Proc. Estonian Acad. Sci. Phys. Math., 1995, 44, 4, 450–458
https://doi.org/10.3176/phys.math.1995.4.04

ON THE $N=1$ LINEARIZED SUPERGRAVITY

Paul SUURVARIK

Tallinna Tehnikaülikool (Tallinn Technical University), Ehitajate tee 5, EE-0026, Tallinn, Eesti (Estonia)

Presented by R.-K. Loide

Received 27 January 1995, accepted 19 June 1995

Abstract. The formalism of superprojectors is used for $N=1$ linearized supergravity. This formalism is based on the traditional and untraditional superprojectors. A modified formulation of $N=1$ linearized supergravity is given.

Key words: untraditional superprojector, supergravity, superfield.

1. INTRODUCTION

In the present paper we demonstrate the using of the formalism [1] of superprojectors for $N=1$ linearized supergravity. In general, the theory of $N=1$ supergravity is well known, but here we point out a quite interesting approach of a version of minimal $N=1$ supergravity. Being supercovariant and basing on the vector and scalar superfield, such an approach has some advantages for finding superpropagators.

Minimal $N=1$ supergravity in superfield formulation is given by Ogievetsky and Sokatchev [2]. It can be considered the theory of real axial vector-superfield $h^{\mu}(x, \theta)$ generated by the supercurrent $V^{\mu}(x, \theta)$ which contains superspins 3/2, 0, 0. The supercurrent obeys the conservation law

$$2\partial_{\mu}(\gamma_{\nu}D)V^{\nu}-D_{\alpha}\bar{D}DV_{\mu}=0. \tag{1}$$

The equation of motion is

$$(d^{\mu}d^{\nu}+3d^{\nu}d^{\mu})h_{\nu}=\varkappa V^{\mu}, \tag{2}$$

where $\varkappa$ is the gravitational coupling constant and $d^{\mu}\equiv Di\gamma^{\mu}\gamma^{5}D$. Equation (2) is invariant under the local gauge transformation

$$\delta h_{\mu}=\bar{D}\gamma_{\mu}\gamma^{\nu}D\bar{D}\varphi_{\nu}-2i\partial^{\nu}\bar{D}\gamma_{\mu}\varphi_{\nu}, \tag{3}$$

where $\varphi_{\nu\alpha}(x, \theta)$ in an arbitrary spinor-vector superfield.

2. MODIFIED FORMULATION

We start from the equation for the massive superfield given by Loide [1] which describes superspins 3/2 and 0. The presence of superspins 0 depends on the choice of parameters a, b, c. In the massless case $m=0$ and by adding the source superfields we obtain [3]

$$\Box \begin{vmatrix} E^{3/2}_{11} - \frac{2}{3} E^{0}_{11} & aE^{0}_{12} \\ bE^{0}_{21} & cE^{0}_{22} \end{vmatrix} \begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix} = -\varkappa \begin{vmatrix} V_1 \\ V_2 \end{vmatrix}, \tag{4}$$

where, E^{y}_{ik} are superspin-projection operators, i.e. superprojectors (see App.), $\psi_1 \equiv h^{\mu}(x, \theta)$ is an axial vector superfield, $\psi_2 \equiv \Phi(x, \theta)$ is a scalar superfield, $\varkappa$ is the gravitational constant, $V_1 \equiv V^{\mu}(x, \theta)$ is an axial source superfield (supercurrent), $V_2 \equiv V(x, \theta)$ is a scalar source superfield (see App.).

In the case of the massless superfield the gauge transformations arise

$$\begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix} \to \begin{vmatrix} \psi_1 + E_{13}\psi_3 \\ \psi_2 + E_{23}\psi_3 \end{vmatrix}, \tag{5}$$

where E_{13}, E_{23} are supercovariant operators and ψ_3 is an arbitrary spinor superfield. This means that the equation operator and gauge operators are related as

$$\begin{vmatrix} E^{3/2}_{11} - \frac{2}{3} E^{0}_{11} & aE^{0}_{12} \\ bE^{0}_{21} & cE^{0}_{22} \end{vmatrix} \begin{vmatrix} E_{13} \\ E_{23} \end{vmatrix} = 0. \tag{6}$$

In addition there exist supercovariant operators E_{31}, E_{32}, satisfying the condition

$$| E_{31}, E_{32} | \begin{vmatrix} E^{3/2}_{11} - \frac{2}{3} E^{0}_{11} & aE^{0}_{12} \\ bE^{0}_{21} & cE^{0}_{22} \end{vmatrix} = 0. \tag{7}$$

From (7) we get the conservation law for source superfields

$$E_{31}V_1 + E_{32}V_2 = 0. \tag{8}$$

Now we are constructing the operators

$$E_{13},\ E_{23},\ E_{31},\ E_{32}. \tag{9}$$

We express these operators by the help of untraditional superprojectors E^{y}_{ik}, $i \neq k$ (see App.). The operators (9), given as a linear combination of E^{y}_{ik}, $(i \neq k)$, have to involve all superspins that are possible by conditions (6) and (7). The coefficients in the expansions of operators (9) are restricted by the requirement that these operators could be localized.

Following this requirement we choose the coefficients so that nonlocalities cancel each other. For gauge operators we may write

$$\begin{aligned} E_{13} &= \alpha_1 E^{0}_{13} + \alpha_2 E^{1}_{13} + \alpha^3 \overset{0\ 1/2}{E}_{13} + \alpha_4 \sqrt{3} \overset{\overline{1}\ 1/2}{E}_{13}, \\ E_{23} &= \alpha_5 E^{0}_{23} + \alpha_6 E^{1/2}_{23}, \end{aligned} \tag{10}$$

where α_i $(i = 1, \ldots, 6)$ are nonzero coefficients. E^{y}_{ik} are superprojectors (see App.). We note that there is a useful relation for superprojectors [1].

$$E^{y}_{ik} E^{y'}_{kj} = \delta_{yy'} E^{y}_{ij} \tag{11}$$

(there is no summation by k). It is easy to see, considering (6) and (11), that Eq. (4) is invariant under gauge transformations (5) on conditions

$$ab=-\frac{2}{3}c,$$

$$\alpha_1=\frac{3}{2}a\alpha_5. \tag{12}$$

Analogously to expressions (10) we can write

$$E_{31}=i\partial(\beta_1 E^{0}_{31}+\beta_2 E^{1}_{31}+\beta_3 E^{0\,1/2}_{31}+\sqrt{3}\beta_4 E^{1/2}_{31}),$$

$$E_{32}=i\partial(\beta_5 E^{0}_{31}+\beta_6 E^{1/2}_{32}), \tag{13}$$

where $\beta_1, \ldots, \beta_6$ are nonzero coefficients. Operators E^{y}_{ik} are for superspins 0, 1/2, 1. The operator $i\partial$ is needed for diminishing nonlocality. From condition (7) we get a constraint for coefficients

$$\beta_1=\frac{3}{2}b\beta_5. \tag{14}$$

By using expressions of E^{y}_{ik} (see App.), the equivalent forms of E_{13} and E_{23} are

$$(E_{13})^{\alpha\mu}=\frac{1}{2\sqrt{2}\,\Box}[(\alpha_1-\alpha_2-\alpha_3-\alpha_4)\bar{D}D\bar{D}^{\alpha}i\partial^{\mu}+(\alpha_2+\alpha_4)\bar{D}D(\bar{D}i\partial\gamma^{\mu})^{\alpha}+$$

$$+2(\alpha_3-\alpha_4)(\bar{D}\partial)^{\alpha}\partial^{\mu}+2\alpha_4\Box(\bar{D}\gamma^{\mu})^{\alpha}], \tag{15}$$

$$(E_{23})^{\alpha}=\frac{1}{2\sqrt{2}\,\Box}[(\alpha_6-\alpha_5)\bar{D}D(\bar{D}i\partial)^{\alpha}+2\alpha_6\Box\bar{D}^{\alpha}].$$

Similarly we can write for E_{31} and E_{32}

$$(E_{31})_{\mu\alpha}=\frac{1}{2\sqrt{2}\,\Box}[(\beta_1-\beta_2+\beta_3-\beta_4)i\partial_{\mu}\bar{D}_{\alpha}\bar{D}D+$$

$$+(\beta_2+\beta_4)(i\partial\gamma_{\mu}D)_{\alpha}\bar{D}D+(\beta_3+\beta_4)2\partial_{\mu}(\partial D)_{\alpha}-2\beta_4\Box(\gamma_{\mu}D)_{\alpha}], \tag{16}$$

$$(E_{32})_{\alpha}=\frac{1}{2\sqrt{2}\,\Box}[(\beta_6-\beta_5)(i\partial D)_{\alpha}\bar{D}D+2\beta_6\Box D_{\alpha}].$$

Now we try to concretize the parameters. Depending on the choice of parameters a, b, two different forms of Eq. (4) are now possible, respectively $a\neq 0$, $b\neq 0$ and $a=b=0$. The first choice gives us a new form of the equation of motion describing minimal $N=1$ supergravity. The second one is entirely equivalent to the equation of minimal $N=1$ supergravity given by Ogievetsky, Eq. (2).

The case of $a\neq 0$, $b\neq 0$. In this case we get minimal $N=1$ supergravity given, at first by the help of two superfields, $h^{\mu}(x, \theta)$ and $\Phi(x, \theta)$. The last one vanishes by fixing the gauge, but as we can see, some constraints that were used in [4] now follow directly from the theory. According to the considerations given before for operators E_{13}, E_{23}, E_{31}, E_{32}, and for simplicity, we take

$a=-2/3,\quad \alpha_1=\alpha_2=-\alpha_3=-\alpha_4,\quad \alpha_5=\alpha_6=\sqrt{3},\quad b=-2/3,\quad \beta_1=-\beta_2=-\beta_3=\beta_4,\ \beta_5=\beta_6=\sqrt{2}$. The equivalents from these operators are now

$$(E_{13})^{\mu\alpha}=(\bar{D}\gamma^\mu)^\alpha, \tag{15'}$$

$$(E_{23})^\alpha=\bar{D}^\alpha$$

and

$$(E_{31})_{\mu\alpha}=(\gamma_\mu D)_\alpha, \tag{16'}$$

$$(E_{32})_\alpha=D_\alpha.$$

All in all, we can write now superfield equation of motion (4) in an apparent form

$$\frac{2}{3}\left[\Box+\frac{1}{4}(\bar{D}D)^2\right]h^\mu-\frac{2}{3}\partial^\mu\partial_\lambda h^\lambda-\frac{1}{6}\varepsilon^\mu{}_{\lambda\rho\sigma}\partial^\rho\bar{D}i\gamma^6\gamma^5Dh^\lambda-$$

$$-\frac{1}{3}\bar{D}Di\partial^\mu\Phi=-\varkappa V^\mu, \tag{17}$$

$$\frac{1}{3}\bar{D}Di\partial_\lambda h^\lambda-\frac{1}{6}(\bar{D}D)^2\Phi=-\varkappa V.$$

The expressions of operators E^{ij}_{ik} (see App.) are used here.

The invariance transformations are

$$h^\mu\to h^\mu+\bar{D}\gamma^\mu\psi, \tag{18}$$

$$\Phi\to\Phi+\bar{D}\psi,$$

and the conservation law for source superfields

$$(\gamma_\mu D)_\alpha V^\mu-D_\alpha V=0. \tag{19}$$

This differential condition (19) singles out superspins 3/2, 0, 0 and is equivalent to condition (3). Really, affecting Eq. (19) by the operator $\bar{D}\gamma_\nu D\eta^\alpha_\beta-D_\beta(\bar{D}\gamma_\nu)^\alpha$, we have the conservation law only for the supercurrent V_μ

$$2\partial_v(\gamma_\mu D)_\alpha V^\mu-D_\alpha\bar{D}DV_\nu=0, \tag{20}$$

which also singles out superspins 3/2, 0, 0. From (19) we get the restriction for $V(x,\theta)$, $\bar{D}\ (\bar{D}D)DV=0$, by which V contains only superspins 0. On the other hand, from (19) we have also

$$2i\partial_\mu V^\mu=\bar{D}DV. \tag{21}$$

Consequently, the supercurrent V_μ contains superspins 3/2, 0, 0, and these superspins 0, 0 are connected with the scalar source V. We see that supercurrent multiplet is given by the axial vector superfield and the scalar superfield with the conservation law (19). The conservation law (19) for currents (V_μ, V) is a differential condition of lower order than the condition for the supercurrent V_μ only.

Fixing the gauge $\Phi'=0$, Eqs. (17) acquire the form

$$\frac{2}{3}\left[\Box+\frac{1}{4}(\bar{D}D)^2\right]h^\mu-\frac{2}{3}\partial^\mu\partial_\lambda h^\lambda-\frac{1}{6}\varepsilon^\mu{}_{\lambda\rho\sigma}\partial^\rho\bar{D}i\gamma^\sigma\gamma^5Dh^\lambda=-\varkappa V^\mu, \tag{22}$$

$$-\frac{1}{3}\bar{D}Di\partial_\lambda h^\lambda=-\varkappa V, \tag{23}$$

with gauge invariance

$$h \to h^{\mu} + \bar{D}\gamma^{\mu}\psi \tag{24}$$

on condition

$$\bar{D}\psi = 0 \tag{25}$$

that we get from (18). We note that Eq. (23) follows from (22) and is therefore inessential.

The gauge fixing leads to the well-known version of minimal linearized supergravity (2). At the same time, the gauge transformations (18) and the differential condition for source (19) derive in the natural way when using superprojectors. Earlier the gauge transformations in form (24) have been given by Ferrara and Zumino [4] without an algorithm for deriving them. Only the constraint $\bar{D}\psi = 0$ was added to (24).

Consequently, Eqs. (17) are a version of minimal supergravity in a modified formulation.

The Lagrangian in superspace corresponding to Eqs. (17) is

$$\mathfrak{L} = \frac{1}{2}\Big[h^{\mu}\Big(E^{3/2}_{11} - \frac{2}{3}E^{0}_{11}\Big)^{\nu}_{\mu} h_{\nu} + h^{\mu}(E^{0}_{12})_{\mu}\Phi + \\ + \Phi(E^{0}_{21})^{V} h_{\nu} + \Phi E^{0}_{22}\Phi\Big] + \varkappa(h^{\mu}V_{\mu} + \Phi V). \tag{26}$$

This Lagrangian is invariant under the gauge transformations (18) by condition (19). It is easy to see that an equivalent form of this Lagrangian follows from (26):

$$\mathfrak{L} = \frac{1}{12}\Big[\frac{1}{4}\bar{D}i\gamma_{\mu}\gamma^{5}Dh^{\mu}\bar{D}i\gamma^{V}\gamma^{5}Dh_{V} + \frac{3}{4}\bar{D}i\gamma^{V}\gamma^{5}Dh^{\mu}Di\gamma_{\mu}\gamma^{5}Dh_{V} + \\ + \bar{D}Dh^{\mu}\bar{D}\gamma_{\mu}D\Phi + \bar{D}D\Phi\bar{D}\gamma^{V}Dh_{V} - \bar{D}D\Phi\bar{D}D\Phi\Big] + \varkappa(h^{\mu}V_{\mu} + \Phi V). \tag{27}$$

Considering the general Euler—Lagrange equation [5], we can get the supersymmetric equation of motion.

The case of $a=0$, $b=0$. Here we get a version of minimal supergravity considered by Ogievetsky and Sokatchev, see (1), only with different gauge transformations in the form given earlier by Ferrara [4]. Indeed, according to (12) and (14), $\alpha_1 = \beta_1 = 0$. Other coefficients have to be chosen by the condition of locality of the operators E_{13} and E_{31}.

For example, $\alpha_2 = -\alpha_3 = \alpha_4 = -\sqrt{2}\,\square$ and $\beta_2 = \beta_3 = -\beta_3 = -2\sqrt{2}\,\square$. Now we may write the equation of motion

$$\Big(E^{3/2}_{11} - \frac{2}{3}E^{0}_{11}\Big)^{\mu}_{\lambda} h^{\lambda} = \varkappa V^{\mu}, \tag{28}$$

with the invariance

$$h^{\mu} \to h_{\mu} + \square\bar{D}\gamma^{\mu}\psi - \frac{1}{2}i\partial^{\mu}\bar{D}D\bar{D}\psi \tag{29}$$

and the conservation law for the supercurrent

$$[i\partial_{\mu}D_{\alpha}\bar{D}D + 2\square(\gamma_{\mu}D)_{\alpha}]V^{\mu} = 0. \tag{30}$$

The gauge operator (29) and differential condition (30) for the supercurrent are of higher order than in the previous case.

3. CONFORMAL SUPERGRAVITY

In the case of conformal supergravity [6], by $m=0$ and the supercurrent $V_\mu(x, \theta)$, the linearized equation of motion is

$$\Box\Box E^{3/2}_{11}\psi_1 = gV_1, \tag{31}$$

where $\psi_1 \equiv h^\mu(x, \theta)$, g is the coupling constant, $V_1 \equiv V^\mu(x, \theta)$. Equation (31) has gauge invariance

$$\psi_1 \to \psi_1 + E_{13}\psi_3, \tag{32}$$

where $\psi_3(x, \theta)$ is an arbitrary spinor superfield. The conservation law for the supercurrent is

$$E_{31}V_1 = 0, \tag{33}$$

which singles out superspin 3/2. By the meaning of operators E_{13}, E_{31} the conditions have to be

$$E^{3/2}_{11}E_{13} = 0 \tag{34}$$

and

$$E_{31}E^{3/2}_{11} = 0. \tag{35}$$

Considering (11), we see that conditions (34) and (35) are satisfied if operators E_{13}, E_{31}, are linear combinations of superprojectors of superspins 0, 1/2, 1. The formal expressions of E_{13}, E_{31} are already given by (10) and (13). The considerations for the coefficients α_i, β_i $(i=1, \ldots, 4)$ given before are the same. In this case only conditions (34), (35) do not give any restrictions for coefficients. Choosing, for example, $\alpha_1 = \alpha_2 = -\alpha_3 = -\alpha_4 = -\sqrt{2}$, $\beta_1 = -\beta_2 = -\beta_3 = \beta_4 = -\sqrt{2}$, the operators E_{13} and E_{31} become explicit forms

$$(E_{13})^{\mu\alpha} = (\bar{D}\gamma^\mu)^\alpha, \tag{36}$$

$$(E_{31})_{\mu\alpha} = (\gamma_\mu D)_\alpha. \tag{37}$$

In conformal supergravity the order of the equations of motion is too high.

4. CONCLUSIONS

Above some expressions of untraditional superprojectors are given, the application of the formalism of superprojectors for $N=1$ supergravity is demonstrated. The structure of superspins of operators becomes apparent.

A modified formulation of $N=1$ linearized supergravity by the help of the vector and scalar superfields is given. Such an approach has advantages on quantization.

APPENDIX. SUPERFIELD PROJECTION OPERATORS

Notations used

$$\frac{1}{2}\{\gamma_\mu, \gamma_\nu\} = \eta_{\mu\nu} = \mathrm{diag}(+---); \quad \gamma^5 = \gamma^0\gamma^1\gamma^2\gamma^3;$$

$$\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]; \quad \partial/\partial x^\mu = \partial_\mu; \quad \Box \equiv \partial_\mu\partial^\mu; \quad \hat{\partial} = \partial_\mu\gamma^\mu;$$

$$\varepsilon^{0123} = 1; \quad \bar{\theta}^\alpha = (C^{-1})^{\alpha\beta}\theta_\beta,$$

where C is a charge conjugate matrix; the covariant derivatives

$$D_\alpha = \partial/\partial\theta^\alpha - (1/2)(\hat{\partial}\theta)_\alpha, \quad \bar{D}^\alpha = (C^{-1})^{\alpha\beta}D_\beta, \quad \{D_\alpha, D_\beta\} = -(i\hat{\partial}C)_{\alpha\beta}.$$

Superfield projection operators, i. e. superprojectors E^Y

1. Scalar superfield.

Superspins $Y = 0, 0, 1/2$ are singled out by

$$E^0_\pm = -\frac{1}{8\Box}[(\bar{D}D)^2 \pm 2i\partial_\nu\bar{D}i\gamma^\nu\gamma_5 D],$$

$$E^{1/2} = 1 + \frac{1}{4\Box}(\bar{D}D)^2. \tag{A.1}$$

Here $+$, $-$ correspond to different chirality of an operator.

2. Spinor superfield.

Superspins $Y = 1, 1/2, 1/2, 0$.

$$E^1 = \frac{3}{4}\left[1 + \frac{1}{4\Box}(\bar{D}D)^2\right] - \frac{1}{8\Box}\partial^\mu\sigma_{\mu\nu}\gamma^5\bar{D}i\gamma^\nu\gamma^5 D,$$

$$E^{1/2}_\pm = -\frac{1}{8\Box}[(\bar{D}D)^2 \pm 2i\partial_\nu\bar{D}i\gamma^\nu\gamma^5 D], \tag{A.2}$$

$$E^0 = \frac{1}{4}\left[1 + \frac{1}{4\Box}(\bar{D}D)^2\right] + \frac{1}{8\Box}\partial^\mu\sigma_{\mu\nu}\gamma^5\bar{D}i\gamma^\nu\gamma^5 D.$$

3. Vector superfield.

Four superspins, 3/2, 1, 1, 1/2, correspond to Poincaré spin 1, which are extracted by

$$(E^{3/2})^\varkappa{}_\lambda = \frac{2}{3}\left[1 + \frac{1}{4\Box}(\bar{D}D)^2\right]\left(\eta^\varkappa_\lambda - \frac{\partial^\varkappa\partial_\lambda}{\Box}\right) - \frac{1}{6\Box}\varepsilon^\varkappa{}_{\lambda\rho\sigma}\partial^\rho\bar{D}i\gamma^\sigma\gamma^5 D,$$

$$(E^1_\pm)^\varkappa{}_\lambda = -\frac{1}{8\Box}[(\bar{D}D)^2 \pm 2i\partial_\nu\bar{D}i\gamma^\nu\gamma^5 D]\left(\eta^\varkappa_\lambda - \frac{\partial^\varkappa\partial_\lambda}{\Box}\right), \tag{A.3}$$

$$(E^{1/2})^\varkappa{}_\lambda = \frac{1}{3}\left[1 + \frac{1}{8\Box}(\bar{D}D)^2\right]\left(\eta^\varkappa_\lambda - \frac{\partial^\varkappa\partial_\lambda}{\Box}\right) + \frac{1}{6\Box}\varepsilon^\varkappa{}_{\lambda\rho\sigma}\partial^\rho\bar{D}i\gamma^\sigma\gamma^5 D,$$

and superspins 0, 0, 1/2 correspond to Poincaré spin 0

$$(E^{1/2}_{0})^{\varkappa}_{\lambda}=\left[1+\frac{1}{4\Box}(\bar{D}D)^2\right]\frac{\partial^{\varkappa}\partial_{\lambda}}{\Box},$$

(A.4)

$$(E^{0}_{0\pm})^{\varkappa}_{\lambda}=-\frac{1}{8\Box}\left[(\bar{D}D)^2\pm 2i\partial_{\nu}\bar{D}i\gamma^{\nu}\gamma^{5}D\right]\frac{\partial^{\varkappa}\partial_{\lambda}}{\Box}.$$

We mark $\psi_1\equiv h^{\varkappa}(x,\theta)$, $\psi_2\equiv\Phi(x,\theta)$, $\psi_3\equiv\Phi_{\alpha}(x,\theta)$. The operator E^{y}_{ik} $(i\neq k)$ is a nontraditional superspin projector, which singles out the superspin Y from the superfield $\Phi_k(x,\theta)$ and transforms it to the superfield Φ_i, i.e. $\Phi_i=E^{y}_{ik}\Phi_k$. The traditional superprojectors we may mark as well E^{y}_{ii}.

The superprojectors satisfy the relations [1]

$$E^{y}_{ij}E^{y'}_{jk}=\delta_{yy'}E^{y}_{ik} \qquad \text{(A.5)}$$

(no summation by j).

For example, the superprojectors of superspins $Y=0,\ 1/2,\ 1$ for superfields ψ_1, ψ_2, ψ_3 are

$$(E^{0}_{12})^{\mu}=\frac{1}{2\Box}i\partial^{\mu}\bar{D}D,\quad (E^{0}_{21})_{\mu}=\frac{1}{2\Box}i\partial_{\mu}\bar{D}D,$$

$$(E^{1/2}_{12})^{\mu}=\frac{1}{\sqrt{\Box}}\left[1+\frac{1}{4\Box}(\bar{D}D)^2\right]\partial^{\mu},$$

$$(E^{1/2}_{21})_{\mu}=\frac{1}{\sqrt{\Box}}\left[1+\frac{1}{4\Box}(\bar{D}D)^2\right]\partial_{\mu},$$

(A.6)

$$(E^{1/2}_{12})^{\mu}=\frac{1}{2\sqrt{3\Box}}\left(\eta^{\mu}_{\ \nu}-\frac{\partial^{\mu}\partial_{\nu}}{\Box}\right)\bar{D}i\gamma^{\nu}\gamma^{5}D,$$

$$(E^{1/2}_{21})_{\mu}=\frac{1}{2\sqrt{3\Box}}\left(\eta_{\mu\nu}-\frac{\partial_{\mu}\partial_{\nu}}{\Box}\right)\bar{D}i\gamma^{\nu}\gamma^{5}D;$$

$$(E^{0}_{23})^{\alpha}=-\frac{1}{2\sqrt{2\Box}}\bar{D}D(\bar{D}i\partial)^{\alpha},\quad (E^{0}_{32})_{\alpha}=-\frac{1}{2\sqrt{2\Box}}D_{\alpha}\bar{D}D,$$

(A.7)

$$(E^{1/2}_{23})^{\alpha}=\frac{1}{2\sqrt{2\Box}}(\bar{D}i\partial)^{\alpha}\bar{D}D,\quad (E^{1/2}_{32})_{\alpha}=\frac{1}{2\sqrt{2\Box}}\bar{D}DD_{\alpha},$$

$$(E^{0}_{13})^{\mu\alpha}=\frac{1}{2\sqrt{2\Box}}i\partial^{\mu}\bar{D}D\bar{D}^{\alpha},\quad (E^{0}_{31})_{\mu\alpha}=\frac{1}{2\sqrt{2\Box}}\frac{\partial_{\mu}\partial_{\nu}}{\Box}(\gamma^{\nu}D)_{\alpha}\bar{D}D,$$

$$(E^{1}_{13})^{\alpha\mu}=\frac{1}{2\sqrt{2\Box}}\bar{D}D(\bar{D}i\sigma^{\nu\mu}\partial_{\nu})^{\alpha}, \qquad \text{(A.8)}$$

$$(E^{1}_{31})_{\alpha\mu}=\frac{1}{2\sqrt{2\Box}}\left(\eta_{\mu\nu}-\frac{\partial_{\mu}\partial_{\nu}}{\Box}\right)(\gamma^{\nu}D)_{\alpha}\bar{D}D;$$

$$(E^{0\,1/2}_{13})^{\alpha\mu}=\frac{1}{2\sqrt{2}\Box}\overline{D}^{\alpha}\overline{D}Di\partial^{\mu},$$

$$(E^{0\,1/2}_{31})_{\mu\alpha}=\frac{1}{2\sqrt{2}\Box}\frac{\partial_{\mu}\partial_{\nu}}{\Box}\overline{D}D(\gamma^{\nu}D)_{\alpha},$$

$$(E^{1\,1/2}_{13})^{\mu\alpha}=\frac{1}{2\sqrt{6}\Box}(\overline{D}i\sigma^{\nu\mu}\partial_{\nu})^{\alpha}\overline{D}D, \tag{A.9}$$

$$(E^{1\,1/2}_{31})_{\mu\alpha}=\frac{1}{2\sqrt{6}\Box}\left(\eta_{\mu\alpha}-\frac{\partial_{\mu}\partial_{\nu}}{\Box}\right)\overline{D}D(\gamma^{\nu}D)_{\alpha}.$$

REFERENCES

1. Loide, R.-K. J. Phys. A: Math. Gen., 1985, 18, 14, 2833—2847.
2. Ogievetsky, V., Sokatchev, E. Nucl. Phys. B, 1977, 124, 2—3, 309—316.
3. Суурварик П. Препринт Ф-32, Тарту, 1985.
4. Ferrara, S., Zumino, B. Nucl. Phys. B, 1978, 134, 2, 301—326.
5. Ogievetsky, V., Sokatchev, E. Preprint JINR E2-11528, Dubna, 1978.
6. Fradkin, E. S., Tseytlin, A. A. Phys. Rep., 1985, 119, 4—5, 233—362.

LINEARISEERITUD $N=1$ SUPERGRAVITATSIOONIST

Paul SUURVARIK

$N=1$ lineariseeritud supergravitatsiooni saamiseks on kasutatud superprojektorite formalismi. See formalism põhineb traditsioonilistel ja mittetraditsioonilistel superprojektoritel. On saadud $N=1$ lineariseeritud supergravitatsiooni modifitseeritud formuleering.

О ЛИНЕАРИЗОВАННОЙ $N=1$ СУПЕРГРАВИТАЦИИ

Паул СУУРВАРИК

Формализм суперпроекторов использован для получения $N=1$ линеаризованной супергравитации. Этот формализм базируется на традиционных и нетрадиционных суперпроекторах. Получена модифицированная формулировка для $N=1$ линеаризованной супергравитации.