Proc. Estonian Acad. Sci. Phys. Math., 1996, **45**, 4, 355–367
https://doi.org/10.3176/phys.math.1996.4.08

AVERAGE PHOTON PATH-LENGTH OF RADIATION EMERGING FROM FINITE ATMOSPHERE

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Presented by A. Sapar

Received 18 September 1996, accepted 8 October 1996

Abstract. The average photon path-lengths of the emergent radiation from atmospheres in the case of isotropic and Rayleigh scattering in different orders of scattering are compared. The finite homogeneous plane-parallel atmosphere is illuminated by a parallel unpolarized beam and only the external axial field of radiation is considered. The calculations show that the impact of polarization on the average path-lengths is quite considerable, increasing towards the larger orders of scattering and decreasing towards optically thicker atmospheres. For small angles of incidence and reflection, and for small orders of scattering ($n < 15$), the average path-lengths of both the reflected and transmitted radiation in the case of Rayleigh scattering are always smaller than those in the case of isotropic scattering, if only the atmosphere is optically not very thick. For large angles of incidence and reflection the situation is vice versa.

As a by-product of the calculations we ascertained the asymptotic behaviour of the average path-lengths which was quite similar for scalar and vector transfer. When increasing the optical thickness of an atmosphere, the average path-lengths l_n^+ approach the value n and they do not depend on the angular variables any more. The smaller the order of scattering, the smaller the optical thickness at which this starts to happen. While for the total radiation emerging from optically thick atmospheres at $\tau = \tau_0$ the average path-length is proportional to τ_0^2, the average path-length in different orders of scattering is a linear function of τ_0.

Key words: average photon path-length, finite atmospheres, polarization, order of scattering.

1. INTRODUCTION

The order-of-scattering approach in the scalar radiative transfer is well known. To the best of our knowledge the pioneer in this field was King [1] who made approximate calculations to explain the brightness and polarization of the blue daylight sky. This idea has been used by Ornstein [2] to calculate the radiation field in the first orders in scattering both in planar and spherical atmospheres. In his work he indicated that this approach was used by him in neutron transfer and published in *Proceedings of the Amsterdam Academy* (1936).

Later this method was used by Hammad and Chapman [3] to find the first two orders of scattering of sunlight in a planar atmosphere. Van de Hulst [4] considered the same problem, assigning the physical meaning to Ambarzumian–Chandrasekhar functions X and Y. Having this meaning in mind, van de Hulst obtained the expressions for single and double scattering in a simple manner. The order-of-scattering approach has been extensively exploited by Bellman et al. [5–8] and Ueno et al. [9, 10], mostly to solve the problems in radiation dosimetry.

Successive orders of scattering as a method for solving the radiation transfer problems in planetary atmospheres has been described by Hansen and Travis [11], van de Hulst [12], and Lenoble [13]. The last author gives an impressive list of references. Hovenier [14] presented exact results for the intensities of radiation emerging from an atmosphere in the first two orders of scattering with polarization included. These results were generalized by Wauben et al. [15] who presented explicit formulae for internal radiation in an atmosphere and who meticulously reported the special cases for equal angles. Kuga [16] described the third- and fourth-order solutions for the vector radiative transfer equation not paying any attention to the case for equal angles. However, this case is important when implementing the method in practical calculations.

An important contribution to understand the behaviour of photons scattered once and twice in a homogeneous plane-parallel atmosphere was made by Irvine [17]. He found the exact expressions of the path-length distribution functions and the average path-lengths for these photons. He also studied the asymptotic behaviour of the average path-length and determined in a rigorous way that in the first two orders of scattering the average path-length for outward radiation at the bottom of the atmosphere is proportional to the optical thickness of the atmosphere.

Discussing the impact of polarization on the photon average path-lengths in different scattering orders of the radiation emerging from an atmosphere we rely on the papers by Fymat and Ueno [18, 19] (cf. Hansen and Travis [11]). The atmosphere under consideration is assumed to be finite, conservative, plane-parallel, and homogeneous.

The photon average path-length in different orders of scattering is defined by the scattering and transmission matrices of the atmosphere, and by their derivatives with respect to the optical thickness of the atmosphere. As in the case of the scalar transfer [20], we proceed from the integro-differential equations for the scattering and transmission matrices in different orders of scattering. Next we substitute all the integrals in these equations by Gaussian sums and if we take into account the symmetry relations [21], we obtain a set of $2N(2N+1)$ ordinary differential equations where N is the order of the Gaussian quadrature. Though these equations are rather stiff (the larger the order of the quadrature, the stiffer the equations), nevertheless the fourth-order Runge–Kutta method [22] did work well if we only kept the step of integration small enough. The length of the step was determined by the trial-and-error method. We tried to use the methods with an adaptive step-size but without much success, since these methods spent drastically long time taking very small steps near $\tau_0 = 0$ in order to meet even the modest requirements of accuracy.

In the following we shall confine ourselves to the case of azimuthal symmetry only. We have accepted Ivanov's [23] notation: bold font is used to denote 2×2 matrices (capital Roman characters) and two-component vectors (small Roman characters).

Throughout this paper we use the (I, Q) representation and we assume that the incident beam of radiation **f** is not polarized, i.e. $\mathbf{f}_I = 1$ and $\mathbf{f}_Q = 0$.

2. RAYLEIGH–CABANNES SCATTERING

One of the possible ways to find the photon average path-lengths is to start from the determination of the photon path-length distribution function which describes the probability that a photon, contributing to the intensity of the radiation at a certain depth in an atmosphere, has travelled an optical path between l and $l + \mathrm{d}l$. In the scalar case the photon path-length distribution functions p_n of the radiation, emerging at the upper and lower boundaries of the atmosphere, are given by the following formulae [12]

$$p_n(0, \mu, \mu_0, l) = L^{-1}\{I_n[0, \mu, \mu_0, \tau_0(1+s)]/[(1+s)^n I_n(0, \mu, \mu_0, \tau_0)]\}, \quad (1)$$

$$p_n(\tau_0, -\mu, \mu_0, l) = L^{-1}\{I_n[\tau_0, -\mu, \mu_0, \tau_0(1+s)]/[(1+s)^n I_n(\tau_0, -\mu, \mu_0, \tau_0)]\}, \quad (2)$$

where the operator L^{-1} means the Laplace inverse transformation with respect to the parameter s and $I_n(0, \mu, \mu_0, \tau_0)$ is the intensity of the n times scattered radiation, emerging from the atmosphere at the upper boundary and $I_n(\tau_0, -\mu, \mu_0, \tau_0)$ is the similar intensity at the lower boundary. Here, τ_0 is the optical thickness of the atmosphere, μ is the cosine of the angle between the photon travel direction and the outward normal to the upper boundary of the atmosphere, and μ_0 is the cosine of the angle between the parallel beam of the incident radiation and the inward normal to the upper boundary of the atmosphere. We may add that the first argument of the intensity is the optical depth τ at which the intensity is observed while $\tau = 0$ at the upper boundary of the atmosphere and $\tau = \tau_0$ at the lower boundary.

If we know the path-length distribution function, then the average path-length can be easily found since it is the first moment of this function. Unfortunately, this straightforward approach is very complicated in practice because of the mathematical difficulties connected with the inverse Laplace transform. However, the properties of the Laplace transform allow us to bypass this procedure completely as shown e.g. by van de Hulst in [12]. The formulae to calculate the average photon path-length l_n of the radiation emerging through the boundaries of an atmosphere for n times scattered photons are the following:

$$l_n^+(\tau_0, \mu, \mu_0) = n - \frac{\partial \ln I_n(0, \mu, \mu_0, \tau_0)}{\partial \ln \tau_0}, \tag{3}$$

$$l_n^-(\tau_0, \mu, \mu_0) = n - \frac{\partial \ln I_n(\tau_0, -\mu, \mu_0, \tau_0)}{\partial \ln \tau_0}. \tag{4}$$

As already mentioned, Eqs. (1) and (2) are valid for scalar radiative transfer. What are their counterparts in the case of polarized radiation? In that case the intensity is described by a two-component Stokes vector (if we consider the axial radiation field only)

$$\mathbf{i}_n(\tau, \mu, \mu_0, \tau_0) = \begin{pmatrix} I_n(\tau, \mu, \mu_0, \tau_0) \\ Q_n(\tau, \mu, \mu_0, \tau_0) \end{pmatrix}, \tag{5}$$

where I_n is the intensity proper of the n times scattered radiation and Q_n characterizes the degree of polarization of the n times scattered radiation, i.e. if $Q_n = 0$, then the radiation is not polarized.

It seems to be quite natural to generalize Eqs. (1) and (2) for the case of polarized radiation in such a way that the scalar intensity in these equations is substituted with the first component of the Stokes vector which physically is also intensity [24]. This means that Eqs.(1)–(4) retain their form also in the case of the vector transfer.

It follows from Eqs. (3) and (4) that all we need to know in order to obtain the average path-lengths are the derivatives of the intensities of the emergent radiation with respect to the optical thickness of the atmosphere. These derivatives are readily found from the principles of invariance originally formulated by Ambarzumian [25] and further elaborated by Chandrasekhar [21]. These principles appeared to be valid for polarized radiation in different orders of scattering as well. If the finite atmosphere is illuminated monodirectionally at the top, then for the intensity vector at the optical depth τ we have (Chandrasekhar [21], Fymat and Ueno [18, 19])

$$\begin{aligned} \mathbf{i}_n(\tau, \mu, \mu_0, \tau_0) &= \mathbf{i}_n(0, \mu, \mu_0, \tau_0 - \tau) \exp(-\tau/\mu_0) \\ &\quad + \frac{1}{2\mu} \sum_{i=1}^{n-1} \int_0^1 \mathbf{S}_{n-i}(\tau_0 - \tau, \mu, \mu') \mathbf{i}_i(\tau, -\mu', \mu_0, \tau_0) \mathrm{d}\mu', \end{aligned} \tag{6}$$

$$\begin{aligned} \mathbf{i}_n(\tau, -\mu, \mu_0, \tau_0) &= \mathbf{i}_n(0, -\mu, \mu_0, \tau_0 - \tau) \\ &\quad + \frac{1}{2\mu} \sum_{i=1}^{n-1} \int_0^1 \mathbf{S}_{n-i}(\tau, \mu, \mu') \mathbf{i}_i(\tau, \mu', \mu_0, \tau_0) \mathrm{d}\mu', \end{aligned} \tag{7}$$

where the scattering function $\mathbf{S}_n(\tau_0, \mu, \mu_0)$ and the transmission function $\mathbf{T}_n(\tau_0, \mu, \mu_0)$ which we shall need later are defined in terms of the emergent radiation as follows:

$$\mathbf{i}_n(0,\mu,\mu_0,\tau_0) = \frac{1}{4\mu}\mathbf{S}_n(\tau_0,\mu,\mu_0), \tag{8}$$

$$\mathbf{i}_n(\tau_0,-\mu,\mu_0,\tau_0) = \frac{1}{4\mu}\mathbf{T}_n(\tau_0,\mu,\mu_0). \tag{9}$$

Differentiating Eqs. (6) and (7) with respect to τ, passing either to the limit $\tau = 0$ or to the limit $\tau = \tau_0$, and making use of the boundary conditions $\mathbf{i}_n(0,\mu,\mu_0,\tau_0) = \mathbf{0}$ and $\mathbf{i}_n(\tau_0,-\mu,\mu_0,\tau_0) = \mathbf{0}$ (there is no scattered radiation incident on the atmosphere) and the equation of radiative transfer, we obtain the following equations:

$$\begin{aligned}\frac{\partial \mathbf{S}_n(\tau_0,\mu,\mu_0)}{\partial \tau_0} = & -\left(\frac{1}{\mu}+\frac{1}{\mu_0}\right)\mathbf{S}_n(\tau_0,\mu,\mu_0) + \mathbf{P}(\mu,-\mu_0)d(n,1)\\ & + \frac{1}{2}\int_0^1 \mathbf{P}(\mu,\mu')\mathbf{S}_{n-1}(\tau_0,\mu',\mu_0)\mathrm{d}\mu'/\mu'\\ & + \frac{1}{2}\int_0^1 \mathbf{S}_{n-1}(\tau_0,\mu,\mu')\mathbf{P}(-\mu',-\mu)\mathrm{d}\mu'/\mu'\\ & + \frac{1}{4}\sum_{i=1}^{n-2}\int_0^1\int_0^1 \mathbf{S}_{n-i-1}(\tau_0,\mu,\mu')\mathbf{P}(-\mu',\mu'')\\ & \times\ \mathbf{S}_i(\tau_0,\mu'',\mu_0)\mathrm{d}\mu''/\mu''\mathrm{d}\mu'/\mu'\end{aligned} \tag{10}$$

and

$$\begin{aligned}\frac{\partial \mathbf{T}_n(\tau_0,\mu,\mu_0)}{\partial \tau_0} = & -\frac{1}{\mu_0}\mathbf{T}_n(\tau_0,\mu,\mu_0) + \mathbf{P}(\mu,-\mu_0)\exp(-\tau_0/\mu)d(n,1)\\ & + \frac{1}{2}\exp(-\tau_0/\mu)\int_0^1 \mathbf{P}(\mu,\mu')\mathbf{S}_{n-1}(\tau_0,\mu',\mu_0)\mathrm{d}\mu'/\mu'\\ & + \frac{1}{2}\int_0^1 \mathbf{T}_{n-1}(\tau_0,\mu,\mu')\mathbf{P}(-\mu',-\mu)\mathrm{d}\mu'/\mu'\\ & + \frac{1}{4}\sum_{i=1}^{n-2}\int_0^1\int_0^1 \mathbf{T}_{n-i-1}(\tau_0,\mu,\mu')\mathbf{P}(-\mu',\mu'')\\ & \times\ \mathbf{S}_i(\tau_0,\mu'',\mu_0)\mathrm{d}\mu''/\mu''\mathrm{d}\mu'/\mu'.\end{aligned} \tag{11}$$

In Eqs. (10) and (11), $d(n,1)$ is the Kronecker delta function which is unity if $n = 1$ and zero otherwise, and $\mathbf{P}(\mu,\mu_0)$ is a superposition of the Rayleigh phase matrix $\mathbf{P}_R$ and the phase matrix of the isotropic scalar scattering $\mathbf{P}_I$:

$$\mathbf{P} = (1-c)\mathbf{P}_I + c\mathbf{P}_R, \tag{12}$$

where

$$\mathbf{P}_I = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{13}$$

and

$$\mathbf{P}_R = \frac{3}{8}\begin{pmatrix} 3-\mu^2-\mu_0^2+3\mu^2\mu_0^2 & (1-3\mu^2)(1-\mu_0^2) \\ (1-\mu^2)(1-3\mu_0^2) & 3(1-\mu^2)(1-\mu_0^2) \end{pmatrix}. \quad (14)$$

Here c is the depolarization parameter which is zero if the scattering is isotropic and unity if there is the Rayleigh scattering present in the atmosphere.

Equations (10) and (11) are to be solved subject to boundary conditions

$$\mathbf{S}_n(0,\mu,\mu_0) = 0, \quad (15)$$

$$\mathbf{T}_n(0,\mu,\mu_0) = 0 \quad (16)$$

for each order of scattering.

3. NUMERICAL RESULTS

In order to solve the system of Eqs. (10) and (11), we replaced all the integrals therein by a Gaussian quadrature formula of the order of N (Chandrasekhar [21], Bellman et al. [26]). As a result we obtained a set of ordinary differential equations. The fourth-order Runge–Kutta method (Press et al. [22]) did work well though we had to keep the step of integration rather small: $\delta\tau_0 = 0.005$. However, the first steps of integration were definitely not very accurate (cf. Viik [20]), to say the least. This means that the results for optically very thin atmospheres ($\sim 10\delta\tau_0$) were not correct and we discarded them in our presentation. Fortunately enough, at larger optical thicknesses the results stabilized and we could report relative accuracy of the order of 10^{-4}.

For all our calculations the order of Gaussian approximation was $N = 15$ and we went as far as fifty orders of scattering for the **S** matrix but the main body of results was obtained for $N = 25$ and $\tau_0 \le 10$. The reason was inadmissibly long time of calculations even on a computer SPARCstation 20.

We have compared the average path-lengths in different orders of scattering in radiation emergent from an atmosphere at $\tau = 0$ and at $\tau = \tau_0$ while the scattering was isotropic or governed by the Rayleigh–Cabannes phase matrix. Though the order of scattering can only be an integer, for better perception we joined the points with a smooth curve – we used the version 3.3b GLE package – as if the order of scattering were real (cf. van de Hulst [12]).

Figure 1 shows the values of l_n^+/n for almost normal reflection ($\mu = \mu_0 = 0.993996$ – this is the last zero of $P_{15}(1-2\mu)$) against finite atmospheres with isotropic and Rayleigh scattering. While for isotropic scattering the curves are strictly monotonic over the whole range of scattering orders considered, the respective curves for Rayleigh scattering may have two inflection points. Almost the same could be said about the behaviour of l_n^- (Fig. 2).

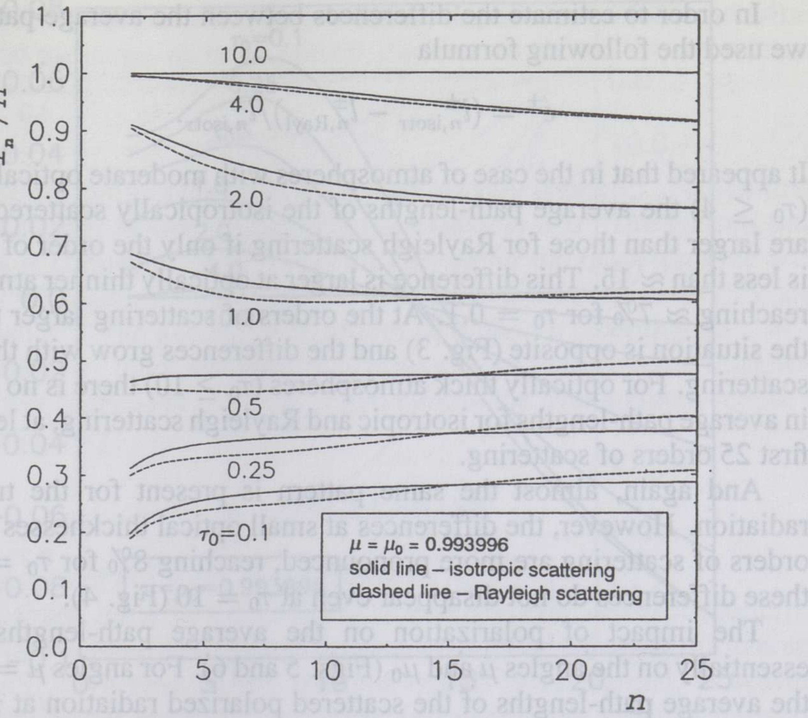

Fig. 1. Average photon path-lengths as functions of the order of scattering for almost normal reflection against optically finite atmospheres.

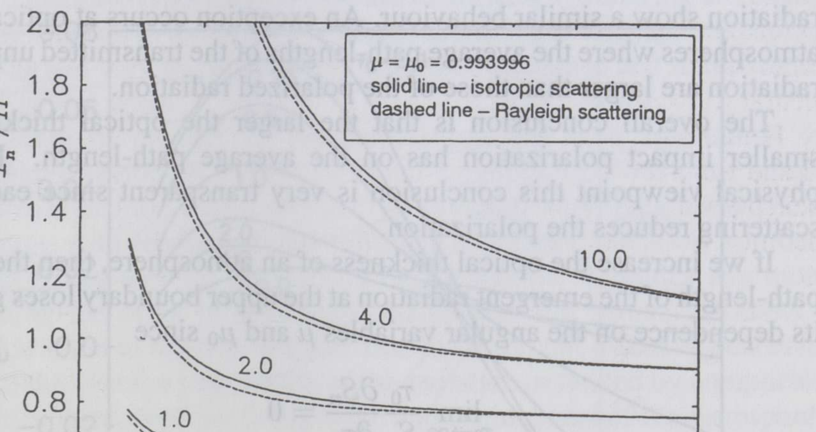

Fig. 2. Same as in Fig. 1 for transmitted radiation.

In order to estimate the differences between the average path-lengths, we used the following formula

$$\epsilon^{\pm} = (l^{\pm}_{n,\mathrm{isotr}} - l^{\pm}_{n,\mathrm{Rayl}})/l^{\pm}_{n,\mathrm{isotr}}. \tag{17}$$

It appeared that in the case of atmospheres with moderate optical thickness ($\tau_0 \le 4$) the average path-lengths of the isotropically scattered radiation are larger than those for Rayleigh scattering if only the order of scattering is less than ≈ 15. This difference is larger at optically thinner atmospheres, reaching $\approx 7\%$ for $\tau_0 = 0.1$. At the orders of scattering larger than ≈ 15 the situation is opposite (Fig. 3) and the differences grow with the order of scattering. For optically thick atmospheres ($\tau_0 \ge 10$) there is no difference in average path-lengths for isotropic and Rayleigh scattering, at least for the first 25 orders of scattering.

And again, almost the same pattern is present for the transmitted radiation. However, the differences at small optical thicknesses and small orders of scattering are more pronounced, reaching 8% for $\tau_0 = 0.1$, and these differences do not disappear even at $\tau_0 = 10$ (Fig. 4).

The impact of polarization on the average path-lengths depends essentially on the angles μ and μ_0 (Figs. 5 and 6). For angles $\mu = \mu_0 = 0.5$ the average path-lengths of the scattered polarized radiation at $\tau = 0$ are larger than those for the unpolarized radiation, and only for optically thick atmospheres ($\tau_0 \ge 10$) the average path-lengths are equal, at least for the orders of scattering less than 25.

For the same angles the average path-lengths of the transmitted radiation show a similar behaviour. An exception occurs at optically thick atmospheres where the average path-lengths of the transmitted unpolarized radiation are larger than those of the polarized radiation.

The overall conclusion is that the larger the optical thickness, the smaller impact polarization has on the average path-length. From the physical viewpoint this conclusion is very transparent since each act of scattering reduces the polarization.

If we increase the optical thickness of an atmosphere, then the average path-length of the emergent radiation at the upper boundary loses gradually its dependence on the angular variables μ and μ_0 since

$$\lim_{\tau_0 \to \infty} \frac{\tau_0}{S_n} \frac{\partial S_n}{\partial \tau_0} = 0 \tag{18}$$

and, respectively,

$$\lim_{\tau_0 \to \infty} l^{+}_{n} = n. \tag{19}$$

The lower the order of scattering, the sooner this isotropization takes place, e.g. if for $n = 3$ we cannot distinguish between the average path-lengths at different angular variables for $\tau_0 \ge 4$, then for $n = 25$ the same situation occurs at $\tau_0 \ge 9$. There are no significant differences for vector transfer in such a behaviour.

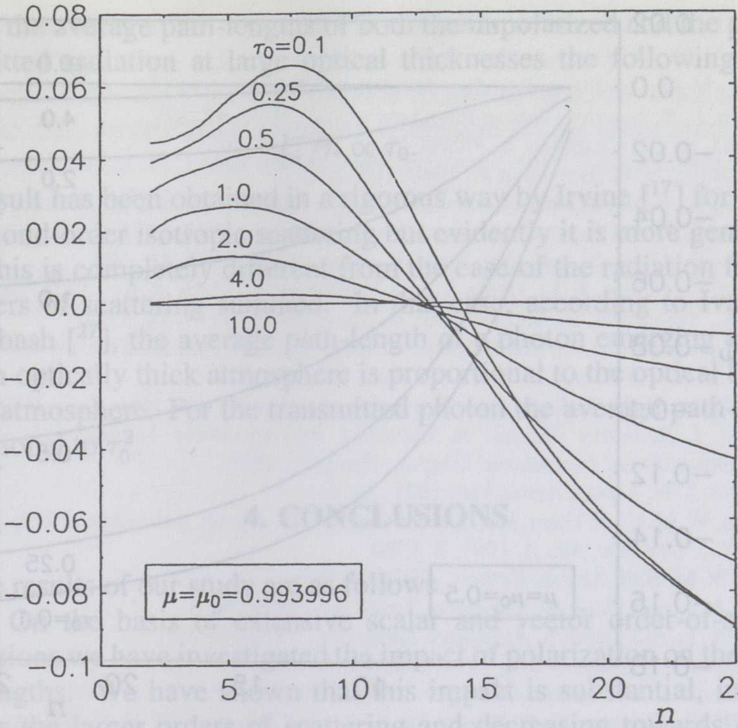

Fig. 3. Relative differences in the average path-lengths for emergent radiation at $\tau = 0$ for isotropically and Rayleigh scattering atmospheres as functions of the order of scattering.

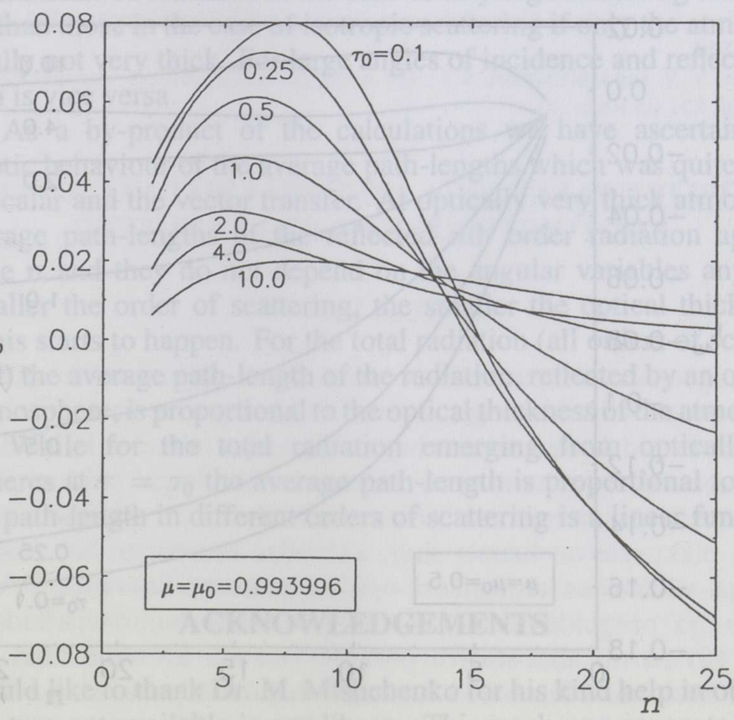

Fig. 4. Same as in Fig. 3 for transmitted radiation.

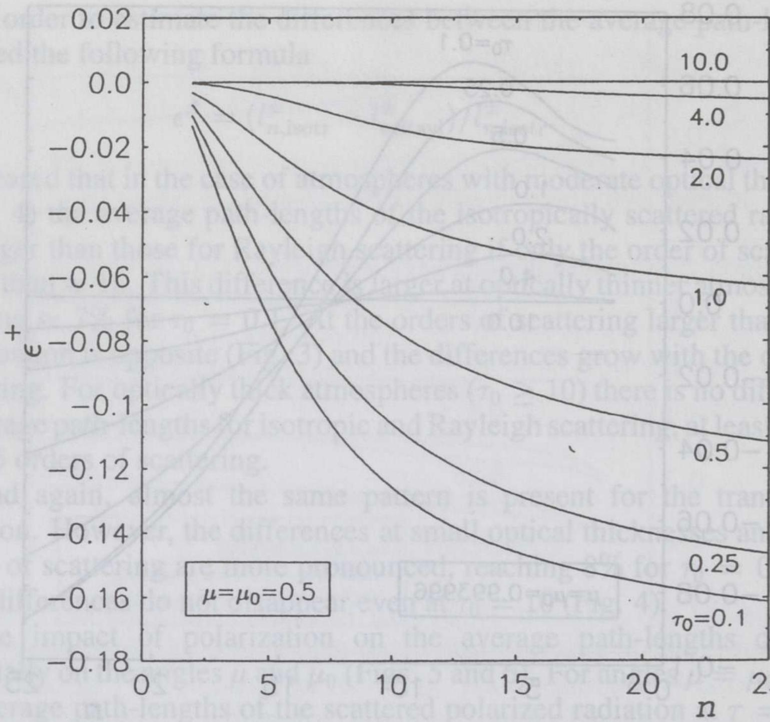

Fig. 5. Same as in Fig. 3 for different angles of incidence and reflection.

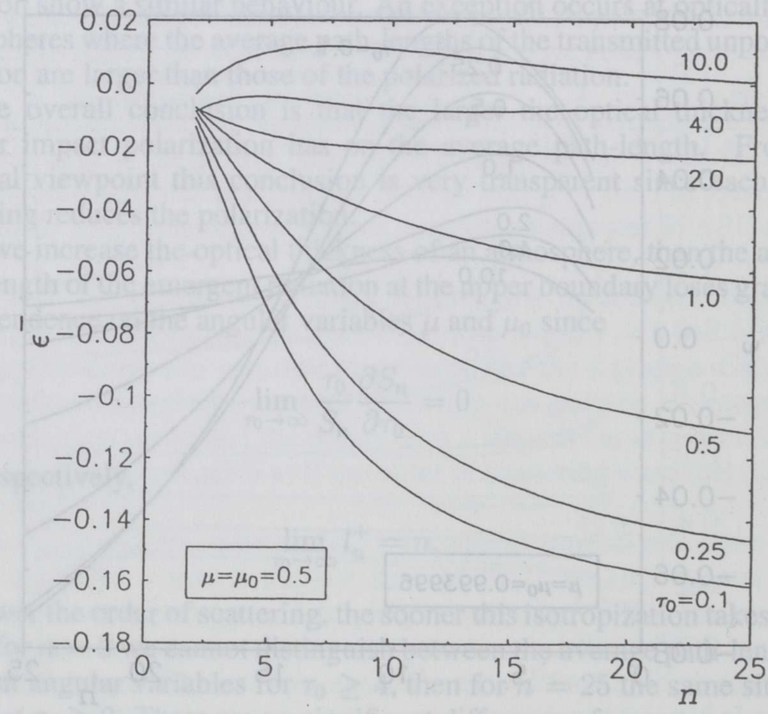

Fig. 6. Same as in Fig. 5 for transmitted radiation.

For the average path-lengths of both the unpolarized and the polarized transmitted radiation at large optical thicknesses the following relation holds:

$$l_n^- / n \propto \tau_0. \tag{20}$$

This result has been obtained in a rigorous way by Irvine [17] for the first- and second-order isotropic scattering but evidently it is more general than that. This is completely different from the case of the radiation field with all orders of scattering summed. In that case, according to Ivanov and Gutshabash [27], the average path-length of a photon emerging at $\tau = 0$ from an optically thick atmosphere is proportional to the optical thickness of that atmosphere. For the transmitted photon the average path-length is proportional to τ_0^2.

4. CONCLUSIONS

The results of our study are as follows.

1. On the basis of extensive scalar and vector order-of-scattering calculations we have investigated the impact of polarization on the average path-lengths. We have shown that this impact is substantial, increasing towards the larger orders of scattering and decreasing towards optically thicker atmospheres.

2. For small angles of incidence and reflection, and for small orders of scattering ($n < 15$) the average path-lengths of both the reflected and the transmitted radiation in the case of Rayleigh scattering are always smaller than those in the case of isotropic scattering if only the atmosphere is optically not very thick. For large angles of incidence and reflection the situation is vice versa.

3. As a by-product of the calculations we have ascertained the asymptotic behaviour of the average path-lengths which was quite similar for the scalar and the vector transfer. At optically very thick atmospheres the average path-lengths of the reflected nth order radiation approach the value n and they do not depend on the angular variables any more. The smaller the order of scattering, the smaller the optical thickness at which this starts to happen. For the total radiation (all orders of scattering summed) the average path-length of the radiation, reflected by an optically thick atmosphere, is proportional to the optical thickness of the atmosphere.

4. While for the total radiation emerging from optically thick atmospheres at $\tau = \tau_0$ the average path-length is proportional to τ_0^2, the average path-length in different orders of scattering is a linear function of τ_0.

ACKNOWLEDGEMENTS

I would like to thank Dr. M. Mishchenko for his kind help in obtaining the literature not available in our library. This work was supported by the Estonian Science Foundation under grants No. 171 and 2006.

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LÕPLIKU PAKSUSEGA ATMOSFÄÄRIST VÄLJUVATE FOOTONITE KESKMISED LENNUTEED

Tõnu VIIK

Homogeenset tasaparalleelset atmosfääri valgustatakse paralleelse polariseerimata kiirtekimbuga. Atmosfäärist väljuvate footonite keskmisi lennuteid võrreldakse erinevates hajumisjärkudes juhtudel, kui atmosfääris toimub isotroopne hajutamine või hajutamine Rayleigh' seaduse järgi.

Arvutused näitavad, et polarisatsiooni mõju footoni keskmise lennutee pikkusele on kaunis suur, kasvades suuremate hajumisjärkude suunas ja vähenedes atmosfääri optilise paksuse kasvades. Väikeste langemis- ja peegeldumisnurkade ning väikeste hajumisjärkude puhul ($n < 15$) on nii atmosfäärilt peegeldunud kui ka atmosfääri läbinud footonite keskmised lennuteed Rayleigh' hajumise puhul alati väiksemad kui isotroopse hajumise korral, kui ainult atmosfäär pole optiliselt liiga paks. Suurte langemis- ja peegeldumisnurkade puhul on asi vastupidi.

Arvutuste kõrvaltulemusena tehti kindlaks, et kui atmosfääri optiline paksus on väga suur, siis atmosfääri ülemiselt pinnalt (sellelt, millele langeb paralleelne kiirtekimp) väljuvate footonite keskmised lennuteed l_n^+ lähenevad hajumiskordsust näitavale suurusele n ega sõltu enam nurgamuutujatest. Mida väiksem on hajumiskordsus, seda väiksemate optiliste paksuste puhul selline käitumine algab. Läbiva kiirguse puhul ilmneb aga huvitav fakt: kui üle kõikide hajumisjärkude summeeritud kogukiirguse puhul keskmine footoni lennutee pikkus on võrdeline atmosfääri optilise paksuse ruuduga, siis erinevates hajumisjärkudes on footoni lennutee pikkus võrdeline vaid atmosfääri optilise paksusega.