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SIMULATION OF VORTEX RINGS INTERACTION BY THE METHOD OF LIQUID PARTICLES

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Abstract. The inviscid interaction of two identical vortex rings is studied numerically. At the initial moment, the two rings are inclined symmetrically to the horizontal plane for an angle β, and they are located sufficiently far apart, so that the effect of their initial mutual influence was reduced at a minimum. Particles are marked uniformly along the central vortex lines of rings and their motion is described according to the Lagrangian approach. Velocities of particles are calculated by using the modified Biot-Savart law. A five-point filtration procedure is used to prevent a calculation instability. When particles belonging to different filaments overlapped, the topology of filaments was redefined to continue the simulation. Positions of particles were calculated at each moment of their motion, which displayed the evolution of the vortex contours with time. A correlation between the initial angle β and the collision angle θ was found. For example, when the initial distance between vortex rings was equal to 4 of their radii, the smallest obtained value for the collision angle was 43^o, and it corresponded to the zero value of the initial angle. Even in the case of such a small initial angle two vortex rings would join into a single filament, and after some time this resulting filament would split back into two other vortex rings.

Key words: inviscid fluids, vortex flows, numerical calculations, Lagrangian code.

1. INTRODUCTION

It was experimentally observed [1–4] and numerically confirmed [5–7] that two vortex rings approaching one another along intersecting paths joined, at first, into a single vortex filament, that split back into two other rings after its certain evolution.

However, the secondary splitting did not always occur. The existence of a critical approach angle below which such a splitting has not been observed, was pointed out by Fohl and Turner [1]. On the other hand, a more recent experiment [2] gave a good example of the secondary splitting of the filament formed out of the rings of a smaller value of the approach angle than that of a critical one. A similar contradictory situation takes place in numerical simulations. While some calculations confirmed the conclusion by Fohl and Turner[5], others had no problems with small approach angles, but for angles more than 45^o, the secondary splitting was not observed [6]. Moreover, simulations of the same experimental situation by means of different methods of calculation led to distinct results in respect to the secondary splitting [7,8]. In addition, difficulties in the interpretation of numerical results are connected with the choice of the initial position of vortex rings, which can be arbitrary in simulations, but not in observations.

The main goal of this paper is to elucidate the influence of values of the initial angle and initial distance between vortex rings in their ability to join or split again. The numerical method is described in the next section of the paper. In the third section, results of various simulations are discussed, and possible explanation for the causes of incompatibility in previous papers are given in the conclusions.

2. NUMERICAL METHOD

Let $\vec{u}(\vec{x},t)$ be the velocity field and $\vec{\omega}(\vec{x},t) = \nabla \times \vec{u}$ be the vorticity field. The three-dimensional vorticity transport equation in an incompressible flow is the following [9]

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{u} \cdot \nabla)\vec{\omega} = (\vec{\omega} \cdot \nabla)\vec{u} + \nu \nabla^2 \vec{\omega} \, , \tag{1}$$

where ν is the kinematic viscosity, t is time. Taking into account the continuity equation

$$\nabla \cdot \vec{u} = 0 \, , \tag{2}$$

and the definition of vorticity

$$\vec{\omega}(\vec{x},t) = \nabla \times \vec{u} \, , \tag{3}$$

we obtain that velocity and vorticity fields are connected by the equation

$$\nabla^2 \vec{u} = -\nabla \times \vec{\omega} \, . \tag{4}$$

The last equation has a solution in the form of the Biot–Savart law [9]

$$\vec{u}(\vec{x}) = -\frac{1}{4\pi} \int \frac{(\vec{x}-\vec{x}') \times \vec{\omega}(\vec{x}')d\vec{x}'}{|\vec{x}-\vec{x}'|^3}. \tag{5}$$

It is well known from the theorems of Helmholtz and Kelvin that tubes of vorticity retain their identity, and move as material entities with constant circulation for a uniform-density inviscid fluid. Accordingly, we define the inviscid flow as the motion of the system of thin vortex elements with fixed circulation. For a single space curve C, the vorticity field has the representation in the thin-filament approximation [10]

$$\vec{\omega}(\vec{x}) = \Gamma \int_C \delta[\vec{x}-\vec{r}(s)] \frac{\partial \vec{r}}{\partial s} ds\,, \tag{6}$$

where $\vec{r}(s)$ is the space curve of the filament parameterized by the arc length s, and Γ is the filament circulation. The velocity field induced by this filament in an unbounded domain is given by [10]

$$\vec{u}(\vec{x}) = -\frac{\Gamma}{4\pi} \int_C \frac{[\vec{x}-\vec{r}(s)] \times \frac{\partial \vec{r}}{\partial s} ds}{|\vec{x}-\vec{r}|^3}. \tag{7}$$

This expression is a good approximation for the velocity field as long as the field point $\vec{x}$ does not approach any part of the curve. As $\vec{x}$ approaches $\vec{r}(s)$ on the space curve, $\vec{u}$ diverges as $1/|\vec{x}-\vec{r}(s)|$. The finiteness of the vortex core must be taken into account to avoid this singularity.

Several methods have been developed to treat the finite core effects in the Biot-Savart law [10,11]. We use the Rosenhead-Moore approximation, which leads to the following evolution equations for the sequence of N vortex elements with associated space curves $\vec{r_i}(s)$ [10]

$$\frac{d\vec{r_i}}{dt} = -\sum_j \frac{\Gamma}{4\pi} \frac{[\vec{r}_i(s,t)-\vec{r}_j(s',t)] \times \frac{\partial \vec{r}_j}{\partial s'} \Delta s'_j}{(|\vec{r}_i-\vec{r}_j|^2 + \alpha\sigma_i^2)^{3/2}}\,, \quad i,j = 1,\ldots,N, \tag{8}$$

where σ is a characteristic core radius and α is a constant. The core sizes σ_i may depend on time, and vary along the filament. The condition of conservation for the total volume of vorticity is used to model their variations

$$\frac{d}{dt}(\sigma_i^2 L_i) = 0\,, \tag{9}$$

where L_i is the instantaneous length of filament i. For any vortex element, its velocity vector (8) is induced both by its own vortex ring and by another one.

The forth order Runge-Kutta time integration scheme was used for numerical simulations. In addition, a five-point filtration procedure was applied by reconstruction of space curves

$$\vec{r}_n = \frac{-\vec{r}_{n-2} + 4\vec{r}_{n-1} + 10\vec{r}_n + 4\vec{r}_{n+1} - \vec{r}_{n+2}}{16} \tag{10}$$

to avoid calculation instabilities.

At an initial moment, two identical vortex rings with an initial radius R_0 were placed at a distance D_0 along a certain axis, say y axis, and inclined for an initial angle β, $0 \leq \beta \leq \pi/2$, to the horizontal plane xy. Vortex contours were parameterized by the relations

$$\begin{aligned} x_1 &= R_0 \cos\frac{2S}{R_0}, \\ y_1 &= \frac{D_0}{2} + R_0 \sin\frac{2S}{R_0}\cos\beta, \\ z_1 &= R_0 \sin\beta + R_0 \sin\frac{2S}{R_0}\sin\beta, \end{aligned} \tag{11}$$

$$\begin{aligned} x_2 &= R_0 \cos\frac{2S}{R_0}, \\ y_2 &= -\frac{D_0}{2} + R_0 \sin\frac{2S}{R_0}\cos\beta, \\ z_2 &= R_0 \sin\beta - R_0 \sin\frac{2S}{R_0}\sin\beta\,. \end{aligned} \tag{12}$$

Introducing dimensionless variables

$$r^* = \frac{r}{R_0},\ t^* = \frac{t\Gamma}{R_0^2}, \tag{13}$$

we obtain only two governing parameters in the formulated problem, namely the initial angle β and the initial relative distance between the rings D_0/R_0. Elucidating the influence of their variation on the interaction process of vortex rings is just the aim of the paper.

Initial values of σ_i/R_0 were taken as 0.1. The value of parameter α was chosen equalling to 0.22, which corresponds to a uniform vorticity distribution in the vortex core [12]. Discretization step L_i was equal to 20% of the initial radii.

3. NUMERICAL RESULTS

Calculations were proceeded in two stages. At first, the motion of vortex rings before their collision was simulated. The change in the topology of the vorticity lines in a reconnection process was modelled by an intervention procedure, i.e. excluding overlapped elements of the vortex rings and connecting their retained parts. The motion of a single vortex filament was calculated after reconnection.

Calculations were made for several initial inclinations of rings to the horizontal plane and relative distances between rings. A typical example of the results is represented in Fig.1. It corresponds to the value of the initial angle $\beta = 30^o$ and to the initial relative distance $D_0/R_0 = 2$. All the basic states of the evolution of vortex contours can be seen in this Figure. At first, two vortex rings driven by induction, approach each other. At a certain moment ($t^* = 0.35$ in this case), the rings touch one another and join by the reconnection of the vortex lines. After that, a single vortex filament begins to evolve due to its self-induction. Such an evolution is finished with the second reconnection (at $t^* = 0.95$ in this case). A clear pyramidal structure before the second reconnection is observed. There is a satisfactory agreement between numerically calculated and experimentally observed [2] forms of vortex contours.

Numerical experiments confirm the absence of the second reconnection for sufficiently small values of colliding angle θ. At the same time, they show that the value of colliding angle depends on both the initial angle β and the initial distance D_0/R_0. In fact, vortex rings change their angular position in respect to each other due to the induction. As an example, the relationship between the initial and colliding angles in the case, when $D_0/R_0 = 4$ is represented in Fig.2. It is seen that in this case the value of the colliding angle $\theta = 43^o$ corresponds to the zero initial angle, which means that the second reconnection will take place even for such a small value of the initial angle between the paths of vortex rings when they are placed sufficiently far from each other. The difference between initial and colliding angles decreases with the increase of the initial angle for the fixed initial distance.

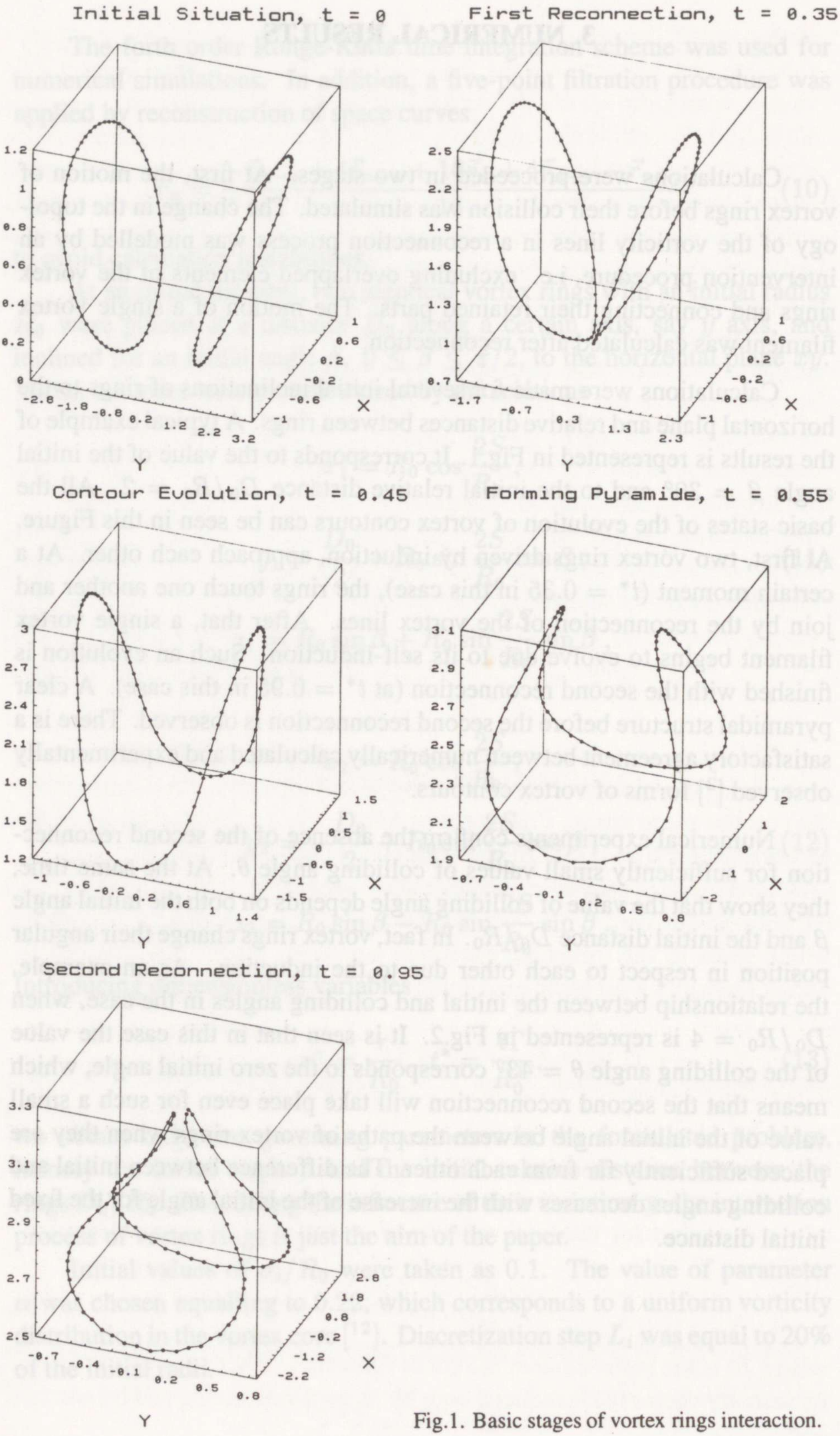

Fig.1. Basic stages of vortex rings interaction.

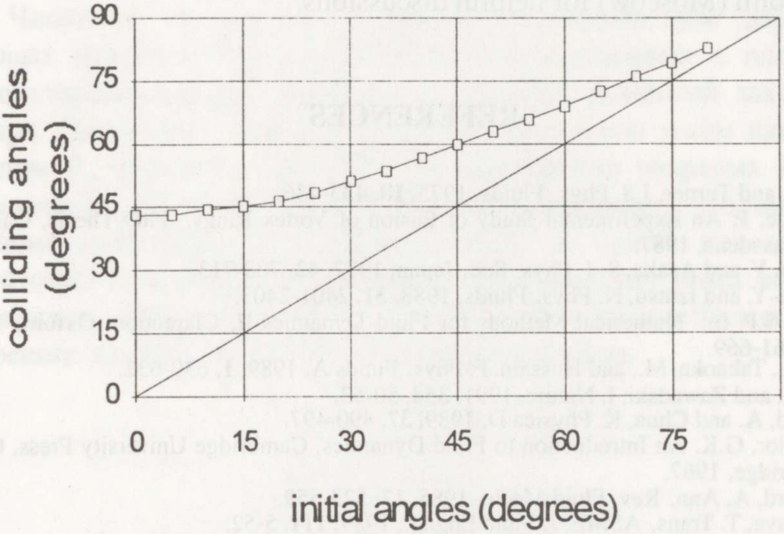

Fig.2. Colliding angle θ as a function of initial angle β in the case $D_0/R_0 = 4$.

4. CONCLUSION

The dependence of a colliding angle on the initial distance allows to give an explanation for the contradictions in previous investigations. In fact, if the initial distance is small, the difference between initial and colliding angles is also small. Consequently, the second reconnection is simply not possible in this case. At the same time, the second reconnection will take place for the same value of the initial angle, if, however, the initial distance between rings will be sufficiently large, because the colliding angle will then be also large.

Thus, the initial distance between vortex rings is a significant parameter determining essential features of their interaction process, such as the presence or absence of second reconnection. Moreover, the case of a very small initial distance used in numerical experiments [13] is unrealistic, because vortex rings cannot be formed suddenly at an arbitrary space. If the real initial distance were taken into account in [13], the result could have been different.

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KEERISRÕNGASTE KOOSTOIMEPROTSESSIDE NUMBRILINE MODELLEERIMINE VEDELATE OSAKESTE DÜNAAMIKA ABIL

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Kasutades Lagrange'i meetodit keerisrõngaste liikumise kirjeldamiseks ja modifitseeritud Biot'-Savart'i seadust märgistatud osakeste kiiruse määramiseks on välja selgitatud kahe identse horisontaaltasandi suhtes sümmeetriliselt kallutatud keerisrõngaste koostoime mehhanism ning leitud seaduspärasus keerisrõnga kaldenurkade vahel algasendis ja rõngaste kokkupuutel.

МОДЕЛИРОВАНИЕ ВЗАИМОДЕЙСТВИЯ ВИХРЕВЫХ КОЛЕЦ С ПОМОЩЬЮ ДИНАМИКИ ЖИДКИХ ЧАСТИЦ

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Численно исследовано невязкое взаимодействие двух одинаковых вихревых колец, симметрично наклоненных к плоскости. Использовался подход Лагранжа и модифицированный закон Био-Савара. Определена зависимость между начальным углом наклона β и углом θ, образующимся при соприкосновении вихревых колец, в зависимости от начального расстояния. Показано, что даже при нулевом начальном угле (кольца лежат в одной плоскости) и достаточно большом начальном расстоянии между ними возможно разделение образующегося в результате их взаимодействия одного вихревого контура на два новых вихревых кольца.