C(Y, U)P(V, X)W = 0, \end{aligned} \quad (22)$$

where $X, Y, U, V, W \in \chi(M)$. Taking $V = \xi$ in (22), we have

$$\begin{aligned} (P(\xi, X) \cdot C)(Y, U)W &= P(\xi, X)C(Y, U)W \\ &\quad - C(P(\xi, X)Y, U)W - C(Y, P(\xi, X)U)W \\ &\quad - C(Y, U)P(\xi, X)W = 0. \end{aligned} \quad (23)$$

Furthermore, substituting (2), (9), (13), (21) into (23) and multiplying with ξ , we get

$$\begin{aligned} &-g(X, C(Y, U)W) - n\eta(C(Y, U)W)\eta(X) - g(X, Y)\eta(C(\xi, U)W) \\ &+ n\eta(Y)\eta(C(X, U)W) - g(X, U)\eta(C(Y, \xi)W) \\ &+ n\eta(U)\eta(C(Y, X)W) + n\eta(W)\eta(C(Y, U)X) \\ &+ \frac{1}{n-1}\{S(X, C(Y, U)W) + S(X, Y)\eta(C(\xi, U)W) \\ &+ S(X, U)\eta(C(Y, \xi)W)\} = 0. \end{aligned} \quad (24)$$

Thus, replacing W with ξ in (24), we have

$$-g(X, C(Y, U)\xi) - n\eta(C(Y, U)X) + \frac{1}{n-1}S(X, C(Y, U)\xi) = 0. \quad (25)$$

Again, taking $Y = \xi$ in (25) and after some calculations, since $n > 3$, we get

$$\begin{aligned} S^2(X, U) &= \left[\frac{\tau}{n-1} - (n-1)^2 - 1 \right] S(X, U) \\ &\quad + (n-1)[\tau - (n-1)]g(X, U) \\ &\quad + n[\tau - n(n-1)]\eta(X)\eta(U). \end{aligned}$$

Our theorem is thus proved. \square

Theorem 2. Let M be an n -dimensional ($n > 3$) LP -Sasakian manifold. If the condition $Z \cdot C = L_C Q(g, C)$ holds on M , then either M is conformally flat or $L_C = \frac{\tau}{n(n-1)} - 1$.

Proof. Let M^n be an LP -Sasakian manifold. So we have

$$(Z(V, X) \cdot C)(Y, U)W = L_C Q(g, C)(Y, U, W; V, X).$$

Then from (19) and (20) we can write

$$\begin{aligned} & Z(V, X)C(Y, U)W - C(Z(V, X)Y, U)W - C(Y, Z(V, X)U)W \\ & \quad - C(Y, U)Z(V, X)W \\ & = L_C[(V \wedge X)C(Y, U)W - C((V \wedge X)Y, U)W \\ & \quad - C(Y, (V \wedge X)U)W - C(Y, U)(V \wedge X)W]. \end{aligned} \quad (26)$$

Therefore, replacing V with ξ in (26), we have

$$\begin{aligned} & Z(\xi, X)C(Y, U)W - C(Z(\xi, X)Y, U)W - C(Y, Z(\xi, X)U)W \\ & \quad - C(Y, U)Z(\xi, X)W \\ & = L_C[(\xi \wedge X)C(Y, U)W - C((\xi \wedge X)Y, U)W \\ & \quad - C(Y, (\xi \wedge X)U)W - C(Y, U)(\xi \wedge X)W]. \end{aligned} \quad (27)$$

Using (20), (9) and taking the inner product of (27) with ξ , we get

$$\begin{aligned} & \left[1 - \frac{\tau}{n(n-1)} - L_C\right] [-g(X, C(Y, U)W) - \eta(C(Y, U)W)\eta(X) \\ & \quad - g(X, Y)\eta(C(\xi, U)W) + \eta(Y)\eta(C(X, U)W) \\ & \quad - g(X, U)\eta(C(Y, \xi)W) + \eta(U)\eta(C(Y, X)W) + \eta(W)\eta(C(Y, U)X)] = 0. \end{aligned} \quad (28)$$

Putting $X = Y$ in (28), we have

$$\begin{aligned} & \left[1 - \frac{\tau}{n(n-1)} - L_C\right] [-g(Y, C(Y, U)W) + \eta(W)\eta(C(Y, U)Y) \\ & \quad - g(Y, Y)\eta(C(\xi, U)W) - g(Y, U)\eta(C(Y, \xi)W)] = 0. \end{aligned} \quad (29)$$

A contraction of (29) with respect to Y gives us

$$\left[1 - \frac{\tau}{n(n-1)} - L_C\right] \eta(C(\xi, U)W) = 0. \quad (30)$$

If $L_C \neq 1 - \frac{\tau}{n(n-1)}$, then Eq. (30) is reduced to

$$\eta(C(\xi, U)W) = 0, \quad (31)$$

which gives

$$S(U, W) = \left(\frac{\tau}{(n-1)} - 1\right)g(U, W) + \left(\frac{\tau}{(n-1)} - n\right)\eta(U)\eta(W). \quad (32)$$

Therefore, M is a η -Einstein manifold. So, using (31) and (32), we have Eq. (28) in the form

$$C(Y, U, W, X) = 0,$$

which means that M is conformally flat.

If $L_C \neq 0$ and $\eta(C(\xi, U)W) \neq 0$, then $1 - \frac{\tau}{n(n-1)} - L_C = 0$, which gives $L_C = 1 - \frac{\tau}{n(n-1)}$. This completes the proof of the theorem. \square

Corollary 3. Every n -dimensional ($n > 3$) nonconformally flat LP-Sasakian manifold satisfies $Z \cdot C = (1 - \frac{\tau}{n(n-1)})Q(g, C)$.

Theorem 4. Let M be an n -dimensional ($n > 3$) LP-Sasakian manifold. M satisfies the condition

$$P \cdot Z - Z \cdot P = 0$$

if and only if M is a η -Einstein manifold.

Proof. Let M satisfy the condition $P \cdot Z - Z \cdot P = 0$. Then we can write

$$\begin{aligned} & P(V, X) \cdot Z(Y, U)W - Z(V, X) \cdot P(Y, U)W \\ &= \frac{1}{n-1} [R(V, X)R_1(Y, U)W - R_1(V, X)R(Y, U)W] \\ & \quad + \frac{\tau}{n(n-1)^2} [R_1(V, X)R_0(Y, U)W - R_0(V, X)R_1(Y, U)W] \\ & \quad + \frac{\tau}{n(n-1)} [R_0(V, X)R(Y, U)W - R(V, X)R_0(Y, U)W] = 0. \quad (33) \end{aligned}$$

Therefore, replacing V with ξ in (33), we have

$$\begin{aligned} & P(\xi, X) \cdot Z(Y, U)W - Z(\xi, X) \cdot P(Y, U)W \\ &= \frac{1}{n-1} [R(\xi, X)R_1(Y, U)W - R_1(\xi, X)R(Y, U)W] \\ & \quad + \frac{\tau}{n(n-1)^2} [R_1(\xi, X)R_0(Y, U)W - R_0(\xi, X)R_1(Y, U)W] \\ & \quad + \frac{\tau}{n(n-1)} [R_0(\xi, X)R(Y, U)W - R(\xi, X)R_0(Y, U)W] = 0. \quad (34) \end{aligned}$$

Using (10), (13), we get

$$\begin{aligned} & \frac{1}{n-1} [S(U, W)g(X, Y)\xi - S(U, W)\eta(Y)X - g(X, U)S(Y, W)\xi \\ & \quad + S(Y, W)\eta(U)X - S(X, R(Y, U)W)\xi + (n-1)g(U, W)\eta(Y)X \\ & \quad - (n-1)g(Y, W)\eta(U)X] \\ & \quad + \frac{\tau}{n(n-1)^2} [g(U, W)g(X, Y)\xi - g(U, W)\eta(Y)X - g(Y, W)g(X, U)\xi \\ & \quad + g(Y, W)\eta(U)X - S(U, W)g(X, Y)\xi + S(U, W)\eta(Y)X \\ & \quad + S(Y, W)g(X, U)\xi - S(Y, W)\eta(U)X] \\ & \quad + \frac{\tau}{n(n-1)} [g(X, R(Y, U)W)\xi + g(Y, W)\eta(U)X - g(U, W)g(X, Y)\xi \\ & \quad + g(Y, W)g(X, U)\xi - g(Y, W)\eta(U)X] = 0. \quad (35) \end{aligned}$$

Again, taking $U = \xi$ in (35), we get

$$\begin{aligned} & \frac{1}{n-1} [(n-1)g(X, Y)\eta(W)\xi - S(Y, W)\eta(X)\xi - S(Y, W)X \\ & + (n-1)g(Y, W)\eta(X)\xi - S(X, Y)\eta(W)\xi + (n-1)g(Y, W)X] \\ & + \frac{\tau}{n(n-1)^2} [g(X, Y)\eta(W)\xi - \eta(W)\eta(Y)X - g(Y, W)\eta(X)\xi - g(Y, W)X \\ & - (n-1)g(X, Y)\eta(W)\xi + (n-1)\eta(W)\eta(Y)X \\ & - S(Y, W)\eta(X)\xi + S(Y, W)X] = 0. \end{aligned} \quad (36)$$

Taking the inner product of (36) with ξ , we find

$$\begin{aligned} & \frac{1}{n-1} [S(X, Y)\eta(W) - (n-1)g(X, Y)\eta(W)] \\ & + \frac{\tau(n-2)}{n(n-1)^2} [g(X, Y)\eta(W) + \eta(X)\eta(Y)\eta(W)] = 0. \end{aligned} \quad (37)$$

Again, taking $W = \xi$ and using (3) in (37), we get

$$\begin{aligned} S(X, Y) &= \left[(n-1) - \frac{(n-2)}{n(n-1)}\tau \right] g(X, Y) \\ & - \left[\frac{(n-2)}{n(n-1)}\tau \right] \eta(X)\eta(Y). \end{aligned} \quad (38)$$

So, M is a η -Einstein manifold.

Conversely, if M^n is a η -Einstein manifold, then it is easy to show that $P \cdot Z - Z \cdot P = 0$. Our theorem is thus proved. \square

Theorem 5. *Let M be an n -dimensional ($n > 3$) LP-Sasakian manifold. M satisfies the condition*

$$P \cdot Z + Z \cdot P = 0$$

if and only if M is an Einstein manifold.

Proof. Let M satisfy the condition $P \cdot Z + Z \cdot P = 0$. Then, from (33) and (34), we can write

$$\begin{aligned} & 2R(\xi, X)R(Y, U)W \\ & - \frac{1}{n-1} [R(\xi, X)R_1(Y, U)W + R_1(\xi, X)R(Y, U)W] \\ & + \frac{\tau}{n(n-1)^2} [R_1(\xi, X)R_0(Y, U)W + R_0(\xi, X)R_1(Y, U)W] \\ & - \frac{\tau}{n(n-1)} [R_0(\xi, X)R(Y, U)W + R(\xi, X)R_0(Y, U)W] = 0. \end{aligned} \quad (39)$$

Using (6), (10), and (13) in (39), we have

$$\begin{aligned}
& 2[g(X, R(Y, U)W)\xi - g(U, W)\eta(Y)X + g(Y, W)\eta(U)X] \\
& - \frac{1}{n-1}[S(U, W)g(X, Y)\xi - S(U, W)\eta(Y)X - S(Y, W)g(X, U)\xi \\
& + S(Y, W)\eta(U)X + S(X, R(Y, U)W)\xi - (n-1)g(U, W)\eta(Y)X \\
& + (n-1)g(Y, W)\eta(U)X] \\
& + \frac{\tau}{n(n-1)^2}[g(U, W)S(X, Y)\xi - (n-1)g(U, W)\eta(Y)X \\
& - g(Y, W)S(X, U)\xi + (n-1)g(Y, W)\eta(U)X + S(U, W)g(X, Y)\xi \\
& - S(U, W)\eta(Y)X - S(Y, W)g(X, U)\xi + S(Y, W)\eta(U)X] \\
& - \frac{\tau}{n(n-1)}[g(X, R(Y, U)W)\xi - 2g(U, W)\eta(Y)X + 2g(Y, W)\eta(U)X \\
& + g(U, W)g(X, Y)\xi - g(Y, W)g(X, U)\xi] = 0. \tag{40}
\end{aligned}$$

Replacing Y with ξ and using (3) in (40), we have

$$\begin{aligned}
& 2[g(X, R(\xi, U)W)\xi + g(U, W)X + \eta(W)\eta(U)X] \\
& - \frac{1}{n-1}[S(U, W)\eta(X)\xi + S(U, W)X - (n-1)g(X, U)\eta(W)\xi \\
& + 2(n-1)\eta(W)\eta(U)X + S(X, R(\xi, U)W)\xi + (n-1)g(U, W)X] \\
& + \frac{\tau}{n(n-1)^2}[(n-1)g(U, W)\eta(X)\xi + (n-1)g(U, W)X \\
& - S(X, U)\eta(W)\xi + (n-1)\eta(W)\eta(U)X + S(U, W)\eta(X)\xi \\
& + S(U, W)X - (n-1)g(X, U)\eta(W)\xi + (n-1)\eta(W)\eta(U)X] \\
& - \frac{\tau}{n(n-1)}[g(X, R(\xi, U)W)\xi + 2g(U, W)X + 2\eta(W)\eta(U)X \\
& + g(U, W)\eta(X)\xi - g(X, U)\eta(W)\xi] = 0. \tag{41}
\end{aligned}$$

Taking the inner product of (41) with ξ and using (7), (10), we get

$$\begin{aligned}
& \left[2 - \frac{2\tau}{n(n-1)}\right] [g(X, U)\eta(W) + \eta(X)\eta(U)\eta(W)] \\
& + \left[\frac{\tau}{n(n-1)^2} - \frac{1}{n-1}\right] [(n-1)g(X, U)\eta(W) + 2(n-1)\eta(X)\eta(U)\eta(W) \\
& + S(X, U)\eta(W)] = 0. \tag{42}
\end{aligned}$$

Again, taking $W = \xi$ and using (3) in (42), we get

$$\begin{aligned}
& \left[\frac{2\tau}{n(n-1)} - 2\right] [g(X, U) + \eta(X)\eta(U)] \\
& - \left[\frac{\tau}{n(n-1)^2} - \frac{1}{n-1}\right] [(n-1)g(X, U) \\
& + 2(n-1)\eta(X)\eta(U) + S(X, U)] = 0. \tag{43}
\end{aligned}$$

Thus, from (43), we have

$$S(X, U) = (n - 1)g(X, U).$$

So, M^n is an Einstein manifold.

Conversely, if M^n is an Einstein manifold, then it is easy to show that $P \cdot Z + Z \cdot P = 0$. Our theorem is thus proved. \square

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Ühest Lorentzi para-Sasaki muutkondade klassist

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On käsitletud Lorentzi para-Sasaki muutkondi, mille puhul $P \cdot C = 0$, $Z \cdot C = L_C Q(g, C)$, $P \cdot Z - Z \cdot P = 0$ või $P \cdot Z + Z \cdot P = 0$, kus C on Weyli konformse kõveruse tensor, P on v -Weyli projektiivne tensor ja Z on kantsirkulaartensor.