Proc. Estonian Acad. Sci. Phys. Math., 1995, 44, 4, 466—475
https://doi.org/10.3176/phys.math.1995.4.06

CYCLOTRON FREQUENCY TECHNIQUE OF THE MEASUREMENT OF BETA DECAY SPECTRA

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Presented by G. Liidja

Received 14 December 1994, accepted 19 June 1995

Abstract. The calculations made show that the energy of a single beta decay electron can be measured in a homogeneous magnetic field by measuring the frequency of its cyclotron radiation. The radioactive source is placed in an oversized waveguide at the liquid N_2 temperature and the signal is directed to a maser amplifier. The technique can be applied to the precise measurement of discrete spectra of low-intensity samples. As detailed description of the method requires some specification of physical conditions, attention is focused on its possible application to continuous spectra of tritium and other nuclei with an aim to narrow down neutrino mass limits. The activities of the end regions of the spectra tend to be rather low for the immediate application of this method to neutrino mass, therefore some preliminary experiments are needed in order to maximally increase the sample surface. Still, some other applications, like the measurement of the mass differences of helium and tritium nuclei, and the testing of molecular and surface corrections to beta decay samples, are feasible free from this problem.

Key words: electron spectroscopy, beta decay spectra, neutrino mass, surface energy, microwave guides, cyclotron radiation.

1. INTRODUCTION

Precise measurement of beta decay spectra is still topical as the resolution hitherto achieved is insufficient for a considerable improvement in the upper limit of the electron antineutrino mass. Direct methods are almost exclusively concentrated on the tritium spectrum because of its low end point energy 28 600 eV. The other possible sources are of considerably higher end point energies and a correspondingly higher relative precision of measurements must be achieved to get the needed resolution.

Two basic figures are required in order to determine the antineutrino mass. First, one needs to know the total energy released during the beta decay process. This can be achieved in two ways. The first one is to extrapolate the β-particles spectrum to the end point neglecting the actual end point that is shifted by the neutrino mass. The other method includes the measurement of the mass difference of the two nuclei of the reaction, e. g. ^{3}T and ^{3}He (a review of the results see [1]). The second

figure needed is the actual end point energy. The neutrino mass equals the difference of these two figures, as being the missing rest mass of the reaction.

Our technique is expected to be directly applicable to the determination of the energy released in the first way, but the improvement of the actual end point limits seems to be promising as well.

There are some circumstances impeding further improvement of the accuracy of end point measurements. Beta decay partial activity of the end point region is dropping proportionally to the third power of the distance from the actual end point and is very low for the energy interval of a few electronvolts. This confines one to using large surfaces of solid sources. Only the Los Alamos group [2] has measured the end point using a tritium gas source, with the greatest resolution achieved so far. Solid sources bring about additional uncertainties due to initial and final state corrections [3]. Large sources require expensive large spectrometers that are sensitive to background radiation and stray magnetic fields.

We introduce a technique (first proposed by the author at the 2nd Tallinn Symposium on Neutrino Physics [4]) where the relativistic total energy of a single electron from either a gaseous or solid source is measured. For the sake of concreteness the energy of tritium beta decay end point is considered in greater detail. Owing to the fact that lower total energies do not depend much on the kinetic energies, this method, perhaps, can also be applied to many other spectra such as ^{35}S, ^{63}Ni, ^{71}Ge, etc.

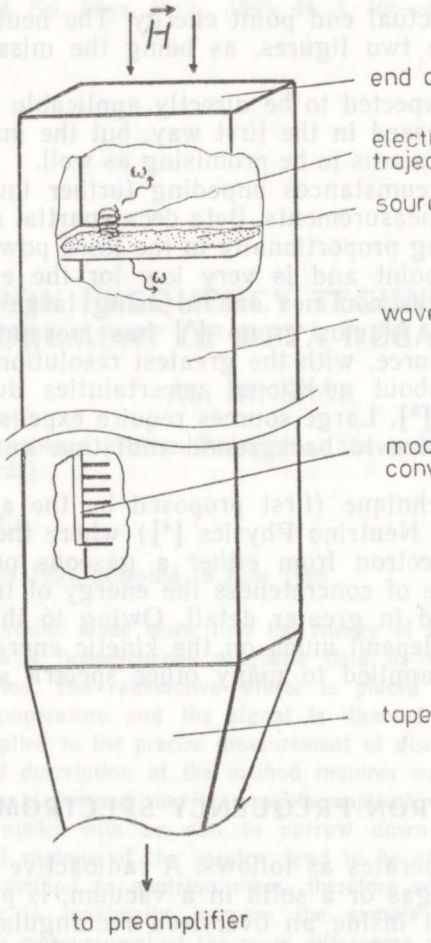
2. CYCLOTRON FREQUENCY SPECTROMETER

The spectrometer operates as follows. A radioactive sample, either in the form of a rarified gas or a solid in a vacuum, is placed in a homogeneous magnetic field inside an oversized rectangular waveguide. An electron emitted in the plane perpendicular to the magnetic field is captured for a while in the field where it keeps circling until leaving the experimental zone. At that it emits cyclotron radiation that can be registered. The frequency of the radiation, known to be independent of the velocity of the particle within the classical limits, depends on its relativistic total energy, being inversely proportional to it. The end point energy corresponds to the lowest frequency. At tritium the difference of relativistic energy from that of classical electrons is about 4%.

The method is based on the fact that, in principle, frequency can be measured with a very high precision. Problems are due to the need of extremely high sensitivity with thermal noise suppressed. The experiments resemble those with a "geonium" [5] where also a single electron was observed in a magnetic field, with the difference that there the axial movement of a totally trapped electron was detected.

The frequency must be chosen as high as possible by adjusting the magnetic field in order to attain a sufficiently good signal-to-noise ratio and a large number of cycles before the electron leaves the experimental zone. The electron cannot be trapped by electric fields, for that would create differences of potential in the field.

As the signal from a single electron is very weak, one has to apply a preamplifier with the possibly lowest noise level. Another requirement mentioned above is high frequency. These can be met by a quantum paramagnetic (maser) amplifier with the working frequency of 40 GHz, noise temperature 30 K, and 25 dB gain [6]. The bandwidth of the amplifier is practically not restricted.



Principal sketch of the cyclotron frequency spectrometer.

The resolution of the measurement is taken 2.5 eV, which choice will be shown as feasible but for some cases it could even be improved. The relative precision is

$$\frac{\Delta\nu}{\nu}=\frac{\Delta E_{\text{res}}}{E_{\text{rest}}+E_{\text{kin}}}=\frac{1}{200\,000}=5\cdot10^{-6}. \tag{1}$$

The magnetic field is determined by cyclotron frequency:

$$B=\frac{2\pi\nu m}{e}=1.48\ \text{T}. \tag{2}$$

Being homogeneous according to (1), it will necessarily be created by a superconducting magnet.

The radius of the orbit of the circling electron is

$$r=\frac{mv}{eB}\approx 0.3\ \text{mm}, \tag{3}$$

where v is the electron's velocity $\approx 0.2c$.

In order to achieve precision (1), the electron must complete at least 200 000 circles before leaving the experimental zone where the magnetic

field has the necessary homogeneity. In this case the length of the trajectory is 400 m and the time of flight $t=5\cdot10^{-6}$ s. The electron's orbit must be perpendicular to the magnetic field and cannot decline from that by more than 10^{-6} rad. All the other electrons leave the experimental zone very quickly, thus giving a signal too broad and weak for detection.

3. SOURCES OF RADIATION

For the calculation of partial activities, the effect of a possible small mass of the electron neutrino can be neglected. The form of the spectrum is known as

$$N(E)=K[\sqrt{E^2-M^2}\cdot E(E_0-E)^2], \tag{4}$$

where E is the electron's total energy, E_0 is the end point energy, and M is the rest mass energy.

For the very end point region the activity integrated from E to E_0 can be approximated to

$$\Delta N=\frac{K}{3}E_0(\Delta E)^3\sqrt{E^2-M^2}, \tag{5}$$

where

$$\Delta E\equiv E_0-E. \tag{6}$$

The relative activity of the end point region is

$$\frac{\Delta N}{N}=C(\Delta E)^3. \tag{7}$$

For tritium

$$E_0=m_0c^2+18.6\ \text{keV}, \tag{8}$$

$$C=3.502\cdot10^{-4}\ \text{keV}^{-3}. \tag{9}$$

It would be preferable to have an experimental zone filled with tritium in gaseous form to avoid the effects of a solid state. The pressure of the gas is limited by free flight: $p\leqslant10^{-6}$ torr. Together with the other restrictions such as the limited emitting angle and the long half-life (12.6 years), the effective partial activity for gaseous tritium end point proves to be rather low. For instance, for the end region of 10 eV one measurable electron would appear in the volume of 1 litre during 1 year. Due to difficulties in collecting the signal, larger volumes are, perhaps, not feasible. Evidently, the problem requires special study. Besides, tritium must be carefully confined, for atoms found in the regions where the magnetic field is lower would imitate states of higher energy.

The situation is better in the case of a solid source. For 100 cm^2 of a solid source one measurable electron will emerge within the following time:

ΔE_{res}	time	(10)
2.5 eV	2 months	
10 eV	1 day	
25 eV	2 hours.	

Regions up to some keV display much more partial activity. The possibility of using these at the measurement of beta decay end points in the

bremsstrahlung technique was considered by the author in [7]. Budick et al. [1] have demonstrated the applicability of these regions for the determination of the sum total of the electron and neutrino energies. In principle, one day would suffice for collecting statistics for the update level measurement of the helium and tritium mass difference by treating the 1 keV end region from a 100 cm^2 tritium solid source.

Solid sources bring along different problems. The electron's orbit is perpendicular to the magnetic flux lines, slowly parting from the source surface. That must also be perpendicular to the flux lines. This means that the electron is emitted at an angle not exceeding 10^{-6} rad to the surface plane. Before the distance increases, the electron can lend some of its energy to surface atoms. Only electrons from the extreme surface atoms are applicable, but the loss of energy may still be too great. The problem can be resolved by making the surface rough. It may fall off in the case of intense discrete spectra where sources can be designed as sets of thin threads. Evidently, after preliminary experiments, some corrections should be applied to figures in (10).

Obviously, the surface area should be made as big as possible. The best way to do it is to use many thin nonconductive layers covered with thin solid tritium sources so that microwaves are not considerably absorbed. The distance between the sources can be 1—2 cm, for longer spirals of the electrons' trajectories are not acceptable due to Doppler distortions of the signal.

4. CYCLOTRON RADIATION OF AN ELECTRON

An electron circling with an orbital frequency ω, as a classical oscillator, radiates electromagnetic waves with a corresponding frequency $\nu=\omega/2\pi$ (cyclotron radiation). The process causes attenuation

$$E=E_i e^{-gt}, \tag{11}$$

where the attenuation constant is

$$g=\frac{e^2\omega^2}{3\pi\varepsilon_0 c^3 m}, \tag{12}$$

for $\nu=40$ GHz one gets

$$g=0.77\ \text{s}^{-1}. \tag{13}$$

The higher harmonics of ω (synchrotron radiation) are negligible, for tritium decay electrons are not very relativistic.

During the time of detection of a single event $t=5\cdot10^{-6}$ s the cyclotron radiation carries away an amount of energy

$$E_t=gtE=0.08\ \text{eV}=460\ \text{quanta} \tag{14}$$

that is small compared to the planned precision of 2.5 eV. If one wants to increase the precision, then more time t is needed for that. That means measuring the electrons that stay longer in the zone, so giving a higher signal. Thus, improvement of the accuracy of the measurement would result in a higher signal but lower counting rate. The theoretical limit of resolution at the frequency 40 GeV is

$$\Delta E_{40}=\sqrt{0.08\cdot2.5}\approx0.5\ \text{eV}. \tag{15}$$

Much lower resolutions are also inconceivable because of a larger bandwidth and, accordingly, greater thermal noises.

5. THERMAL NOISES AND MINIMUM SIGNAL

In a closed volume the radiation field is determined by the black body radiation

$$N=\frac{\Delta\nu t}{e^{h\nu/kt}-1} \text{ quanta per } \lambda^2. \tag{16}$$

Being below maximum intensity frequency, this can be approximated to

$$N=\frac{\Delta\nu}{\nu}\frac{kT}{h}t\ \frac{\text{quanta}}{\lambda^2}. \tag{17}$$

Inserting $\Delta\nu/\nu=5\cdot10^{-6}$ and $t=5\cdot10^{-6}$ s, one will get

$$N=\left(0.5\frac{1}{\text{K}}\right)T. \tag{18}$$

Comparing the noise with the number of quanta in signal (8), one can see that the input circuits should be at the nitrogen temperature

$$T=77 \text{ K}, \qquad N=39. \tag{19}$$

The noise temperature of a maser preamplifier can be $\approx$30 K that adds 15 quanta. The noise of the following mixer $\approx$400 K referred to the input of the preamplifier with $\approx$20 dB would add 4 quanta. If the input waveguide is at the nitrogen temperature, then the total noise would be

$$N_{\text{noise}}\leqslant 60 \text{ quanta}. \tag{20}$$

As the noise is expressed in rather small discrete figures, we need the calculation of the noise fluctuations in equal time intervals t. The level of the signal at the input port of the preamplifier must exceed the fluctuations of noise during a certain experimenting period τ.

The probability for the counting rate i of a random event within a certain time interval t in case the average rate is n, is given by the Poisson distribution

$$\Pi_{i,n}=e^{-n}n^i/i! \ . \tag{21}$$

One has to find the number I for which

$$\left(\sum_{i=I}^{\infty}\Pi_{i,n}\right)^{-1}t=\tau. \tag{22}$$

We can choose τ as equal to either one day or one year, for example. With an error less than 10% Eq. (22) takes the τ-independent approximated solution

$$I=50+n \tag{23}$$

which in our case (20)

$$n=N_{\text{noise}}=60 \tag{24}$$

gives

$$I=110. \tag{25}$$

Thus, in order to avoid false signals from noise, one must have a pure signal exceeding 50 quanta, together with the noise background $>$110 quanta. The power of this signal is

$$P=6\cdot10^{-16} \text{ W}=-122 \text{ dBm}. \tag{26}$$

6. CONVERSION OF WAVE MODES

By comparing 50 quanta with (8), one can see that at least 11% of the total number of quanta must enter the preamplifier. A problem arises in connection with the fact that according to the dimensions of the waveguide the whole energy of the radiation is distributed across a number of different wave modes but only one of them, TE_{01}, can enter the input port.

It is known that all the modes in an oversized waveguide have different phase velocities as well as wavelengths. Irregularities in the waveguide cause mutual conversion of the modes. The process has such a peculiarity that conversion is greatly preferred into the mode for which the irregularities are periodic with the period of half the wavelength [8]. This can be used for a directed conversion of the energy of the wave modes.

The wavelength of the modes TE_{mn} and TM_{mn} in a rectangular waveguide of dimensions a and b is

$$\lambda = \frac{\lambda_0}{\sqrt{1-(k_c/k_0)^2}}, \tag{27}$$

where λ_0 is the wavelength in the open vacuum, k_0 is the wave number

$$k_0 = \frac{\omega}{c}, \tag{28}$$

and k_c is the cutoff wave number

$$k_c = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}. \tag{29}$$

Our experiments have shown that by applying a carefully chosen and adjusted plastic comb or a spiral wire of about 10 wavelengths long, one can convert the mode TE_{10} into TE_{01} with the efficiency of 90% and more. All the wrong modes can several times pass the oversized region of the waveguide being reflected from both the end cap and the port taper. At that they are gradually converted into TE_{01}. Losses due to attenuation should be tested experimentally.

7. MEASUREMENT OF ^{35}S SPECTRUM

Sulphur ^{35}S is characterized by a considerably higher top energy

$$E_0 = 167.5 \text{ keV} \tag{30}$$

and shorter half-life,

$$T_{1/2} = 87.4 \text{ days}, \tag{31}$$

than tritium. Though the kinetic energy is nine times higher, the corresponding total energy is merely 28% higher and this, unlike other spectrometries, creates no problems in the case of our method.

The magnetic field needed is $B = 1.9$ T, and the time of measurement for a 100 cm^2 source would be

ΔE_{res}	time
2.5 eV	6.5 years
10 eV	1 month
25 eV	2 days.

(32)

Cyclotron radiation carries considerably more energy

$$\Delta E = 0.28 \text{ eV} = 1660 \text{ quanta}, \tag{33}$$

still notably exceeding the contribution of the higher harmonics (synchrotron radiation).

The detectable signal is the same as in the case of tritium but it constitutes only 3% of the total number of quanta generated. The signal-to-noise ratio is so good that the experiment can be carried out at room temperature.

8. SUMMARY AND OUTLOOK

Radiation of a single electron has been first detected at the "geonium" experiments [5]. The exquisite results obtained encourage us to attempt to detect the cyclotron radiation of a β-decay electron instead of a trapped electron's axial radiation. Precise measurement of this higher frequency would enable one to measure the energy as the relativistic mass excess with a resolution of some electronvolts. At that LN_2 temperatures and noiseless amplifiers are needed.

Maser is an amplifier with the lowest possible noise temperature. Unfortunately, the upper limit of its working frequencies is at ≈ 42 GHz [6]. In principle, there are also possibilities to avoid a preamplifier, by applying the most effective kinds of frequency mixers. Such would be a SIS structure that in laboratory experiments has yielded an input noise level as low as 9.5 K in the frequency region of 85—116 GHz [9]. For this reason it is of interest to consider the effect of raising the frequency ν. That would change the magnetic field B, length of trajectory, time of flight t, angle of declination, amount of measurable electrons, and attenuation rate. The theoretical limit of resolution increases:

$$\Delta E_{\text{res }\nu} \sim \sqrt{\nu} \tag{34}$$

but the signal-to-noise ratio is proportional to ν. That means that at $\approx$ 100 GHz one can detect tritium end point energy at room temperature!

At our technique resolution is rather a prerequisite than the result of an experiment. The improvement of resolution begins with an improvement of the homogeneity of the magnetic field. Ratio (1) has to drop. This means an increase in time t. The effective activity is adversely affected through (i) change of ejection angle (proportional), (ii) restriction of the effective end point region (proportional to the cube of ΔE (8)), (iii) pressure of the gas (in the case of a gaseous $^3H_2$). Of these only the increase of t would give a directly proportional gain. All that together would lead to lengthening of the experimenting time and/or a larger volume of the β-decay region.

The signal is proportional to t but, as to the thermal noise, the increase of t is compensated by the dropping bandwidth $\Delta\nu$. The signal-to-noise ratio would increase.

The only factor limiting resolution is the loss of energy (14) that may not exceed the resolution figure. In this sense lower frequencies would get some preference which, evidently, would require liquid helium temperatures throughout the whole device to suppress thermal noises.

Comparing cyclotron spectrometry with the conventional ones, we may conclude that:

1. The problem of the measurement of the energy of a particle is reduced to the measurement of a frequency that can be done with very high precision, in principle.

2. No ionization-based detector is applied, which means that, in effect, no other particle of a different origin can imitate a measurable signal, and the device may need no shielding against background radiation.

3. The volume of the homogeneous magnetic field is small compared to that of conventional beta spectrometers. It is not difficult to cancel the effects of geomagnetic and random fluxes.

4. The electrons are not collimated and the use of larger source surfaces is feasible with little changes in design. The device is compact enough to be installed in any laboratory.

5. Additional perspectives seem to be open depending on electronics.

6. For possible measurements of the end points of more energetic spectra, such as of ^{14}C, ^{35}S, ^{63}Ni, ^{71}Ge, etc. the method has obvious advantages over the others.

7. The energy of the electron is measured where it is born. This considerably restricts the possibilities of using the spectrometer for applications with sources other than radioactive ones.

In the case of discrete spectra the method is applicable with less difficulty than at the end points of continuous spectra, especially when the intensity of the spectral line allows the source to be made one-dimensional. As our method yields best results with gaseous sources, we envisage applications to the investigation of molecular and solid state corrections at different surfaces by a comparison of various solid sources with gaseous ones.

In order to judge the applicability of the method to neutrino mass experiments, one has to carry out experiments with electrons emitted nearly parallel to the surfaces of various solid sources. Problems of collecting the signal from larger volumes and the conversion of wave modes also need some experimental testing.

ACKNOWLEDGEMENTS

I would like to thank Prof. E. Lippmaa, J. Past, J. Puskar, N. T. Cherpak, and Ch. Weinheimer for valuable discussions.

This research was done when the author was working at the Institute of Chemical Physics and Biophysics, Estonian Academy of Sciences, Tallinn.

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BEETA-LAGUNEMISE SPEKTRITE MÕÕTMISE TSÜKLOTRONSAGEDUSE MEETOD

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Arvutustega on näidatud, et üksiku beeta-lagunemisel tekkinud elektroni energia on mõõdetav homogeenses magnetväljas tema tsüklotronsageduse mõõtmise teel. Radioaktiivne allikas on paigutatud ülemõõdulise lainejuhi sisse vedela lämmastiku temperatuuril ja signaal juhitakse maservõimendisse. See meetod on rakendatav väikese intensiivsusega diskreetsete spektrite mõõtmiseks. Kuna meetodi detailne käsitlus nõuab füüsikaliste tingimuste täpsustamist, siis on tähelepanu pööratud võimalikele rakendustele triitiumi ja teiste tuumade pidevate spektrite puhul eesmärgiga täiendavalt piirata neutriino massi võimalikku väärtust. Spektrite tipupiirkondade aktiivsus võib olla liiga väike neutriino massi testimiseks. Seepärast on vaja spetsiaalseid katseid, et maksimaalselt suurendada allika pinda. Mõned teised rakendused, nagu heeliumi ja triitiumi massi diferentsi mõõtmine ja beetaaktiivsete allikate molekulaarsete ja pinnakorrektsioonide uurimine, on teostatavad, ilma et mainitud probleem esile tuleks.

МЕТОД ЦИКЛОТРОННОЙ ЧАСТОТЫ ДЛЯ ИЗМЕРЕНИЯ СПЕКТРОВ БЕТА-РАСПАДА

Айн АЙНСААР

С помощью вычислений показано, что энергию единичного электрона бета-распада можно измерить в гомогенном магнитном поле методом определения его циклотронного излучения. Радиоактивный источник введен в волновод при температуре жидкого азота и сигнал направлен в мазерный усилитель. Эта техника применима для точного измерения дискретных спектров препаратов малой интенсивности. Так как детальное описание метода требует уточнений физических условий, то внимательно изучена возможность применения его для непрерывных спектров трития и других ядер с целью ограничить пределы массы нейтрино. Активности конечных регионов спектра могут быть низкими для непосредственного определения массы нейтрино, в связи с чем необходимы предварительные эксперименты для увеличения поверхности источника. Однако возможны и другие варианты, как измерение разности масс гелия и трития, а также проверка молекулярных и поверхностных коррекций бета-активных источников, которые не испытывают этот недостаток.