# BOUNDARY LAYER EFFECTS IN THE DUSTY GAS FLOW OVER A BLUNT BODY 

Yuri TSIRKUNOV, Natalia TARASOVA, and Aleksei VOLKOV

Балтийский государственный технический университет (Baltic State Technical University), 198005 Санкт-Петербург, I Красноармейская ул. 1, Россия (Russia)

Received July 13, 1994; accepted November 14, 1994


#### Abstract

Particle flow patterns over a body with variable surface temperature are studied by an example of a supersonic uniform dusty gas flow around a sphere. The particle concentration is accepted to be negligibly small. The mathematical formulation of the particle motion includes the description of the gas-flow field in the shock layer, taking into account the boundary layer on a sphere and considering additional components to the usual drag force in the interphase interaction inside boundary layer, namely the lift Saffman force, the thermophoretic force and the "wall effect" force. The analysis of fine particle flow patterns inside boundary layer near the stagnation point is carried out in the case of a "hot" surface. It was found that the boundary layer influenced weakly the global particle flow field.


Key words: two-phase flows, boundary layer, gas-particle interaction.

## 1. INTRODUCTION

In two-phase gas-particle aerodynamics one of the most interesting problems is the influence of the wall temperature on the particle motion and mass transfer. This problem is important in many applications: in heatexchange apparatus with a gas-particle mixture as a heat-transfer medium, in powder spraying by jets, etc.

The majority of investigators do not take into account the boundarylayer effects in the problems of dusty gas flows over bodies and obstacles. But for fine particles the boundary layer can play an important role in shaping particle trajectories near the wall and hence forming the particlephase flux to a body surface (see for example [ ${ }^{1}$ ]). Among mentioned effects the influence of the surface temperature on the motion and deposition of particles is of great interest for practical purposes. The aim of the present investigation is to make a detailed study of fine particle flow patterns over a blunt body particularly inside a boundary layer. A mass concentration of particles is presupposed to be low enough, so their influence on the carrier gas flow and particle-particle collisions are
negligible. Estimates for a particle concentration in the undisturbed flow when both mentioned assumptions are valid have been obtained in $\left[{ }^{2}\right]$. It should be noted that when a particle concentration in the undisturbed flow increases, particle-particle collisions become essential before particles begin to affect considerably the carrier gas flow. So, taking into account the inverse influence of the particle phase on the gas-phase flow without simultaneous consideration of particle-particle collisions is physically incorrect.

When the effect of particles on a gas flow is negligibly small the problem of the particle motion over a body is simplified considerably. In the present investigation carrier gas in the flow over a sphere is assumed to be viscous. A practically important case of high Reynolds numbers is considered. The constructing of a gas-flow field in this case is considered in Section 2.1.

The two-phase boundary-layer structure has been studied by many authors (see for example [ ${ }^{3-6}$ ]), but the present physical and mathematical setting of the problem differs from previous ones. The distinction refers to the simulation of the force acting on a particle inside boundary layer. Conditions of gas flow over a particle inside a boundary layer greatly differs from the ones in the region far from the body surface where gas flow is practically inviscid. In the inviscid flow the usual drag force is the dominant one and other components are negligible. Inside a boundary layer in addition to a drag force other components of the interphase force must be taken into account. This question is discussed in detail in Section 2.2.

In Section 3, numerical results characterizing the particle flow fields in the boundary layer near the stagnation point and in the whole shock layer of a sphere are presented.

## 2. MATHEMATICAL MODELLING

Let us consider a supersonic uniform dusty gas flow with the velocity $V_{\infty}$ over a sphere of radius $a$. The following assumptions for the analysis are used:
(i) the gas is viscous both in the flow over a sphere and in its interaction with the particles;
(ii) the gas is a continuum perfect gas with constant specific heats;
(iii) particles are solid spheres with a constant material density;
(iv) the mass concentration of particles is low and the effect of particles on the gas flow is neglected;
(v) the Brownian motion of particles is neglected;
(vi) particles do not interact with each other;
(vii) no phase change takes place.

Owing to the assumption (iv), the particle motion can be studied by sequential solving of two separate problems (1): the determination of the carrier gas-flow field and (2): the calculation of the particle trajectories in the gas-flow field.

### 2.1. Modelling of the gas flow

We consider the case when the Reynolds number $\operatorname{Re}_{\infty}\left(\operatorname{Re}_{\infty}=\right.$ $\rho_{\infty} V_{\infty} a / \mu_{\infty}$, where $\rho_{\infty}$ and $\mu_{\infty}$ are the density and the viscosity of the gas in the undisturbed flow, respectively) is high enough so that the shock layer can be divided into two areas: the outer area where gas viscosity and thermal conductivity in the flow over a sphere are of no importance, and the thin boundary layer.

In the outer area we use the tabulated numerical solution for inviscid flow [ ${ }^{7}$ ] when the vicinity of the stagnation streamline is considered.

For the analysis of the flow in the whole shock layer we solve the Euler equations by using the TVD scheme developed in $\left.{ }^{8}\right]$.

For solving of the laminar compressible boundary layer equations near the stagnation point we use semi-self-similar variables [ ${ }^{9}$ ] without using the classic Lees-Dorodnitsyn transformation which is inconvenient for the description of the carrier gas flow in the physical space. The continuous gas-flow field near the stagnation streamline in a shock layer is constructed by tailoring together inviscid and boundary layer profiles of $u$, $v, \rho$ and $T(u, v$ are the velocity components in curvilinear boundary-layer coordinates (xy), $\rho$ and $T$ are the density and the temperature of the gas, respectively).

When we study the flow in the whole shock layer, we solve the boundary layer equations by using a cubic-spline-finite-difference scheme. For constructing the continuous flow field in this case we use the composed technique of asymptotic expansions $\left[{ }^{10}\right]$.

It is obvious that the boundary layer flow depends on the temperature factor $T_{w} / T_{0}$ ( $T_{w}$ and $T_{0}$ are the body surface temperature and the adiabatic stagnation temperature in the critical streamline, respectively). The difference between gas-flow fields in the boundary layer for the different values of $T_{w} / T_{0}$ leads to the different pictures of fine particle streamlines.

### 2.2. Modelling of the particle motion

Consider the question of modelling of the force the carrier gas acts with on a single particle. We suppose that particles which impinge on the body surface are absorbed by it.

In the present investigation fine particles with a radius less than the critical one are of great interest, because the boundary layer on a sphere influences considerably just their trajectories. In the inviscid area of the shock layer the gas flow over such particles can be assumed to be nearly uniform at a distance equal to the particle velocity relaxation length. It allows to use the results for the particle drag force obtained in the unbounded uniform flow. Moreover, simple estimates show [ ${ }^{11}$ ] that in the dusty-gas flow where the density ratio $\rho_{p} / \rho$ is of the order $10^{3}-10^{4}$ the gas flow over a particle is quasi-statfonary. Here we use the drag coefficient $C_{D}$ given by Henderson [ ${ }^{12}$ ]. It takes into account inertial, compressibility and rarefaction effects as well as a temperature difference
between the gas and a particle. For fine particles a temperature difference is neglected. The Henderson formulae are very cumbersome and not presented here, but they are in good agreement with the experimental data over the wide range of the Mach number $M_{p}$ and the Reynolds number $\operatorname{Re}_{p}$ defined as

$$
M_{p}=\left|\vec{V}-\vec{V}_{p}\right| / c, \quad \operatorname{Re}_{p}=2 r_{p} \rho\left|\vec{V}-\vec{V}_{p}\right| / \mu
$$

Here $\left|\vec{V}-\vec{V}_{p}\right|$ is the particle velocity relative to the gas, $c$ the sound velocity, $r_{p}$ the particle radius. In the limit when $M_{p} \rightarrow 0$ and $\operatorname{Re}_{p} \ll 1$, the Henderson formulae reduce to the Stokes-Oseen drag law.

Inside the boundary layer on a body surface the action of the carrier gas on a particle has a few specific features. Let us consider them in detail.

Due to the shear flow in the boundary layer the lift force and the particle rotation appear. Saffman received the formula for the lift force, acting on a freely rotational particle in the uniform slow shear flow $\left[{ }^{13}\right]$. We use it in the form

$$
\begin{equation*}
f_{s}=6.46 \mu r_{p}^{2}\left(u-u_{p}\right)\left(\frac{\rho \partial u}{\mu \partial y}\right)^{1 / 2} . \tag{1}
\end{equation*}
$$

This formula is valid when the particle Reynolds number $\operatorname{Re}_{p}$ and the shear parameter $æ$ satisfy the inequalities

$$
\operatorname{Re}_{p} \ll\left(æ \operatorname{Re}_{p}\right)^{1 / 2} \ll 1, \quad æ=(\partial u / \partial y) r_{p} / u .
$$

It should be noted that although the Saffman's result requires the particle Reynolds number to be very small it cannot be received from the Stokes equations for a creeping flow.

The second addition to the usual drag force that can be significant inside a boundary layer near the surface is connected with the so-called "wall effect" due to the hydrodynamic interaction between a particle and a wall. In the vicinity of the wall the particle drag force is considerably greater than the one given by the Stokes law. This problem has been analyzed for a Stokesean particle by many investigators. We use the approximate asymptotic solution from the paper $\left[{ }^{18}\right]$ and the result of Goldman et al. $\left[{ }^{15}\right]$.

Due to the linearity of the Stokes equations and boundary conditions we may obtain the solution for the arbitrary motion of a spherical particle in the Couette flow near a rigid wall by superposing of the cited results. In this case due to the "wall effect" $x$ - and $y$-components of the additional force take the form

$$
\begin{gather*}
f_{w x}=\frac{9}{16} \frac{r_{p}}{y_{p}}\left(1+\frac{9}{16} \frac{r_{p}}{y_{p}}\right) f_{\mu x},  \tag{2}\\
f_{w y}=\frac{9}{8} \frac{r_{p}}{y_{p}}\left(1+\frac{9}{8} \frac{r_{p}}{y_{p}}\right) f_{\mu y}, \tag{3}
\end{gather*}
$$

where $y_{p}$ is the distance from the wall to the particle centre, and $f_{\mu x}$ and $f_{\mu y}$ are the $x$ - and $y$-components of the usual Stokes drag force which can be written in the form

$$
\begin{gather*}
f_{\mu x}=\varphi\left(u-u_{p}\right)  \tag{4}\\
f_{\mu y}=\varphi\left(v-v_{p}\right)  \tag{5}\\
\varphi=\frac{1}{2} C_{D} \pi r_{p}^{2} \rho\left(\left(u-u_{p}\right)^{2}+\left(v-v_{p}\right)^{2}\right)^{1 / 2} \tag{6}
\end{gather*}
$$

Up to the terms written out in (2) and (3) the "wall effect" force is independent of the angular velocity of a particle. On the contrary, terms of the higher order depend on the particle angular velocity and also on the curvature of a $u$-velocity profile in the boundary layer. Hence the further refinement of the expressions (2) and (3) is of no importance if we want to describe the dusty-laden boundary-layer flow on a curved shape body.

The last addition to the usual drag force considered in the present study is the thermophoretic force. The literature on thermophoresis is rather extensive but, as it has been mentioned in $\left[{ }^{16}\right]$, there is a number of conflicting results both theoretical and experimental. The cited paper contains a critical review of the problem as well as arguments in support of the theory of Brock [ ${ }^{17}$ ] with an improved value for the thermal slip coefficient. The modified Brock's formula for the thermophoretic force agrees within $20 \%$ or less with the majority of the available data over the entire range of the particle Knudsen number $0<K n_{p}<\infty\left[{ }^{16}\right]\left(K n_{p}=l / r_{p}\right.$, $\underline{l}=2 \mu / \rho \bar{c}$ is the mean free path of molecules in gas with $\bar{c}=(8 R T / \pi)^{1 / 2}$ the mean molecular speed and $R$ the specific gas constant). This formula is used in the present study. It takes the form

$$
\begin{equation*}
f_{t}=\frac{-12 \pi \mu^{2} r_{p}}{\rho T} \frac{C_{s}\left(\lambda / \lambda_{p}+C_{t} l / r_{p}\right)}{\left(1+3 C_{m} l / r_{p}\right)\left(1+2 \lambda / \lambda_{p}+2 C_{t} l / r_{p}\right)} \frac{\partial T}{\partial y} \tag{7}
\end{equation*}
$$

in which the constants have the values $C_{s}=1.17, C_{t}=2.18, C_{m}=1.14$. Here $\lambda$ and $\lambda_{p}$ are the thermal conductivities of the gas and particle, respectively.

From the above discussion the Newton's equation in (xy) coordinates for the particle motion inside a boundary layer may be written in the following form

$$
\begin{gather*}
m\left(\frac{d u_{p}}{d t}+\frac{u_{p} v_{p}}{a+y_{p}}\right)=f_{\mu x}+f_{w x}  \tag{8}\\
m\left(\frac{d v_{p}}{d t}+\frac{u_{p}^{2}}{a+y_{p}}\right)=f_{\mu y}+f_{w y}+f_{s}+f_{t} . \tag{9}
\end{gather*}
$$

Here $m=\frac{4}{3} \rho_{p} \pi r_{p}^{3}$ is the mass of a particle. Components $f_{w x}, f_{w y}, f_{s}$ and $f_{t}$ must be omitted outside the boundary layer.

Adding to (1)-(9) the kinematic relations

$$
\begin{gather*}
\frac{d x_{p}}{d t}=\frac{a u_{p}}{a+y_{p}}  \tag{10}\\
\frac{d y_{p}}{d t}=v_{p} \tag{11}
\end{gather*}
$$

we receive a closed set of equations which describes the particle motion.
The particle velocity in the undisturbed flow is assumed to be equal to that of the gas and the initial conditions for the set (1)-(11) are accepted in the following form

$$
\begin{gathered}
t=0: \quad y_{p}=y_{s h}, \quad x_{p}=a \arcsin \left(z_{p \infty} /\left(a+y_{s h}\right)\right), \\
u_{p}=V_{\infty} \sin \left(x_{p} / a\right), \quad v_{p}=-V_{\infty} \cos \left(x_{p} / a\right) .
\end{gathered}
$$

Here $z_{p_{\infty}}$ is the distance from the particle centre to the symmetry axis in the undisturbed flow, $y_{s h}$ the bow shock coordinate corresponding to $z_{p \infty}$.

One additional comment is necessary. The additive form of the expression for the force acting on a particle is the well-known tradition in simulating of two-phase gas-particles flows. But this approach is not so obvious as it seems at first glance. It is based on a hypothesis that the total force may be represented as a simple sum of the components caused by different phenomena. Unfortunately this additive form is often physically incorrect. The additive form of the force expression is valid only when the governing equations and boundary conditions in the problem of the gas flow over a particle are linear. Such situation is realized when the flow is either potential or creeping. In general the carrier gas flow over a particle is neither potential nor creeping. Because of this, the additive form of the expression for the total force must be justified in every concrete gasparticle flow. In the present study very small particles are considered, because it is just they that are of great interest from the standpoint of boundary-layer effects. One might expect the carrier gas flow over such particles to be creeping or nearly creeping and therefore the additive form for the total force expression to be correct. It should be pointed out that in that case formulae (1), (2) and (3) for $f_{s}, f_{w x}$ and $f_{w y}$ are valid.

## 3. NUMERICAL RESULTS

Computational investigations have been carried out in dimensionless variables which were obtained by dividing all linear sizes to the radius of a sphere $a$, velocity components to $V_{\infty}$, physical density of each phase to $\rho_{\infty}$, temperature to $V_{\infty}^{2} /\left(\gamma c_{v}\right)$, viscosity to $\mu_{\infty}$. The special numerical



Fig. 1. Particle streamlines in the boundary layer near the stagnation point for particles of different size, $T_{w} / T_{0}=2.0 . a-f$ correspond to $\left(r_{p} / a\right) \times 10^{6}=4.0,4.21,4.37,4.39,4.41$, 4.42 .
method has been used to solve the set of ordinary differential equations from Section 2.2 which became stiff for very small particles. In calculations, there have been taken the following values of parameters: $M_{\infty}=2, \operatorname{Re}_{\infty}=10^{7}$, the Prandtl number $\operatorname{Pr}=0.75$, the specific heat ratio $\gamma \stackrel{\infty}{=} 1.4, T_{\infty} \stackrel{\infty}{=} 273 \mathrm{~K}, \mu_{\infty}=1.71 \times 10^{-5} \mathrm{~N} \cdot \mathrm{~s} / \mathrm{m}^{2}, R=286.7 \mathrm{~J} /(\mathrm{kg} \cdot \mathrm{K})$, $\lambda_{p}=2 \mathrm{~W} /(\mathrm{m} \cdot \mathrm{K}), \rho_{p}=2300 \mathrm{~kg} / \mathrm{m}^{3}, a=1 \mathrm{~m}$. Parameters $T_{w} / T_{0}$ and $r_{p}$ were variable.

The typical particle flow patterns for $T_{w} / T_{0}>1$ are shown in Fig. 1. Curves $1-15$ correspond to $\left(z_{p \infty} / a\right) \times 10^{3}=0.2,1,2,3,4,5,6,7$, $8,9,10,11,12,13,14$. The particle size has been changed in the vicinity of the critical one. We see that the pattern of particle streamlines changes very considerably when the particle radius varies from $r_{p} / a=4.0 \times 10^{-6}$ to $4.42 \times 10^{-6}$. In the cases $(a)-(c)$ there is a continuous particle-free region near the surface. But cases (a) and (c) differ from each other qualitatively. In the case (c) particle streamlines intersect one another in spite of the equal size of the particles. When passing from the case ( $d$ ) to ( $e$ ), the particle-free region in the vicinity of the stagnation gas streamline decreases and disappears in the case $(f)$. In all cases except $(f)$, there is a limiting line with a high concentration of particles. It can be assumed that
in its neighbourhood the model of noninteracting particles becomes incorrect. This question requires to be studied more closely.

Figs. 2 and 3 illustrate that the particle Knudsen number on the trajectories in the typical cases shown in Fig. $1 a, c$ is less than 0.06 , and therefore particles move in a carrier gas as in the $a$ continuum medium. Moreover, we can conclude that gas is incompressible because of $M_{p} \leq 0.01$ and the flow over a particle is nearly creeping because of $R{ }^{p}{ }_{p} \leq 1$. The last result indicates that the accepted model of the force interphase interaction inside the boundary layer is justified.


Fig. 2. Variation of the particle Knudsen number, Mach number and Reynolds number along the particle streamlines shown in Fig. $1 a$.


Fig. 3. Variation of the particle Knudsen number, Mach number and Reynolds number along the particle streamlines shown in Fig. 1c.

It is interesting to compare the components of the total force acting on a particle. As indicated by Fig. 4, all the components taken into account are important.

Fig. 5. shows particle streamlines in the whole shock layer of a sphere. The streamlines are plotted for $T_{w} / T_{0}=0.4$. But calculations carried out for $T_{w} / T_{0}=5.0$ have given practically the same pictures of streamlines. It means that the boundary-layer effects are negligible from the standpoint of the global particle flow pattern.



Fig. 4. Comparison of $y$-components of the force acting on a particle inside boundary layer. $T_{w} / T_{0}=2, r_{p} / a=4.37 \times 10^{-6}, z_{p \infty} / a=0.002$.

## 4. CONCLUSIVE REMARKS

In this research it was assumed that particles did not interact with each other. But in crossing particle streamlines the appearance of a limiting line with a high particle concentration and the polydispersity of real particle admixture can lead to the necessity to consider more complex models for a particle phase. This question is one of the most important for the future investigation.

This work was supported in part by the Russian Federation Foundation for Fundamental Research (Grant N 94-01-01338) and in part by RF Higher School State Committee (Grant N 94-4.100-81).


Fig. 5. Streamlines of particles in the shock layer of a sphere, $T_{w} / T_{0}=0.4 . a-r_{p} / a=10^{-5}$, $b-r_{p} / a=10^{-6}$.

## REFERENCES

1. Циркунов Ю. М., Тарасова Н. В. Моделирование в механике, 1990, 2, 141-148.
2. Циркунов Ю. М. Моделирование в механике, 1993, 2, 151-194.
3. Спокойный Ф. Е., Горбис 3. Р. Теплофизика высоких температур, 1981, 1, 182-199.
4. Fernandez de la Mora, J. Acta Mechanika, 1982, 36, 261-265.
5. Осипцов А. Н., Шапиро Е. Г. Изв. АН СССР. Механика жидкости и газа, 1986, 5, 55-62.
6. Агранат В. М., Милованова А. В. Изв. АН СССР. Механика жидкости и газа, 1990, 6, 169-172.
7. Любимов Ф. Н., Русанов В. В. Обтекание затупленных тел газовым потоком. Часть 2. Наука, Москва, 1970.
8. Йи Х. С., Хартен А. Аэрокосмическая техника, 1987, 11, 11-21.
9. Циркунов Ю. М., Тарасова Н. В. Течение газов в каналах и струях. Изд-во СПбГУ, Санкт-Петербург, 1993.
10. Найфэ А. Х. Методы возмущений. Мир, Москва, 1976.
11. Нигматулин Р. И. Основы механики гетерогенных сред. Наука, Москва, 1978.
12. Henderson, C. B. AIAA J., 1976, 14, 259.

# PIIRIKIHI MÕJU KERA UHTMISELE DISPERSSE VOOLUSEGA 

Juri TSIRKUNOV, Natalia TARASSOVA, Aleksei VOLKOV

Kasutades arvmodelleerimist on uuritud osakeste vooluse parameetrite sõltuvust kera pinnatemperatuurist kera uhtmisel ühtlase kiirusväljaga ülehelikiirusega dispersse voolusega. Osakeste kontsentratsioon on väike ja seda ei arvestata osakeste liikumise kirjeldamisel rõhukihis kera läheduses. Arvestatud on takistusjõu, Saffmani jõu, termoforeesi jõu ja "seinaefekti" mõju. On täheldatud piirikihi nõrka toimet osakeste vooluse väljale.

## ВЛИЯНИЕ ПОГРАНИЧНОГО СЛОЯ НА ОБТЕКАНИЕ ЗАТУПЛЕННОГО ТЕЛА ЗАПЫЛЕННЫМ ГАЗОВЫМ ПОТОКОМ

Юрий ЦИРКУНОВ, Наталия ТАРАСОВА, Алексей ВОЛКОВ

Исследуются характеристики потока частиц в зависимости от температуры поверхности сферы при обтекании ее сверхзвуковым однородным запыленным газовым потоком. Концентрация частиц предполагается пренебрежительно малой. Движение частиц не учитывается в ударном слое вблизи сферы. В пограничном слое сферы наряду с силой сопротивления учитываются сила Саффмена, сила термофореза и "эффект стенки". Обнаружена слабая зависимость поля потока частиц от пограничного слоя.

