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TWO-GRID METHOD FOR THE SOLUTION OF WEAKLY SINGULAR INTEGRAL EQUATIONS BY PIECEWISE POLYNOMIAL APPROXIMATION

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Abstract. To solve weakly singular integral equations by the piecewise polynomial collocation method it is necessary to solve large linear systems. In this paper a two-grid iteration method is presented for the solution of such systems and the convergence rate of this method is discussed. The efficiency of the method is shown.

Key words: weakly singular integral equation, collocation method, two-grid method.

1. INTRODUCTION

Consider the weakly singular integral equation

$$u(x) = \int_G K(x,y)u(y)dy + f(x),\ x \in G, \tag{1}$$

where

$$G = \{x = (x_1, \ldots, x_n) : 0 < x_k < b_k,\ k = 1, \ldots, n\}$$

is an n-dimensional parallelepiped. Such an equation can be effectively solved by a collocation method. In this case the parallelepiped $\overline{G}$ is partitioned into small parallelepipeds (cells) and the approximate solution is searched in the form of a function which on every cell is a polynomial of the same degree. Such a piecewise polynomial collocation method is discussed, e.g., in [1–3]. It is shown there how to choose the nonuniform grid so that the method would have the best convergence rate for weakly singular equations. To calculate the approximate solution, large linear systems must be solved. In the present paper a two-grid iteration method is used for the solution of such a system. At every step of this method it is necessary to solve a smaller linear system corresponding to a coarse grid. The main results of the study are presented in the last section where the convergence rate of the two-grid method is proved.

2. INTEGRAL EQUATION

For the integral equation (1) we shall make the following assumptions (A1)–(A3).

(A1) The kernel $K(x,y)$ is m times ($m \geq 1$) continuously differentiable with respect to x and y for $x, y \in G,\ x \neq y$, and there exists a real number $\nu \in (-\infty, n)$ such that the estimate

$$|D_x^\alpha D_{x+y}^\beta K(x,y)| \leq \text{const} \left\{ \begin{array}{ll} 1, & \nu + |\alpha| < 0 \\ 1 + |\log|x-y||, & \nu + |\alpha| = 0 \\ |x-y|^{-\nu-|\alpha|}, & \nu + |\alpha| > 0 \end{array} \right\}, \quad x, y \in G,$$

holds for all multi-indices $\alpha \in (\alpha_1, \ldots, \alpha_n) \in \mathbb{Z}_+^n$ and $\beta = (\beta_1, \ldots, \beta_n) \in \mathbb{Z}_+^n$ with $|\alpha| + |\beta| \leq m$. Here

$$|\alpha| = \alpha_1 + \ldots + \alpha_n, \quad |x| = (x_1^2 + \ldots + x_n^2)^{1/2},$$

$$D_x^\alpha = \left(\frac{\partial}{\partial x_1}\right)^{\alpha_1} \cdots \left(\frac{\partial}{\partial x_n}\right)^{\alpha_n},$$

$$D_{x+y}^\beta = \left(\frac{\partial}{\partial x_1} + \frac{\partial}{\partial y_1}\right)^{\beta_1} \cdots \left(\frac{\partial}{\partial x_n} + \frac{\partial}{\partial y_n}\right)^{\beta_n}.$$

(A2) $f \in C^{m,\nu}(G)$ with the same ν as in (A1), i.e., $f(x)$ is m times continuously differentiable on G and the estimates

$$|D^\alpha f(x)| \leq \text{const} \left\{ \begin{array}{ll} 1, & |\alpha| < n - \nu \\ 1 + |\log \rho(x)|, & |\alpha| = n - \nu \\ \rho(x)^{n-\nu-|\alpha|}, & |\alpha| > n - \nu \end{array} \right\},$$

$$\left|\frac{\partial^l f(x)}{\partial x_k^l}\right| \leq \text{const} \left\{ \begin{array}{ll} 1, & l < n - \nu \\ 1 + |\log \rho_k(x)|, & l = n - \nu \\ \rho_k(x)^{n-\nu-l}, & l > n - \nu \end{array} \right\}$$

hold for $x \in G,\ |\alpha| \leq m,\ l = 1, \ldots, m,\ k = 1, \ldots, n$, where $\rho_k(x) = \min\{x_k, b_k - x_k\}$ and $\rho(x) = \min_{1 \leq k \leq n} \rho_k(x)$ is the distance from x to ∂G.

(A3) Equation (1) has a unique solution $u \in L^\infty(G)$.

From (A1)–(A3) it follows that, for the solution u of (1), we have $u \in C^{m,\nu}(G)$ [3].

3. PIECEWISE POLYNOMIAL COLLOCATION METHOD

To define the partition of $\overline{G}$ into cells, we choose the vector $N = (N_1, \ldots, N_n)$ of natural numbers and introduce in the interval $[0, b_k]$, $k = 1, \ldots, n$, the following $2N_k + 1$ grid points

$$x_{k,N}^{j_k} = \frac{b_k}{2}\left(\frac{j_k}{N_k}\right)^r, \quad j_k = 0, 1, \ldots, N_k,$$

$$x_{k,N}^{N_k+j_k} = b_k - x_{k,N}^{N_k-j_k}, \quad j_k = 1, \ldots, N_k. \tag{2}$$

Here $r \in \mathbb{R}$, $r \geq 1$, characterizes the degree of the nonuniformity of the grid. If $r = 1$, then the grid points are uniformly located.

Making use of points (2), we introduce the partition of $\overline{G}$ into closed cells

$$G_{j,N} = \{x = (x_1, \ldots, x_n) : x_{k,N}^{j_k-1} \leq x_k \leq x_{k,N}^{j_k}, \quad k = 1, \ldots, n\} \subset \overline{G},$$

$$j \in J_N = \{j = (j_1, \ldots, j_n) : j_k = 1, \ldots, 2N_k, \quad k = 1, \ldots, n\}.$$

We determine the collocation points in the following way. We choose m points $\eta_1, \ldots, \eta_m$ in the interval $[-1, 1]$:

$$-1 \leq \eta_1 < \eta_2 < \ldots < \eta_m \leq 1.$$

By affine transformations we transfer them into the interval $\left[x_{k,N}^{j_k-1}, x_{k,N}^{j_k}\right]$:

$$\xi_{k,N}^{j_k,q_k} = x_{k,N}^{j_k-1} + \frac{\eta_{q_k}+1}{2}\left(x_{k,N}^{j_k} - x_{k,N}^{j_k-1}\right),$$

$$q_k = 1, \ldots, m, \quad j_k = 1, \ldots, 2N_k, \quad k = 1, \ldots, n.$$

Note that $\xi_{k,N}^{j_k,m} = \xi_{k,N}^{j_k+1,1} = x_{k,N}^{j_k}$ if $\eta_1 = -1$ and $\eta_m = 1$, $j_k = 1, \ldots, 2N_k - 1$. We assign the collocation points

$$\xi_N^{j,q} = \left(\xi_{1,N}^{j_1,q_1}, \ldots, \xi_{n,N}^{j_n,q_n}\right),$$

$$q \in Q = \{q = (q_1, \ldots, q_n) : q_k = 1, \ldots, m, \ k = 1, \ldots, n\},$$

to the cells $G_{j,N}$, $j \in J_N$.

For a function $u: \overline{G} \to \mathbb{R}$ we construct a piecewise polynomial interpolation function $\mathcal{P}_N u: \overline{G} \to \mathbb{R}$ as follows. On every cell $G_{j,N}$, $j \in J_N$, $\mathcal{P}_N u$ is a polynomial of the degree not exceeding $m-1$ with respect to any of arguments $x_1, \ldots, x_n$, whereby $\mathcal{P}_N u$ interpolates u at points $\xi_N^{j,q}$, $q \in Q$. Thus, the interpolant $\mathcal{P}_N u$ is uniquely defined in every cell separately and may have jumps on hyperplanes $x_k = x_{k,N}^{j_k}$, $j_k = 1, \ldots, 2N_k - 1$, $k = 1, \ldots, n$. We may treat

$\mathcal{P}_N u$ as a multivalued function on these hyperplanes. Note that $\mathcal{P}_N u$ is a continuous function on $\overline{G}$ in the case $\eta_1 = -1$ and $\eta_m = 1$.

We can define the interpolation projector $\mathcal{P}_N$ by the formula

$$(\mathcal{P}_N u)(x) = \sum_{q\in Q} u\left(\xi_N^{j,q}\right) \varphi_N^{j,q}(x), \quad x \in G_{j,N}, \quad j \in J_N, \tag{3}$$

where

$$\varphi_N^{j,q}(x) = \varphi_{1,N}^{j_1,q_1}(x_1)\dots\varphi_{n,N}^{j_n,q_n}(x_n)$$

and $\varphi_{k,N}^{j_k,q_k}(x_k)$ are the polynomials of one variable of the degree $m-1$ such that

$$\varphi_{k,N}^{j_k,q_k}\left(\xi_{k,N}^{j_k,p_k}\right) = \begin{Bmatrix} 1, & p_k = q_k \\ 0, & p_k \neq q_k \end{Bmatrix}, \quad p_k = 1,\dots,m. \tag{4}$$

Let us denote by E_N the range of the projector $\mathcal{P}_N$. This is the finite dimensional space of piecewise polynomial functions u_N on $\overline{G}$ which on any cell $G_{j,N}$, $j \in J_N$, are polynomials of the degree not exceeding $m-1$ with respect to any of arguments $x_1,\dots,x_n$.

We determine the approximate solution $u_N \in E_N$ of the integral equation (1) from the collocation condition:

$$\left[u_N(x) - \int_G K(x,y)u_N(y)dy - f(x)\right]_{x=\xi_N^{i,p}} = 0, \quad p \in Q, \quad i \in J_N. \tag{5}$$

By the representation (3), we can find $u_N \in E_N$ in the form

$$u_N(x) = \sum_{q\in Q} c^{j,q}\varphi_N^{j,q}(x), \quad \text{if } x \in G_{j,N}, \quad j \in J_N,$$

where, as it follows from (4),

$$c^{j,q} = u_N\left(\xi_N^{j,q}\right).$$

Now the collocation conditions (5) will take the following form of a system which determines the coefficients $c^{j,q} = u_N\left(\xi_N^{j,q}\right)$:

$$u_N\left(\xi_N^{i,p}\right) = \sum_{j\in J_N}\sum_{q\in Q} a_N^{i,p,j,q} u_N(\xi_N^{j,q}) + f(\xi_N^{i,p}), \quad p \in Q, \quad i \in J_N, \tag{6}$$

where

$$a_N^{i,p,j,q} = \int_{G_{j,N}} K\left(\xi_N^{i,p}, y\right)\varphi_N^{j,q}(y)dy.$$

If $\eta_1 > -1$ or $\eta_m < 1$, then all collocation points $\xi_N^{j,q}$, $q \in Q$, $j \in J_N$, are different and there are $(2m)^n N_1 \dots N_n$ collocation points. Thus, in this case the system (6) has $(2m)^n N_1 \dots N_n = \dim E_N$ equations and the same number of unknowns. If $\eta_1 = -1$ and $\eta_m = 1$, then part of the collocation points will coincide. In this case the corresponding equations will coincide as well. The number of different collocation points is $[2N_1(m-1)+1] \dots [2N_n(m-1)+1] = \dim E_N$ and the system (6) has the same number of equations and unknowns.

The collocation method described above is thoroughly discussed in Vainikko [3]. The convergence of this method under assumptions (A1)–(A3) and the estimation of the convergence rate are proved there.

To apply the method, it is necessary to solve the linear system (6). We write this system in the form

$$\overline{u}_N = \mathcal{K}_N \overline{u}_N + \overline{f}_N, \tag{7}$$

where $\overline{u}_N = \left(u\left(\xi_N^{j,q}\right)\right)_{j \in J_N, q \in Q}$, $\overline{f}_N = \left(f\left(\xi_N^{j,q}\right)\right)_{j \in J_N, q \in Q}$ are vectors and $\mathcal{K}_N = (a^{i,p,j,q})_{i,j \in J_N, p,q \in Q}$ is a matrix.

Usually the system (7) has very many equations, which makes it rather cumbersome to solve. An effective method to solve this system is a two-grid iteration method.

4. TWO-GRID METHOD

In addition to the original grid corresponding to $N = (N_1, \dots, N_n)$, we define another grid, the coarse grid, corresponding to $M = (M_1, \dots, M_n)$, where M_k, $k = 1, \dots, n$, are the integers so that $M_k = N_k/\mu_k$ with μ_k the integers greater than 1. Then every cell of the original grid $G_{j,N}$, $j \in J_N$, is fully contained in some cell $G_{i,M}$ of the coarse grid.

To solve the system (7), the following two-grid iteration method is used:

$$\begin{aligned}
\overline{v}_N^l &= \overline{u}_N^l - \mathcal{K}_N \overline{u}_N^l - \overline{f}_N, \\
\overline{w}_M^l &= (I_M - \mathcal{K}_M)^{-1} \mathcal{R}_{NM} \mathcal{K}_N \overline{v}_N^l, \\
\overline{u}_N^{l+1} &= \overline{u}_N^l - \overline{v}_N^l - \mathcal{P}_{MN} \overline{w}_M^l, \quad l = 0, 1, \dots,
\end{aligned} \tag{8}$$

where $\overline{u}_N^0$ is the initial guess of $\overline{u}_N$, I_M is the identity matrix, $\mathcal{R}_{NM}\colon I\!R^{d_N} \to I\!R^{d_M}$ ($d_N = \dim E_N$) is the restriction operator defined by

$$\left(\mathcal{R}_{NM} \mathcal{K}_N \overline{v}_N^l\right)\left(\xi_M^{i,p}\right) = \sum_{j \in J_N} \sum_{q \in Q} \int_{G_{j,N}} K\left(\xi_M^{i,p}, y\right) \varphi_N^{j,q}(y) dy\, v_N^l\left(\xi_N^{j,q}\right)$$

and $\mathcal{P}_{MN}\colon I\!R^{d_M} \to I\!R^{d_N}$ is the prolongation operator defined by

$$\left(\mathcal{P}_{MN} \overline{w}_M^l\right)\left(\xi_N^{i,p}\right) = \sum_{q \in Q} w_M^l\left(\xi_M^{j,q}\right) \varphi_M^{j,q}\left(\xi_N^{i,p}\right), \text{ if } \xi_N^{i,p} \in G_{j,M}.$$

Similar two-grid iteration methods for weakly singular integral equations are considered in [2–5].

We write the integral equation (1) in the form

$$u = \mathcal{K}u + f, \tag{9}$$

where

$$(\mathcal{K}u)(x) = \int_G K(x,y)u(y)dy.$$

It is easy to see that the approximate solution $u_N \in E_N$ of (1), determined from the collocation condition (5), is the solution of the equation

$$u_N = \mathcal{P}_N \mathcal{K} u_N + \mathcal{P}_N f. \tag{10}$$

For the investigation of the convergence of the method (8) we use the following result.

Lemma. *Let the assumption* (A1) *hold. Then*

$$||\mathcal{K} - \mathcal{P}_N\mathcal{K}||_{L^\infty(G)\to L^\infty(G)} \le \text{const}\ \varepsilon_{\nu,h_N},$$

where

$$h_N = \max_{1\le k\le n} \frac{b_k}{N_k}$$

and

$$\varepsilon_{\nu,h_N} = \begin{cases} h_N, & \nu < n-1 \\ h_N(1+|\log h_N|), & \nu = n-1 \\ h_N^{n-\nu}, & \nu > n-1 \end{cases}.$$

Proof. Let $x \in G_{j,N}$ and $u \in L^\infty(G)$. Then

$$[(\mathcal{K} - \mathcal{P}_N\mathcal{K})u](x) = \int_G K(x,y)u(y)dy - \sum_{q\in Q} \varphi_N^{j,q}(x) \int_G K\left(\xi_N^{j,q}, y\right) u(y)dy$$

$$= \sum_{q\in Q} \varphi_N^{j,q}(x) \int_G \left[K(x,y) - K\left(\xi_N^{j,q}, y\right)\right] u(y)dy.$$

At first we estimate $\left|\varphi_N^{j,q}(x)\right|$. The change of the variable

$$x_k = x_{k,N}^{j_k-1} + \frac{\eta+1}{2}\left(x_{k,N}^{j_k} - x_{k,N}^{j_k-1}\right)$$

gives

$$\varphi_{k,N}^{j_k,q_k}(x_k) = \prod_{\substack{s=1\\ s\neq q_k}}^{m} \frac{x_k - \xi_{k,N}^{j_k,s}}{\xi_{k,N}^{j_k,q_k} - \xi_{k,N}^{j_k,s}} = \prod_{\substack{s=1\\ s\neq q_k}}^{m} \frac{\eta - \eta_s}{\eta_{q_k} - \eta_s}.$$

Denote

$$c = \max_{1\leq q_k\leq m} \max_{-1\leq \eta \leq 1} \left| \prod_{\substack{s=1\\ s\neq q_k}}^{m} \frac{\eta - \eta_s}{\eta_{q_k} - \eta_s} \right|.$$

Then

$$\left|\varphi_N^{j,q}(x)\right| = \left|\varphi_{1,N}^{j_1,q_1}(x_1)\right| \ldots \left|\varphi_{n,N}^{j_n,q_n}(x_n)\right| \leq c^n \text{ for } x \in G_{j,N}. \tag{11}$$

Thus, for $x \in G_{j,N}$, we get

$$|[(\mathcal{K} - \mathcal{P}_N\mathcal{K})u](x)| \leq c^n \int_G \left|K(x,y) - K(\xi_N^{j,q}, y)\right| |u(y)| dy.$$

In the same way as in the proof of Lemma 2.3 in [3], from the assumption (A1) it follows that

$$\int_G \left|K(x,y) - K(\xi_N^{j,q}, y)\right| |u(y)| dy \leq \text{const} ||u||_{L^\infty(G)} \varepsilon_{\nu,|x-\xi_N^{j,q}|}.$$

Therefore, if x and $\xi_N^{j,q}$ are placed in $G_{j,N}$, then

$$\left|x - \xi_N^{j,q}\right| \leq \text{diam} G_{j,N} \leq \text{const} h_N.$$

The assertion of the lemma follows from these results.

We are now ready to prove the following result about the convergence of the two-grid method (8).

Theorem. *Let the assumptions* (A1)–(A3) *hold. Then there exists* M_0 *so that, for* $N_k \geq M_0, k = 1, \ldots, n$, *the system* (7) *has a unique solution* $\overline{u}_N$ *and the two-grid iteration method* (8) *with* $M_k = N_k/\mu_k \geq M_0,\ k = 1, \ldots, n$, *converges to this solution with the rate*

$$||\overline{u}_N^{l+1} - \overline{u}_N|| \leq \text{const} \varepsilon_{\nu,h_M} ||\overline{u}_N^{l} - \overline{u}_N||, \ l = 0, 1, \ldots, \tag{12}$$

where

$$||\overline{u}_N|| = \max_{j\in J_N, q\in Q} \left|u_N(\xi_N^{j,q})\right|.$$

Proof. We use the approach of [6] and consider the iteration method, corresponding to (8), in functional spaces.

Define the operators $\mathcal{R}_{\infty N}: E_N \to I\!R^{d_N}$ ($d_N = \dim E_N$) and $\mathcal{P}_{N\infty}: I\!R^{d_N} \to E_N$ by the equalities

$$(\mathcal{R}_{\infty N} g)(\xi_N^{j,q}) = g(\xi_N^{j,q}), \quad j \in J_N, \quad q \in Q,$$

and

$$(\mathcal{P}_{N\infty}\overline{u}_N)(x) = \sum_{q\in Q} u_N(\xi_N^{j,q})\varphi_N^{j,q}(x), \quad x \in G_{j,N}, \quad j \in J_N.$$

These operators define one-to-one correspondence between the elements of E_N and $I\!R^{d_N}$. In the following we use the operator $\mathcal{R}_{\infty N}$ for all functions defined in the collocation points $\xi_N^{j,q}$.

Denote

$$u_N^l = \mathcal{P}_{N\infty}\overline{u}_N^l, \; v_N^l = \mathcal{P}_{N\infty}\overline{v}_N^l, \; w_M^l = \mathcal{P}_{M\infty}\overline{w}_M^l.$$

Making use of the identities

$$\mathcal{R}_{\infty N}\mathcal{P}_{N\infty} = I_N, \quad \mathcal{P}_{N\infty}\mathcal{R}_{\infty N} = \mathcal{P}_N,$$

$$\mathcal{K}_N = \mathcal{R}_{\infty N}\mathcal{K}\mathcal{P}_{N\infty}, \quad \mathcal{P}_{N\infty}\mathcal{P}_{MN} = \mathcal{P}_{M\infty},$$

we write the formulas (8) as follows:

$$\begin{aligned} v_N^l &= u_N^l - \mathcal{P}_N\mathcal{K}u_N^l - \mathcal{P}_N f, \\ w_M^l &= (I - \mathcal{P}_M\mathcal{K})^{-1}\mathcal{P}_M\mathcal{K}v_N^l, \\ u_N^{l+1} &= u_N^l - v_N^l - w_M^l, \quad l = 0, 1, \ldots \end{aligned} \tag{13}$$

Whereas $u_N^0 = \mathcal{P}_{N\infty}\overline{u}_N^0 \in E_N$, we also have $v_N^l \in E_N$, $w_M^l \in E_M \subset E_N$, and $u_N^{l+1} \in E_N$, $l = 0, 1, \ldots$ Therefore the methods (8) and (13) are equivalent. At the same time the method (13) is an iteration method to solve (10), and (10) is equivalent to (7) with $u_N = \mathcal{P}_{N\infty}\overline{u}_N$, $\overline{u}_N = \mathcal{R}_{\infty N}u_N$.

By the lemma,

$$||\mathcal{K} - \mathcal{P}_M\mathcal{K}|| = ||\mathcal{K} - \mathcal{P}_M\mathcal{K}||_{L^\infty(G)\to L^\infty(G)} \leq \text{const}\varepsilon_{\nu,h_M} \to 0,$$

if $h_M \to 0$. Due to (A1) and (A3) there exists the inverse operator $(I - \mathcal{K})^{-1}$ from $L^\infty(G)$ to $L^\infty(G)$. Therefore there occurs M_0 such that for $M_k \geq M_0$, $k = 1, \ldots, n$, inverse operators $(I - \mathcal{P}_M\mathcal{K})^{-1}$ exist and are uniformly bounded:

$$||(I - \mathcal{P}_M\mathcal{K})^{-1}|| \leq \text{const}.$$

It then follows that (7) has a unique solution for every $N_k \geq M_0$, $k = 1, \ldots, n$, and the formulas (8) and (13) define unique sequences $\overline{u}_N^l$ and u_N^l, $l = 1, 2, \ldots$, for $M_k \geq M_0$, $k = 1, \ldots, n$.

From (13) it is easy to derive the identity

$$u_N^{l+1} - u_N = (I - \mathcal{P}_M\mathcal{K})^{-1}(\mathcal{P}_N - \mathcal{P}_M)\mathcal{K}(u_N^l - u_N),$$

where u_N is the solution of (10). By the lemma,

$$||(\mathcal{P}_N - \mathcal{P}_M)\mathcal{K}|| \leq ||\mathcal{K} - \mathcal{P}_M\mathcal{K}|| + ||\mathcal{K} - \mathcal{P}_N\mathcal{K}|| \leq \text{const}\varepsilon_{\nu,h_M},$$

and therefore, for $M_k \geq M_0,\ k = 1, \ldots, n$, we have

$$||u_N^{l+1} - u_N|| \leq \text{const}\varepsilon_{\nu,h_M}||u_N^l - u_N||. \tag{14}$$

The estimation (12) follows from (14), whereas

$$||\overline{u}_N^{l+1} - \overline{u}_N|| = ||\mathcal{R}_{\infty N}(u_N^{l+1} - u_N)|| \leq ||u_N^{l+1} - u_N||$$

and

$$||u_N^l - u_N|| = ||\mathcal{P}_{N\infty}(\overline{u}_N^l - \overline{u}_N)|| \leq \text{const}||\overline{u}_N^l - \overline{u}_N||.$$

The last inequality is the consequence of the expression of $\mathcal{P}_{N\infty}$ and of the estimation (11).

From (12) we see that the two-grid iteration method (8) converges quite quickly provided that $M_k,\ k = 1, \ldots, n$, are chosen sufficiently large.

To solve the system (7) with the direct methods of Gauss type, it is necessary to do $O(d_N^3)$ arithmetical operations. Arguing analogously to [4], we see that the approximate solution of the system (7) of suitable accuracy can be found by the two-grid method (8) applying $O(d_N^2)$ arithmetical operations. For this end, we need to choose M so that $d_M = d_N^\tau,\ 0 < \tau < 2/3$. A good strategy will be $d_M \approx d_N^{1/2}$.

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KAHEVÕRGUMEETOD NÕRGALT SINGULAARSETE INTEGRAALVÕRRANDITE LAHENDAMISEKS TÜKITI POLÜNOMIAALSE APROKSIMATSIOONI ABIL

Enn TAMME

Nõrgalt singulaarse integraalvõrrandi lahendamisel tükiti polünomiaalse kollokatsioonimeetodiga tuleb lahendada suuri lineaarseid võrrandisüsteeme. Käesolevas töös on esitatud selliste süsteemide lahendamiseks kahevõrgu iteratsioonimeetod ja selgitatud selle koonduvuskiirus, millest järeldub meetodi efektiivsus.