Proc. Estonian Acad. Sci. Phys. Math., 1997, **46**, 4, 251–261
https://doi.org/10.3176/phys.math.1997.4.03

MODEL MATCHING PROBLEM FOR NONLINEAR RECURSIVE SYSTEMS

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Received 20 May 1997, in revised form 3 September 1997

Abstract. The model matching problem for a discrete-time nonlinear system, described by a higher-order difference equation relating the input, the output, and a finite number of their time shifts is studied. A local solution around a regular equilibrium point of the system is given for the case when the system is $(d_1, \ldots, d_p)$-forward time-shift right invertible. The necessary and sufficient conditions for solvability of the problem are derived in terms of the delay orders of the system and the model. The paper does not give any algorithm which explicitly constructs the controller; it only describes how to obtain such controller. The solution proposed in the paper relies on the application of the implicit function theorem and is a natural extension of the corresponding solution for nonlinear systems in the state space form.

Key words: nonlinear recursive systems, model matching problem, delay order, right invertibility.

1. INTRODUCTION

The definition of the nonlinear model matching problem (MMP), now recognized as classical, was introduced in [1]. Given a system and a model, one is looking for a compensator such that the outputs of the compensated system depend on the closed-loop system inputs in the same manner as the model outputs depend on the model inputs. This is a generalization of the linear MMP in the transfer matrix approach. In general, the model is also assumed to be nonlinear. Often, the system, the model, and the compensator are required to be causal. The MMP is a typical design problem in the sense that it plays a key role in various other control problems like the disturbance decoupling problem, the input–output linearization and decoupling problem, and the model reference adaptive control.

For nonlinear systems, the MMP has mainly been studied for systems described by state equations (see, for example, [2,3] and the references therein). There are only few papers concerning the problem for nonlinear systems which are represented by input–output models. In [4,5] the MMP is studied for the systems described by the Volterra series. In [6] the MMP is considered in the differential algebraic setting where the input–output representation of the system is defined by differential field extension. This is in remarkable contrast to the wide application of *linear* higher-order input–output difference equations in digital control. The success of this approach motivates our search for feedback design methods that can be directly applied to nonlinear higher-order input–output difference equations.

The purpose of this paper is to study the MMP for the nonlinear system described by a higher-order difference equation relating the inputs, the outputs, and a finite number of their time shifts:

$$y(t) = F(y(t-1), \dots, y(t-n), u(t-1), \dots, u(t-n)). \tag{1}$$

Systems of the form (1) are called recursive nonlinear systems (RNSs) [7–9]. This representation has obvious advantages if the model of the system has to be obtained via identification, either using the traditional approaches [9] or the neural networks [10,11].

One may think that once the equations of the RNS have been obtained via identification, one can transform these equations into the state space form and then apply the conventional state space control theory. However, this idea is not feasible in practice, because the application of the realization algorithms is not an easy task, even for linear systems. In the nonlinear case the situation is much more complicated. Firstly, the realization problem of a given input–output nonlinear system is not completely solved. Secondly, it has been shown [12,13] that a nonlinear input–output discrete-time system cannot, in general, be realized by state space equations. To realize the recursive input–output model, one needs to introduce generalized state equations whose dynamics involves the control variables and a finite number of their time shifts [12]. In other words, the class of recursive input–output models describes a broader class of systems than the class of the state space models. Thus there is an obvious need to study several fundamental issues for RNS, including the MMP that is critical to the advancement of the discrete nonlinear control design techniques.

In this paper we investigate the MMP only for a special subclass of RNSs – $(d_1, \dots, d_p)$-forward time-shift (FTS) right invertible systems [14,15] – and follow the approach used in the state space formulation, where the necessary and sufficient conditions for local solvability of the MMP via regular dynamic state feedback were formulated in terms of the delay orders. We show that the above result has its direct counterpart for the RNSs. We concentrate on the local solutions around a regular equilibrium point of the system. However, the paper does not give any algorithm which explicitly constructs the solution; it only presents necessary and sufficient conditions under which the feedback locally exists, the structure of the controller, and describes how to obtain it. The solution given in the paper relies on the application of the implicit function theorem.

To our best knowledge, there exist only a few short papers dealing with the synthesis problems for the RNS: [7] studies the output dead-beat control and [16] the output tracking problem. Both papers consider only the single-input single-output systems.

2. RECURSIVE NONLINEAR SYSTEMS

Consider the RNS (1) at non-negative time steps in a finite time interval $0 \leq t \leq t_F$ under the initial conditions

$$x(0) = [y^T(-1), \ldots, y^T(-n), u^T(-1), \ldots, u^T(-n)]^T.$$

The mapping F is supposed to be C^ω, the inputs $u(t) \in U$, an open subset of R^m, the outputs $y(t) \in Y$, an open subset of R^p. In this section we introduce some preliminary material [14,15].

Definition 2.1. Equilibrium point. *The pair of constant values* (u^0, y^0) *is called the equilibrium point of the RNS* (1) *if* (u^0, y^0) *satisfies the equality* $y^0 = F(y^0, \ldots, y^0, u^0, \ldots, u^0)$.

Throughout the paper we shall adopt a local viewpoint. More precisely, we work around an equilibrium point (u^0, y^0) of the system (1). Let us denote by $\mathcal{U}^0$ (resp. $\bar{U}$) the set of control sequences $\mathbf{u} = \{u(t); 0 \leq t \leq t_F\}$ (resp. $\{u^T(t-1), \ldots, u^T(t-n)\}$) such that the controls $u(t)$ for every t are sufficiently close to u^0, i.e., $\| u(t) - u^0 \| \leq \delta$ for some $\delta > 0$. Analogously, let us denote by $\mathcal{Y}^0$ (resp. $\bar{Y}$) the set of output sequences $\{y(t); 0 \leq t \leq t_F\}$ (resp. $\{y^T(t-1), \ldots, y^T(t-n)\}$) such that the outputs $y(t)$ for every t are sufficiently close to y^0, i.e., $\| y(t) - y^0 \| < \epsilon$ for some $\epsilon > 0$. Denote by $\mathcal{Y}_i^0$ the set of sequences $\{y_i(t); 0 \leq t \leq t_F\}$ such that $\| y_i(t) - y_i^0 \| < \epsilon_i$ for some $\epsilon_i > 0$. Denote by x^0 a $n(p+m)$-dimensional vector $(y^{0,T}, \ldots, y^{0,T}, u^{0,T}, \ldots, u^{0,T})^T$. Finally, let us denote by X^0 the neighbourhood of x^0 such that for every $x \in X^0$, $\| x - x^0 \| < \gamma$ for some $\gamma > 0$.

For RNSs, the delay orders $d_i, i = 1, \ldots, p$, with respect to the control have been defined [14,15], one for each output component, as follows. Apply the one-step forward shift operator to Eq. (1) and replace in the latter $y(t)$ via the initial conditions, i.e., via the right-hand side of (1) in order to obtain

$$\begin{aligned}
y(t+1) &= F(y(t), \ldots, y(t-n+1), u(t), \ldots, u(t-n+1)) \\
&= F(F(y(t-1), \ldots, y(t-n), u(t-1), \ldots, u(t-n)), \ldots, \\
&\qquad y(t-n+1), u(t), \ldots, u(t-n+1)) \\
&\triangleq F^1(y(t-1), \ldots, y(t-n), u(t), \ldots, u(t-n)).
\end{aligned}$$

Denote the ith component of F^1 by F_i^1 and compute for $i = 1, \ldots, p$ the derivative

$$\frac{\partial}{\partial u(t)} F_i^1(y(t-1), \ldots, y(t-n), u(t), \ldots, u(t-n)).$$

From the analyticity of the system (1) it follows that either the vector $\partial F_i^1(\cdot)/\partial u(t)$ is nonzero for all $(y(t-1),\ldots,y(t-n),u(t),\ldots,u(t-n))$ belonging to an open and dense subset O_i of $\bar{Y}\times\bar{U}$ or this vector vanishes for all $(y(t-1),\ldots,$ $y(t-n),u(t),\ldots,u(t-n))\in\bar{Y}\times\bar{U}$. In the first case we define $d_i=1$ whereas in the latter case we continue by observing that the function F_i^1 does not depend on $u(t)$, i.e., it depends completely on the initial conditions and so we may write

$$y_i(t+1)=F_i^1(y(t-1),\ldots,y(t-n),u(t-1),\ldots,u(t-n)). \tag{2}$$

Apply again a forward shift operator to Eq. (2) and replace in the latter $y(t)$ via the right-hand side of (1). Compute in an analogous fashion the derivative

$$\frac{\partial}{\partial u(t)}F_i^2(y(t-1),\ldots,y(t-n),u(t),\ldots,u(t-n)).$$

If this vector is nonzero on an open and dense subset O_i of $\bar{Y}\times\bar{U}$, we set $d_i=2$; otherwise we continue with

$$y_i(t+2)=F_i^2(y(t-1),\ldots,y(t-n),u(t-1),\ldots,u(t-n)). \tag{3}$$

In this way the number d_i – if it exists – determines the inherent delay between the inputs and the ith output.

These system structural parameters tell us how many inherent delays there are between the ith component y_i of the output and the control, or equivalently, for how many first time instants y_i is completely defined by the initial conditions and which is the first time instant for which the possibility arises to change y_i arbitrarily (which does not necessarily realize in every case).

The RNS (1) with delay orders $d_i, i=1,\ldots,p$, admits a representation of the form

$$\begin{array}{rcl} y_1(t+d_1) & = & F_1^{d_1}(x(t),u(t)) \\ & \vdots & \\ y_p(t+d_p) & = & F_p^{d_p}(x(t),u(t)) \end{array} \tag{4}$$

or in the vector form

$$\begin{bmatrix} y_1(t+d_1) \\ \vdots \\ y_p(t+d_p) \end{bmatrix} = A(x(t),u(t)), \tag{5}$$

where

$$x(t)=[y^T(t-1),\ldots,y^T(t-n),u^T(t-1),\ldots,u^T(t-n)]^T.$$

Definition 2.2. $(d_1, \ldots, d_p)$-FTS right invertibility. *The RNS* (1) *is called locally $(d_1, \ldots, d_p)$-FTS right invertible in a neighbourhood of its equilibrium point (u^0, y^0) if there exist sets $\mathcal{U}^0$, $\mathcal{Y}^0$, and X^0 such that given $x(0) \in X^0$, we are able to find for any sequence $\{y_{ref}(t); 0 \leq t \leq t_F\} \in \mathcal{Y}^0$ a control sequence $\{u_{ref}(t); 0 \leq t \leq t_F\} \in \mathcal{U}^0$ (not necessarily unique) yielding*

$$y_i(t, x(0), u_{ref}(0), \ldots, u_{ref}(t)) = y_{ref,i}(t), d_i \leq t \leq t_F,\ i = 1, \ldots, p.$$

The above definition says that for the ith output component it is possible to reproduce locally all sequences $\{y_{ref,i}(t); 0 \leq t \leq t_F\}$ from $\mathcal{Y}_i^0$, beginning from the time instant d_i. But $(d_1, \ldots, d_p)$-FTS right invertibility does not allow us to reproduce the first d_i terms in the arbitrary sequence $\{y_{ref,i}(t); 0 \leq t \leq t_F\} \in \mathcal{Y}_i^0$.

Consider the RNS (1) with delay orders $d_i < \infty$, $i = 1, \ldots, p$, i.e., the system, described by Eqs. (4). The so-called decoupling matrix $K(x, u)$ for the system (1) is defined as

$$K(x, u) = \frac{\partial}{\partial u} \begin{bmatrix} F_1^{d_1}(x, u) \\ \vdots \\ F_p^{d_p}(x, u) \end{bmatrix}.$$

From the definition of the d_i's the rows of the matrix $K(x, u)$ are nonzero vector functions around (u^0, y^0). It is obvious that the rank of $K(x, u)$ is, in general, input and output dependent. However, we shall assume that $K(x, u)$ has a constant rank around (u^0, y^0). This assumption is formalized in the notion of regularity of an equilibrium point.

Definition 2.3. Regularity of an equilibrium point. *We call the equilibrium point (u^0, y^0) of the system* (1) *regular with respect to $(d_1, \ldots, d_p)$-FTS right invertibility if the rank of the decoupling matrix $K(x, u)$ of the system* (1) *is constant around (u^0, y^0).*

Theorem 2.4. *Assume that for the system* (1) *$d_i < \infty$, $i = 1, \ldots, p$. Then the RNS* (1) *is locally $(d_1, \ldots, d_p)$-FTS right invertible around a regular equilibrium point (u^0, y^0) if and only if rank $K(x^0, u^0) = p$.*

3. THE FORMULATION AND THE SOLUTION OF THE MODEL MATCHING PROBLEM

Consider the RNS S, described by (1), and a discrete-time nonlinear model M of the similar form:

$$y^M(t) = F^M(y^M(t-1), \ldots, y^M(t-n_M), u^M(t-1), \ldots, u^M(t-n_M)), \tag{6}$$

where the outputs $y^M(t)$ belong to an open subset Y^M of R^p, and the inputs $u^M(t)$ belong to an open subset U^M of R^m, all for $0 \leq t \leq t_F$. The mapping F^M is supposed to be smooth. Denote

$$x^M(t) = [y^{M,T}(t-1), \dots , y^{M,T}(t-n_M), u^{M,T}(t-1), \dots , u^{M,T}(t-n_M)]^T.$$

The dynamic output compensator C, to be designed to control the system S, is assumed to be a discrete-time nonlinear system described by equations of the form

$$\begin{aligned} y^C(t) &= F^C(y^C(t-1), \dots , y^C(t-n_C), u^M(t-1), \dots , u^M(t-n_C)), \\ u(t) &= \alpha(y(t-1), \dots , y(t-n), u(t-1), \dots , u(t-n), \\ &\quad y^C(t-1), \dots , y^C(t-n_C), u^M(t-1), \dots , u^M(t-n_C), u^M(t)), \end{aligned} \tag{7}$$

with the output $y^C(t) \in Y^C$, an open subset of R^p, and F^C being a smooth mapping. Notice that $u(t)$ does not depend on the future outputs $y(t+1), \dots , y(t+d-1)$, but only on outputs up to the time instant t, i.e., only on variables that are available on the time instant t, and is therefore implementable. Denote

$$x^C(t) = [y^{C,T}(t-1), \dots , y^{C,T}(t-n_C), u^{C,T}(t-1), \dots , u^{C,T}(t-n_C)]^T.$$

The composition of (1) and (7) (i.e., the closed-loop system), initialized at $(x(0), x^C(0))$, is denoted by $S \circ C$.

Recall that we are assumed to work in a neighbourhood of an equilibrium point of the system (1). We say that the equilibrium point (u^{M0}, y^{M0}) of the model M corresponds to the equilibrium point (u^0, y^0) of the system S if $y^0 = y^{M0}$.

Let us give now the formal definition of the local MMP for $(d_1, \dots , d_p)$-FTS right invertible system.

Definition 3.1. Local MMP. *Given the $(d_1, \dots , d_p)$-FTS invertible system S defined by Eq. (1) around a regular equilibrium point (u^0, y^0), the model M defined by Eq. (6) around an equilibrium point (u^{M0}, y^{M0}) corresponding to (u^0, y^0), and the point $(x(0), x^M(0))$, find, if possible, a compensator C defined by equations of the form (7), together with the initial conditions $x^C(0)$, the equilibrium point (u^{M0}, y^{C0}), the neighbourhoods $V_1 = X^0 \times X^{C0} \times U^{M0}$ of (x^0, x^{C0}, u^{M0}) in $X \times X^C \times U^M$ and V_2 of u^0 in U being the domain and the range of C, respectively, as well the neighbourhood X^{M0} of x^{M0} and the map $\xi : X^{M0} \to X^{C0}$, with the property that*

$$y_i^{S \circ C}(t, x(0), \xi(x^M(0)), u^M(0), \dots , u^M(t-1))$$

$$= y_i^M(t, x^M(0), u^M(0), \dots , u^M(t-1)), \;\; d_i \leq t \leq t_F, \quad i = 1, \dots , p$$

for all $(x(0), x^M(0)) \in X^0 \times X^{M0}$ and for all $u^M(t)$ in the domain of C.

Note that we do not require the first terms in the output sequences of $S \circ C$ and M (which are completely defined by the corresponding initial conditions) to be the same.

Theorem 3.2. *Consider the system* (1) *around a regular equilibrium point* (u^0, y^0) *with respect to* $(d_1, \ldots, d_p)$*-FTS right invertibility, and the model* (6) *around an equilibrium point* (u^{M0}, y^{M0})*, corresponding to* (u^0, y^0)*. Suppose that the system* (1) *is locally* $(d_1, \ldots, d_p)$*-FTS right invertible around* (u^0, y^0)*. Then the MMP is locally solvable if and only if the delay orders of the model* (6) *are equal to or greater than those of the original system:* $d_i^M \geq d_i,\ i = 1, \ldots, p$.

Proof. Suppose the system (1) is locally $(d_1, \ldots, d_p)$-FTS right invertible around a regular equilibrium point (u^0, y^0). Then the decoupling matrix $K(x, u)$ has rank p around the point (x^0, u^0). Consider the equation

$$\begin{bmatrix} y_1^M(t+d_1) \\ \vdots \\ y_p^M(t+d_p) \end{bmatrix} = A(x(t), u(t)). \tag{8}$$

Observe that the Jacobian matrix of the right-hand side of (8) with respect to $u(t)$ equals $K(x(t), u(t))$. So we may apply the implicit function theorem yielding locally $u(t)$ as an analytic function of $x(t)$ and $y_1^M(t+d_1), \ldots, y_p^M(t+d_p)$, i.e.,

$$u(t) = \alpha(x(t), y_1^M(t+d_1), \ldots, y_p^M(t+d_p)) \tag{9}$$

such that

$$\begin{bmatrix} y_1^M(t+d_1) \\ \vdots \\ y_p^M(t+d_p) \end{bmatrix} = A(x(t), \alpha(x(t), y_1^M(t+d_1), \ldots, y_p^M(t+d_p))). \tag{10}$$

Then from (9) and (10) it follows that if and only if we apply a compensator C given by Eq. (9) to the system (1), then the outputs of the model and the compensated system coincide (starting from the time instant $t = d_i$ for the ith output component), i.e.,

$$y_i^{S \circ C}(t) = y_i^M(t)\,,\ d_i \leq t \leq t_F\,,\ i \in \{1, \ldots, p\}$$

as long as $(x(t), y_1^M(t+d_1), \ldots, y_p^M(t+d_p)) \in X^0 \times Y^0$ and $u(t) \in U^0$.

The compensator (9) can be given in the form (7) if and only if the delay orders d_i^M of the model are equal to or greater than the corresponding delay orders of the system (1), that is, $d_i^M \geq d_i,\ i = 1, \ldots, p$. In that case, defining the functions $F_i^{M,d_i}(x^M)$ analogously to the functions $F_i^{d_i}(x)$ (see (4)), we obtain

$$y_i^M(t+d_i) = F_i^{M,d_i}(x^M(t), u^M(t)),\quad i = 1, \ldots, p\,, \tag{11}$$

and substituting (11) into (9) gives

$$u(t) = \alpha(x(t), F_i^{M,d_i}(x^M(t), u^M(t)), \quad i = 1, \ldots, p) \triangleq \varphi(x(t), x^M(t), u^M(t)).$$

Note that the outputs of the model y^M (i.e., the components of x^M) in this control law can be computed from the dynamic equations of the model which thus determine the dynamic part of the compensator.

So, the compensator C together with its initial condition $x^C(0) = x^M(0)$, solving the MMP is the following

$$\begin{aligned} y^M(t) &= F^M(y^M(t-1), \ldots, y^M(t-n_M), u^M(t-1), \ldots, u^M(t-n_M)), \\ u(t) &= \varphi(x(t), x^M(t), u^M(t)) \end{aligned} \tag{12}$$

and since $x^C(0) = x^M(0)$, $\xi = \mathrm{Id}$ (identity map).

From (11) we can see that $(x(t), y_1^M(t+d_1), \ldots, y_p^M(t+d_p^M))$ belong to $X^0 \times Y^0$ as long as $(x(t), x^C(t), u^M(t))$ belong to some neighbourhood V_1 of (x^0, x^{C0}, u^{M0}). This completes the proof.

Let us remark that the proof of Theorem 3.2, up to application of the implicit function theorem, is constructive. If we know how to solve Eq. (8), the proof indicates how to find the equations of the compensator which solves the MMP.

4. EXAMPLES

Example 1. Consider the single-input single-output RNS S

$$y(t) = y(t-3)y(t-2)u(t-3) + y^2(t-3)u(t-1), \quad t \geq 0,$$

which does not have a state space realization [13], and the model M described by

$$y^M(t) = ay^M(t-1) + u^M(t-1). \tag{13}$$

Since $d = d^M = 1$, the MMP is solvable around an equilibrium point (u^0, y^0) such that $y^0 \neq 0$. In order to obtain the equations of the compensator (see the proof of Theorem 3.2), we have first to solve the equation

$$y^M(t+1) = y(t-2)y(t-1)u(t-2) + y^2(t-2)u(t)$$

for $u(t)$ and then replace $y^M(t+1)$ in the obtained equation by $a^2y^M(t-1) + au^M(t-1) + u^M(t)$:

$$u(t) = \\ \{a^2y^M(t-1) + au^M(t-1) + u^M(t) - y(t-2)y(t-1)u(t-2)\}/y^2(t-2).$$

The dynamics of the compensator is defined by (13).

Example 2. Consider the system

$$\begin{aligned} y_1(t) &= u_1(t-1) + u_2(t-2)y_2(t-1), \\ y_2(t) &= u_2(t-2) - u_2(t-3)y_1(t-2) + y_3(t-1), \\ y_3(t) &= -u_2(t-1) - u_3(t-2)y_2(t-2) \end{aligned}$$

and the model

$$y_i^M(t) = u_i^M(t-2), \; i = 1, 2, 3.$$

According to (9), in order to find the compensator that solves the MMP, we have to solve the following system of equations for $u(t)$:

$$\begin{aligned} y_1^M(t+1) &= u_1(t) + u_2(t-1)[u_2(t-2) - u_2(t-3)y_1(t-2) \\ &\quad + y_3(t-1)], \\ y_2^M(t+3) &= -u_2(t)u_1(t) - [u_2(t)u_2(t-1) + u_3(t)][u_2(t-2) \\ &\quad - u_2(t-3)y_1(t-2) + y_3(t-1)], \\ y_3^M(t+1) &= -u_2(t) - u_3(t-1)y_2(t-1). \end{aligned}$$

Unfortunately, doing this we do not get the compensator of the form (7), since

$$y_2^M(t+3) = u_2^M(t+1).$$

The reason is that one of the delay orders of the model is greater than the corresponding delay order of the system: $3 = d_2 > d_2^M = 2$.

5. CONCLUSIONS

The model matching problem (MMP) is studied for a class of recursive nonlinear systems (RNSs), i.e., for systems, modelled by recursive nonlinear input–output equations involving only a finite number of input values and a finite number of output values. The solution of the problem via the dynamic state feedback, known for the discrete-time nonlinear systems in the state space form, is extended to the RNS. The necessary and sufficient conditions for local solvability of the problem as well as the procedure for constructing a dynamic output feedback compensator are proposed for the subclass of $(d_1, \ldots, d_p)$-FTS invertible RNSs. The proposed solution has the following properties:

1. The model matching is achieved by using only input and output variables.
2. The control $u(t)$ does not depend on future outputs, but only on variables that are available at time t, and is therefore implementable.

Of course, it would be extremely interesting to relate the concepts and the results obtained in this paper to the known concepts and results of nonlinear systems in the state space form, by first realizing the RNS in a state space representation. Since the RNS, in general, cannot be realized by the standard state equations [12,13], this comparison is not an easy task and can be done only for a subclass of realizable systems. We leave this topic for the future research.

ACKNOWLEDGEMENT

The author gratefully acknowledges that this work was partly supported by the Estonian Science Foundation under grant No. 2243.

REFERENCES

1. Di Benedetto, M. D. and Isidori, A. The matching of nonlinear models via dynamic state feedback. *SIAM J. Contr. Optim.*, 1986, **24**, 1063–1075.
2. Moog, C. H., Perdon, A. M., and Conte, G. Model matching and factorization of nonlinear systems: A structural approach. *SIAM J. Contr. Optim.*, 1991, **29**, 769–785.
3. Kotta, Ü. *Inversion Method in the Discrete-Time Nonlinear Control Systems Synthesis Problems*. Springer-Verlag, London, 1995.
4. Yamanaka, O., Ito, H., Ohmori, H., and Sano, A. Exact model matching scheme for finite Volterra series systems. In *Proc. of the 33rd Conf. on Decision and Control*. Lake Buena Vista, 1994, 1794–1796.
5. Kotsios, S. and Kalouptsidis, N. The model matching problem for a certain class of nonlinear systems. *Internat. J. Contr.*, 1993, **57**, 881–919.
6. Rudolph, J. A differential algebraic approach to the classical right model matching problem. In *Prepr. of 1st IFAC Workshop "New Trends in Design of Control Systems"*. Smolenice, 1994, 112–117.
7. Bastin, G., Jarachi, F., and Mareels, I. M. Y. Dead beat control of recursive nonlinear systems. In *Proc. 32nd Conf. on Decision and Control*. San Antonio, Texas, 1993, 2965–2971.
8. Hammer, J. Nonlinear systems: Stability and rationality. *Internat. J. Contr.*, 1984, **40**, 1–35.
9. Leontaritis, I. J. and Billings, S. A. Input–output parametric models for non-linear systems. Part I: Deterministic nonlinear systems. *Internat. J. Contr.*, 1985, **41**, 303–328.
10. Levin, A. U. and Narendra, K. S. Recursive identification using feedforward neural networks. *Internat. J. Contr.*, 1995, **61**, 533–547.
11. Levin, A. U. and Narendra, K. S. Control of nonlinear dynamical systems using neural networks. Part II: Observability, identification and control. *IEEE Trans. Neural Networks*, 1996, **7**, 30–42.
12. Fliess, M. Automatique en temps discret et algèbre aux différences. *Forum Math.*, 1990, **2**, 213–232.
13. Kotta, Ü., Liu, P., and Zinober, A. State-space realization of input–output nonlinear difference equations. In *Proc. of European Control Conference*. Brussels, 1997.
14. Kotta, Ü. On right invertibility of nonlinear recursive systems. In *Proc. of the 13th IFAC World Congress*. San Francisco, 1996, **F**.
15. Kotta, Ü. On right invertibility of nonlinear recursive systems. *Proc. Estonian Acad. Sci. Phys. Math.*, 1996, **45**, 4, 279–291.

16. Narendra, K. S. and Cabrera, J. B. D. Input–output representation of discrete-time dynamical systems – nonlinear ARMA models. In *Proc. of the 33rd Conf. on Decision and Control*. Lake Buena Vista, 1994, 1118–1119.

MITTELINEAARSETE REKURSIIVSETE SÜSTEEMIDE MUDELIGA SOBITAMISE ÜLESANNE

Ülle KOTTA

On uuritud mudeliga sobitamise ülesannet mittelineaarsete rekursiivsete süsteemide puhul, mis on kirjeldatavad sisendeid ja väljundeid siduvate kõrgemat järku diferentsvõrranditega. $(d_1, \ldots, d_p)$-nihkega paremalt pööratavate süsteemide alamklassi tarvis on leitud ülesande lokaalne lahend juhtimisobjekti regulaarse tasakaalupunkti ümbruses. Ülesande lahenduvuse tarvilikud ja piisavad tingimused on formuleeritud kahe süsteemi – juhtimisobjekti ja mudeli – teatud täisarvuliste struktuuriparameetrite, nn. hilistumisjärkude abil. Et lahend põhineb teoreemil ilmutamata funktsioonist, siis ei esita artikkel kompensaatori konstrueerimise algoritmi, vaid ainult kirjeldab, kuidas seda leida. Esitatud tulemused üldistavad teadaolevaid tulemusi olekumudeliga kirjeldatava mittelineaarse süsteemi jaoks.