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BOUNDARY CONDITIONS OF THE MASS, MOMENTUM AND ENERGY TRANSFER EQUATIONS IN A TURBULENT TWO-PHASE FLOW

Alexander SHRAIBER[a] and Vladimir NAUMOV[b]

[a] Інститут проблем енергозбереження Національної Академії наук України (Institute of Energy Saving Problems, Ukrainian Academy of Sciences), 254070 Київ, вумеця Покровська 11, Украина (Ukraine)
[b] Калининградский технический институт (Kaliningrad Technical Institute), 236000 Калининград, Советский проспект 1, Россия (Russia)

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Abstract. A consistent and correct method of the boundary conditions for transport equations of mass, momentum and energy of particles in two-phase turbulent flows is developed. Two ensembles of particles moving to and from the wall are considered. The different methods of averaging, approximation of particle distribution along the lateral velocities and properties of interacting surfaces are under analysis.

Key words: dispersed phase, boundary condition, averaging procedure.

1. INTRODUCTION

For integration of two-dimensional Euler equations of confined particle-laden gas flows, it is necessary to have two boundary conditions (BCs) on the transverse coordinate y. The formulation of the first BC is not difficult. For example, on the axis of an axisymmetric channel we have $\partial\bar{\varphi}/\partial y = 0$, where $\varphi = \rho, \bar{\vec{\mathbf{U}}}, \bar{T}, \ldots$; where ρ is the distributed density of the particles, $\vec{\mathbf{U}}, T$ are their velocity and temperature, respectively, and the bar means averaging. For a boundary layer flow, the particles' parameters on its external boundary are usually known (e.g., from calculations of the inviscid flow core).

At the same time, the formulation of the second BC is rather difficult. Johansen [1] supposes that the mean longitudinal slip velocity on the axis of a vertical flow is equal to particles' terminal velocity (it would be true for the case of absence of transverse particles' mixing). Moreover, such a BC does not take into account the peculiarities of particle-to-wall interaction. Therefore it is advisable to formulate the second BC just on the wall and in order to do so, some authors use non-substantiated hypotheses to determine the particles' mass flow rate onto the surface, momentum and energy losses due to the wall. In [2, 3], similarly to the carrying gas, it is supposed that $\bar{u}_w = 0$ (u is the axial velocity, index w refers to particles' parameters near the wall), in [4, 5] a formal, but physically non-justifiable analogy with a rarefied gas flow is used. In [6], the BC is obtained from the averaged equations of gas and particles' axial motion

$$\bar{\rho}_g \left(\bar{u}_g \frac{\partial \bar{u}_g}{\partial x} + \bar{v}_g \frac{\partial \bar{u}_g}{\partial y} \right) = (\alpha - 1) \frac{\partial \bar{p}}{\partial x} - \bar{F} +$$

$$+ \frac{1}{y} \frac{\partial}{\partial y} \left[y \, (1 - \bar{\alpha}) \, (\mu_g + \mu_{gt}) \frac{\partial \bar{u}_g}{\partial y} \right]; \quad (1)$$

$$\rho \left(\bar{u}_g \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} \right) = -\bar{\alpha} \frac{\partial \bar{p}}{\partial x} + \bar{F} + \frac{1}{y} \frac{\partial}{\partial y} \left[y \bar{\alpha} \, (\mu + \mu_t) \frac{\partial \bar{u}}{\partial y} \right] + \bar{G}_x. \quad (2)$$

Here v is the transverse velocity, p is the pressure, α is the particles' volume concentration, μ, μ_t are the laminar and turbulent dynamic viscosities, respectively, F is the force of interphase interaction, G is the external force, symbols with an index g refer to the gas, and without an index – to the particles. Further on, the authors [6] integrate the sum of Eqs. (1) and (2) over the channel cross-section; for a stabilized flow $(\partial(\ )/\partial x = 0)$, provided that $\bar{\alpha} = \text{const}$, $\mu_t = 0$ and $v = 0$ near the wall, they obtain the BC

$$\left. \frac{\partial \bar{u}}{\partial y} \right|_w = \frac{1}{\mu \alpha} \left[\frac{R}{2} \left(\frac{\partial \bar{p}}{\partial x} - \bar{G}_x \right) - (1 - \bar{\alpha}) \, \mu_g \frac{\partial \bar{u}_g}{\partial y} \right]_w, \quad (3)$$

where R is the channel radius. In addition to accepted hypotheses (see below), Eq. (3) has an important demerit: here the dynamics of particle-to-wall impact and their physical properties are not taken into account.

For the conservation equation of the particles' mass, the simplest BC either $\bar{\rho}_w = 0$ (perfectly absorbing wall) or $(\partial \bar{\rho} / \partial y)_w = 0$ (perfectly reflecting surface) is usually used. In [7] one can find an analysis of applicability of these assumptions. Obviously, to construct strict and well-grounded BC, it is necessary to study the particle-to-wall interaction in detail. As far as we know, there are only two groups of papers in this field to be considered below.

2. THE SOLUTION BY KONDRATYEV AND SHOR

Kondratyev and Shor [8] consider two ensembles of spherical particles: one of them moving to the wall, and the other – from it (their parameters we shall mark by indices 1 and 2). The translational and angular (ω) velocities of particles 1 and 2 are subordinate to the equations of a single impact without sliding [9]:

$$u_2 = (5+2k_\tau)\,u_1/7 + (1-k_\tau)\,\omega_1 d/7; \quad v_2 = -k_n v_1;$$
$$\omega_2 = (5k_\tau+2)\,\omega_1/7 + 10\,(1-k_\tau)\,u_1/(7d)\,, \tag{4}$$

where d is the particle diameter, k_n and k_τ are the restitution coefficients for normal and tangential velocity components at collision ($0 \le k_n \le 1$; $-1 \le k_\tau \le 1$), respectively; in Eq. (4) it is supposed that the only non-zero component of the vector $\vec{\omega}$ is perpendicular to the x-y plane. The particles' distribution for every ensemble is approximated by a δ- function

$$f_i\,(u, v, \omega) = \delta\,(u-u_i)\,\delta\,(v-v_i)\,\delta\,(\omega-\omega_i) \tag{5}$$

and it is supposed that the BCs do not depend upon the form of approximation (see below). Since for a reflecting wall the normal components of the mass flow rates of particles 1 and 2 have equal magnitudes ($J_{n1} = -J_{n2}$), the distributed densities are

$$\rho_1 = \bar{\rho}_w k_n/(1+k_n)\,; \quad \rho_2 = \bar{\rho}_w/(1+k_n)\,. \tag{6}$$

Then with the aim of a formula

$$\bar{\varphi}_w = (\rho_1+\rho_2)^{-1}\,(\varphi_1\rho_1 + \varphi_2\rho_2)\,, \tag{7}$$

the mean values of the particles' parameters near the wall are calculated

$$\bar{u}_w = M_u u_1; \quad \bar{v}_w = M_v k_n v_1; \quad \bar{\omega}_w = M_\omega u_1/d;$$

$$M_u = \frac{1}{7\,(1+k_n)} \times \left[\,7k_n + 5 + 2k_\tau - \frac{10\,(1-k_\tau)^2 k_\omega}{7+k_\omega\,(5k_\tau+2)}\right]; \quad M_v = 0;$$
$$M_\omega = \frac{10\,(1-k_\tau)\,(1-k_n k_\omega)}{(1+k_n)\,[\,7+k_\omega\,(5k_\tau+2)\,]}; \quad k_\omega = -\frac{\omega_1}{\omega_2}. \tag{8}$$

The authors [8] suppose that k_ω (as k_n or k_τ) is a constant, but from Eq. (4) we can see that k_ω depends upon u_1/d and ω_1. Similarly to (8), the mean square of the particles' transverse velocity fluctuations and the Reynolds stresses in the disperse phase are found

$$\left(\overline{v'^2}\right)_w = Nk_n v_1^2; \quad \left(\overline{u'v'}\right)_w = R\left(\overline{v'^2}\right)_w^{1/2} \bar{u}_w; \quad N = 1;$$

$$R = \frac{2\sqrt{k_n}\,(1-k_\tau)\,(1+k_\omega)}{7\,(k_n + k_\tau k_\omega) + 5\,(1 + k_n k_\tau k_\omega) + 2\,(k_\tau + k_n k_\omega)}. \tag{9}$$

Finally, the BC for the equation of the axial motion of particles has the form

$$R\left(\overline{v'^2}\right)_w^{1/2} \bar{u}_w = -\nu_t\,(\partial \bar{u}/\partial y)_w, \tag{10}$$

where ν is the kinematic viscosity. From Eq. (7) it is seen that the particles' parameters are averaged here similarly to the known Favre's method in the theory of single-phase turbulent flows, while the transfer equations are derived in [8], using conventional time averaging. From our view-point, this is a wall (very interesting for practice) that is not considered in [8], and for the particles temperature the simplest BC $(\partial \bar{T}/\partial y)_w = 0$ is used.

3. THE SOLUTION BY DEREVICH ET AL.

An attempt to derive more strictly the BC for the equation of the particles' mass transfer is undertaken in [7]. To describe the interaction with the wall, a coefficient χ is introduced ($\chi = 0$ for a perfectly absorbing and $\chi = 1$ for a perfectly reflecting surface). For a kinetic layer near the wall, a one-dimensional equation for the particles' distribution function f by transverse velocities ($d\rho = f dv$) is written

$$K(f) = \tau v \frac{\partial f}{\partial y} + (v_g + \tau \bar{G}_y)\frac{\partial f}{\partial v} = \frac{D}{\tau}\frac{\partial^2 f}{\partial v^2} + \frac{\partial v f}{\partial v} \equiv L(f), \tag{11}$$

where τ is the relaxation time, D is the particles' turbulent diffusivity. Similarly to the known Chapman–Enskog method, an approximate solution of Eq. (11) is constructed as a series $f = \sum_{n=0}^{\infty} f^{(n)}$, where only two members are taken into account. In the zero approximation, the left-hand-side of Eq. (11) is omitted, whereupon the solution obtains the Maxwellian form

$$f^{(0)} = \left(\frac{\tau}{2\pi D}\right)^{1/2} \bar{\rho} \exp\left(-\frac{\tau v^2}{2D}\right). \tag{12}$$

The next term $f^{(1)}$ is determined from the equation $L(f^{(1)}) = K(f^{(0)})$:

$$f^{(1)} = f^{(0)} v (B + C v^2), \tag{13}$$

where B and C depend upon ρ, D, τ, $\overline{v}_g$ and $\overline{G}_y$. The character of the function f is illustrated in Fig. 1,*a*, but the solution does not agree with the second formula (4). Moreover, for low χ the flow rate of the reflected particles

$$J_{n2} = -\chi J_{n1} - \int_{-\infty}^{0} v f dv \tag{14}$$

must be small enough by its modulus (or $J_{n2} = 0$ if $\chi = 0$), so the f function must be negative for some v, but such f values have no physical sense. (In consequent paper [10] more realistic, binormal distribution at $k_n = 1$ is used). From (14) we have

$$J_{nw} = J_{n1} - J_{n2} = \frac{1-\chi}{1+\chi} \left(\frac{2D}{\pi \tau} \right)^{1/2} \rho_w. \tag{15}$$

It should be noted that in the BC (15) the influence of k_n is not taken into account, but physically it is clear that this coefficient must have an effect on the BC.

The case of a perfectly reflecting wall at arbitrary k_n values is considered in [11]. The analysis similar to Eqs. (11)–(14) gives the BC in the form of (15), where χ is replaced by k_n. It should be noted that there seems to be a mistake in these considerations: the authors [11] obtain the relation $J_{n2} = -k_n J_{n1}$, but at proper reflection, it must obviously be

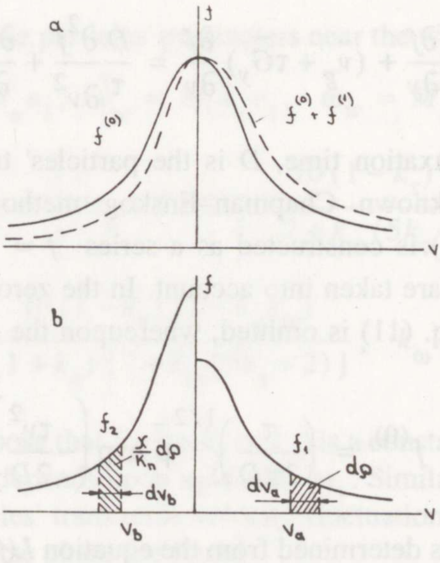

Fig. 1. The character of the particles' distribution by transverse velocities.

$J_{n2} = -J_{n1}$. In this paper, the BC for the equation of axial motion of non-rotating particles is also obtained; this BC can be written in the form (10), where R is determined by formula

$$R = \left(\frac{2}{\pi}\right)^{1/2} \frac{1 - k_n(5 + 2k_\tau)/7}{1 + k_n(5 + 2k_\tau)/7}. \tag{16}$$

We shall return to the analysis of Eq. (16) later. In [12] the BC for the equation of the particles' heat transfer (similar to (16)) is built. It should be emphasized that all considerations (11)–(16) are carried out in the frame of an averaging procedure.

4. NON-ROTATING PARTICLES. FAVRE'S AVERAGING

From the results described above it is clear that all the known BCs for the equations of particles' transfer have some demerits. So it is advisable to return to this question and to formulate the BCs for various cases which can be met with in practice. From our viewpoint, the approach [8] better describes the real physical picture of the particle-to-wall interaction than other variants. We shall use a method similar to [8] but for different averaging procedures, approximations of the $f(v)$ function, properties of interacting surfaces, and arbitrary χ values. For simplification, the particles' rotation is not taken into account. We ascribe one magnitude of the axial velocity and the temperature (u_1, u_2, T_1, T_2) to all particles of each ensemble. The change of particle temperature during collision is described by the formula

$$T_2 = T_1 + k_q(T^0 - T_1), \tag{17}$$

where T^0 is the wall temperature, $0 \le k_q \le 1$.

Let us assume that the particles distribution by transverse velocities is similar to (5), i.e.

$$f_i(v) = \rho_i\, \delta(v - v_i), \quad (i = 1, 2). \tag{18}$$

From (14) it follows that the distributed densities of two ensembles are

$$\rho_1 = \rho_w k_n / (\chi + k_n); \quad \rho_2 = \rho_w \chi / (\chi + k_n). \tag{19}$$

The calculation of the mean values of the particles' parameters near the wall by formula (7) with the account of Eqs. (4) (where $\omega_1 = 0$) and (17) gives

$$M_u = \frac{7k_n + \chi(5 + 2k_\tau)}{7(\chi + k_n)}; \quad M_v = \frac{1 - \chi}{\chi + k_n}; \tag{20}$$

$$\bar{T}_w = \frac{[k_n + \chi(1-k_q)]T_1 + \chi k_q T^0}{\chi + k_n}.$$

Here the meaning of M_u, M_v is the same as in (8). Then we find the correlations of the particles' fluctuation parameters:

$$N = \frac{\chi(1+k_n)^2}{(\chi+k_n)^2}; \quad R = \frac{2\sqrt{\chi k_n}(1-k_\tau)}{7k_n + \chi(5+2k_\tau)}; \tag{21}$$

$$\overline{v'T'} = S\left(\overline{v'^2}\right)_w^{1/2}(\bar{T}_w - T^0); \quad S = k_q\sqrt{\chi k_n}\,[k_n + \chi(1-k_q)]^{-1},$$

where N, R are determined like (9). Obviously, the BC for the equation of motion has the form (10) with the R value from (21). For mass and energy transfer we have

$$J_{nw} = Q\left(\overline{v'^2}\right)_w^{1/2}\bar{\rho}_w; \quad Q = \sqrt{\frac{k_n}{\chi}}\frac{1-\chi}{1+k_n}$$

$$S\left(\overline{v'^2}\right)_w^{1/2}(\bar{T}_w - T^0) = -a_t\left(\frac{\partial \bar{T}}{\partial y}\right)_w, \tag{22}$$

where a is the thermal diffusivity. According to (22) $(\partial T/\partial y)_w = 0$ (as in [8]) only if $k_q = 0$. The Q function substantially depends upon k_n, i.e. upon the conditions of the particles' transverse motion (compare with (15)). If $\chi = 1$ Eqs. (19)–(21) are reduced to (6), (8), (9), where $k_\omega = 0$.

Now let us assume (see Fig. 1, *b*)

$$f_1(v) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}_w k_n}{v_0(\chi+k_n)}\exp\left(-\frac{v^2}{2v_0^2}\right);$$

$$f_2(v) = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}_w \chi}{v_0 k_n(\chi+k_n)}\exp\left(-\frac{v^2}{2k_n^2 v_0^2}\right) \tag{23}$$

(at $k_n = 1$ approximation (23) coincides with [10]). It is clear that (23) describes in a better way the particle-to-wall impact than (12) and (13) do – some particles from the first ensemble with the velocities v_a, dv_a after collision transform into group v_b, dv_b ($v_a > 0$, $v_b = -k_n v_a$), therefore the f

function must have a discontinuity at $v = 0$. The mean transverse velocity of the particles 1 is $v_1 = \rho_1^{-1}\int_0^\infty f_1 v dv = v_0\sqrt{2/\pi}$. The mean values of the particles' parameters in this case are

$$\bar{\varphi}_w = \bar{\rho}_w^{-1}\left(\int_0^\infty f_1\varphi dv + \int_{-\infty}^0 f_2\varphi dv\right). \tag{24}$$

As a result we obtain

$$Q = \sqrt{\frac{2k_n}{\pi N^0}}\frac{1-\chi}{\chi+k_n}; \quad N^0 \equiv \left(\overline{v'^2}\right)_w / (k_n v_0^2) =$$

$$= \frac{1+\chi k_n}{\chi+k_n} - \frac{2k_n(1-\chi)^2}{\pi(\chi+k_n)^2}; \quad R = \frac{2}{7}\sqrt{\frac{2k_n}{\pi N^0}}\frac{\chi(1+k_n)(1-k_\tau)}{M_u(\chi+k_n)^2}; \tag{25}$$

$$S = \sqrt{\frac{2k_n}{\pi N^0}}\frac{\chi k_q(1+k_n)}{(\chi+k_n)[k_n+\chi(1-k_q)]}$$

(M_u, Q, R, S are the same, as in (9), (20), (22)).

For comparison's sake, some other approximations of the f function were considered. For uniform distribution

$$f_1(v) = \begin{cases} \bar{\rho}_w k_n / [v_0(\chi+k_n)], & 0 \le v \le v_0, \\ 0, & v > v_0; \end{cases}$$

$$f_2(v) = \begin{cases} 0, & v < -v_0 k_n, \\ \bar{\rho}_w \chi / [v_0 k_n(\chi+k_n)], & -v_0 k_n \le v < 0 \end{cases} \tag{26}$$

we have

$$Q = \sqrt{\frac{k_n}{N^0}}\frac{1-\chi}{2(\chi+k_n)}; \quad N^0 = \frac{4\chi(1+k_n)^2 + k_n(1-\chi)^2}{12(\chi+k_n)^2};$$

$$R = \frac{\chi\sqrt{k_n}(1-k_\tau)(1+k_n)}{7(\chi+k_n)^2 M_u (N^0)^{1/2}}; \quad S = \sqrt{\frac{k_n}{N^0}}\frac{\chi k_q(1+k_n)}{2(\chi+k_n)[k_n+\chi(1-k_q)]}. \tag{27}$$

For log-normal distribution

$$f_1 = \frac{\bar{\rho}_w k_n}{(\chi + k_n)\sqrt{2\pi} v \ln\sigma} \exp\left[-\frac{\ln^2(v/v_0)}{2\ln^2\sigma}\right];$$

$$f_2 = \frac{\bar{\rho}_w \chi}{(\chi + k_n)\sqrt{2\pi} v \ln\sigma} \exp\left[-\frac{\ln^2(v/(k_n v_0))}{2\ln^2\sigma}\right], \tag{28}$$

the BC for the equation of motion has the form

$$R = \frac{2}{7}\sqrt{\frac{k_n}{N^0}} \frac{\chi(1+k_n)(1-k_\tau)}{(\chi+k_n)^2 M_u} \exp\left(\frac{1}{2}\ln^2\sigma\right);$$

$$N^0 = \left[\frac{1+\chi k_n}{\chi + k_n} \exp(\ln^2\sigma) - k_n\left(\frac{1-\chi}{\chi+k_n}\right)^2\right] \exp(\ln^2\sigma). \tag{29}$$

At $\ln\sigma = 0$ Eqs. (29) are reduced to (21). If distributions

$$f = A v^k \exp(-v^2/v_0^2), \quad (k = 1, 2) \tag{30}$$

are used, the formulae for R are similar to (27), but with other numerical coefficients.

5. NON-ROTATING PARTICLES. AVERAGING BY TIME

If instead of Favre's method we use conventional averaging by time, the distribution function $f(v)$ obtains another sense: in this case $f dv$ being not the density of v, dv group, but the probability of some particle to have transverse velocities in the range v, dv (i.e. $\int f dv = 1$). Since the probability of particles' i passing through some point is proportional to the specific mass flow rate $\rho_i|\vec{U}_i|$, formulae (7), (24) must be replaced by

$$\bar{\varphi}_w = \frac{\varphi_1\rho_1 U_1 + \varphi_2\rho_2 U_2}{\rho_1 U_1 + \rho_2 U_2};$$

$$\bar{\varphi}_w = (\rho_1 U_1 + \rho_2 U_2)^{-1}\left(\rho_1\int_0^\infty U_1 f_1 \varphi dv + \rho_2\int_{-\infty}^0 U_2 f_2 \varphi dv\right). \tag{31}$$

In the same approximation of boundary layer $v_i << u_i$, and we can replace U_i in (31) by u_i. If approximation (18) is used, similarly to (20)–(23) we have

$$M_u = \left[49k_n + \chi (5 + 2k_\tau)^2 \right] / (7\phi);$$

$$\bar{T}_w = \{ [7k_n + \chi(1 - k_q)(5 + 2k_\tau)] T_1 + \chi k_q (5 + 2k_\tau) T^0 \} / \phi;$$

$$N = \frac{7\chi(1 + k_n)^2 (5 + 2k_\tau)}{\phi^2}; \quad Q = \sqrt{\frac{k_n}{N}} \frac{1 - \chi}{k_n + \chi}; \tag{32}$$

$$R = \frac{2\sqrt{k_n N}(1 - k_\tau)}{7(1 + k_n) M_u}; \quad S = \frac{\sqrt{7\chi k_n (5 + 2k_\tau) k_q}}{7k_n + \chi(1 - k_q)(5 + 2k_\tau)};$$

$$\phi \equiv 7k_n + \chi(5 + 2k_\tau).$$

As in Eq. (20), the mean transverse velocity differs from zero (except in the case $\chi = 1$, $k_n = 1$, when $M_v = 0$). That is why the hypothesis $v_w = 0$ used while obtaining Eq. (3) is incorrect in a general case. At $k_\tau = 1$ Eqs. (32) for M_v, $\bar{T}_w$, N, Q and S turn into (20)–(22) because here the averaging procedure (31) is reduced to (7), (24).

For approximation (23) one can obtain

$$R = 2\sqrt{\frac{2}{\pi}} \frac{\chi\sqrt{k_n}(1 + k_n)(1 - k_\tau)(5 + 2k_\tau)}{\phi^2 M_u \sqrt{N^0}}; \quad N^0 = \frac{7 + \chi k_n (5 + 2k_\tau)}{\phi} -$$

$$-\frac{2}{\pi} \frac{k_n [7 - \chi(5 + 2k_\tau)]^2}{\phi^2}; \quad S = \sqrt{\frac{2}{\pi}} \frac{7\chi k_q \sqrt{k_n}(1 + k_n)(5 + 2k_\tau)}{\phi [7k_n + \chi(1 - k_q)(5 + 2k_\tau)] \sqrt{N^0}} \tag{33}$$

(formula for Q is the same as in (25) but with N^0 from (33)). The BCs on the basis of approximation (26), (28), (30) are derived in a similar way; these formulae are not presented here.

6. ROTATING PARTICLES

The results obtained can be generalized for the case of the motion of rotating particles. For Favre's averaging and $\chi = 1$ from Eqs. (4), (7) we obtain the mean values of axial and angular velocities

$$\bar{u}_w = \frac{1}{7(1+k_n)}[(7k_n + 5 + 2k_\tau)u_1 + (1-k_\tau)d\omega_1];$$

$$\bar{\omega}_w = \frac{1}{7(1+k_n)}[(7k_n + 5k_\tau + 2)\omega_1 + 10(1-k_\tau)u_1/d]. \tag{34}$$

The solution of the system (34) gives us the values of u_1 and ω_1. According to Eqs. (8), (9) in this case $\bar{v}_w = 0$, $(\overline{v'^2})_w = k_n v_1^2$. After calculating the $\overline{u'v'}$ correlation, we obtain the BC for the equation of motion

$$\frac{\sqrt{k_n}(1-k_\tau)}{49(1+k_n)(k_n+k_\tau)}\left(\overline{v'^2}\right)_w^{1/2}[2(7k_n + 10k_\tau - 3)\bar{u}_w +$$

$$+ (7k_n + 4k_\tau + 3)d\bar{\omega}_w] = -\nu_t(\partial\bar{u}/\partial y)_w. \tag{35}$$

7. DISCUSSION

Comparison of formulae (21), (22), (25), (27), (29), (32), (33), (35) shows that the BCs depend upon the form of $f(v)$ approximation. This question is considered in detail for the R values from Eqs. (21), (25), (27), (29) where, for simplification, we set $\chi = 1$. In this case the formulae mentioned (except (29) !), and the dependencies for R corresponding to the distribution (30) differ from each other only by numerical coefficients: 2 for (21); $2\sqrt{2/\pi}$ for (25); $\sqrt{3}$ for (27); $\sqrt{\pi}$ at $k = 1$ and $4\sqrt{2/(3\pi)}$ at $k = 2$ for relations following from (30). Thus the R values differ from each other by not more than 25%. At the same time, Eq. (29) substantially differs from other formulae because if $\ln\sigma$ is large enough according to (29), the R values can be very small. Certainly, the distribution (28) and especially a large $\ln\sigma$ do not describe any real process; nevertheless, this example demonstrates the dependence of R upon the form of $f(v)$. Simple physical considerations show that for real flows the particles' distribution by transverse velocities must be close to (23) (see Fig. 1, *b*). That is why we shall consider only such results which correspond to (23) and to the simplest approximation (18).

Fig. 2 illustrates the influence of the approximation on the R value. Here a qualitative difference between curves attracts our attention – according to (21), (25) R increases with the growth of k_n, yet, according to (16), it decreases. The first type of pattern seems to be more realistic.

Besides, according to (16), for R to be equal to zero it is necessary that not only $k_\tau = 1$, but $k_n = 1$, too. The second condition is superfluous because on a perfectly smooth wall the Reynolds stresses or the momentum losses are equal to zero at arbitrary k_n. While using time averaging, the character of the R dependence upon the form of approximation (Fig. 3, $\chi = 1$) is similar to that considered earlier. At $k_\tau < 0$ the method of averaging has a noticeable influence on the BC. The results for partially absorbing the wall are presented in Fig. 4 – here the divergences between various curves are somewhat greater than for the case $\chi = 1$.

Fig. 5 illustrates the dependence of the BC for the mass conservation equation upon k_n, k_τ, τ and the method of calculation. It can be seen that a significant difference exists between Q values obtained by various formulae. It should be noted that the approximation (18) does not allow to analyze an extreme case $\chi = 0$ (perfect absorbtion): here $N = 0$,

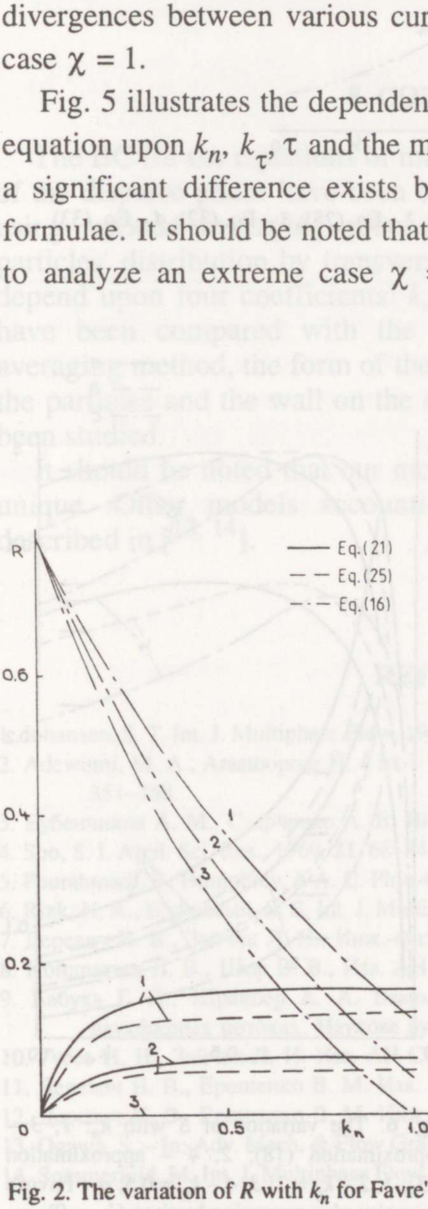

Fig. 2. The variation of R with k_n for Favre's averaging: *1*– $k_\tau = 0$; *2*–$k_\tau = 0.5$; *3*–$k_\tau = 1$.

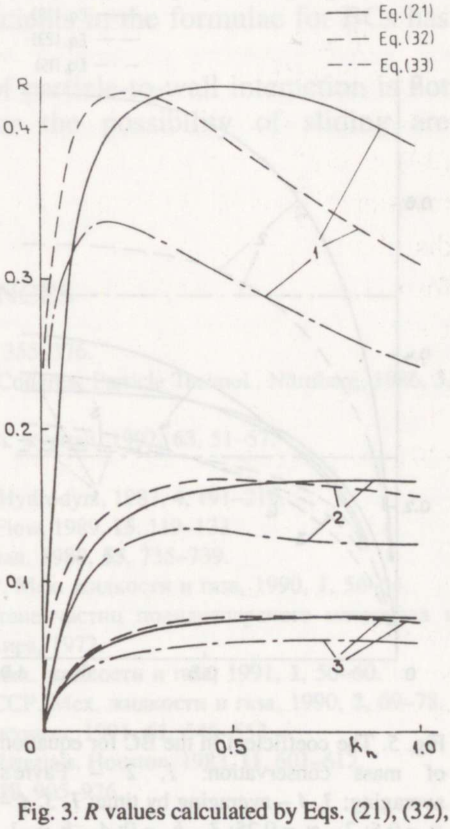

Fig. 3. R values calculated by Eqs. (21), (32), and (33): *1*–$k_\tau = 1$; *2*–$k_\tau = 0$; *3*–$k_\tau = 0.5$.

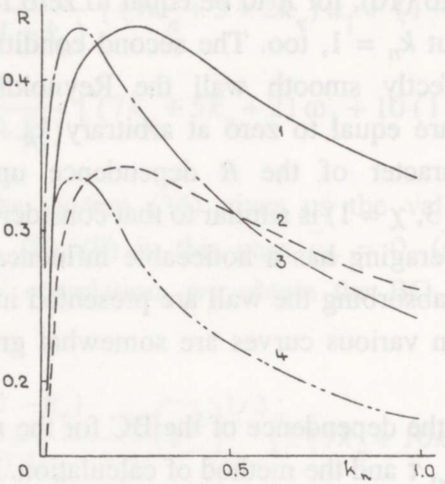

Fig. 4. $k(k_n)$ at $\lambda = 0.5$ and $k_\tau = -1$: *1* – Eq. (21); *2* – Eq. (25); *3* – Eq. (32); *4* – Eq. (33).

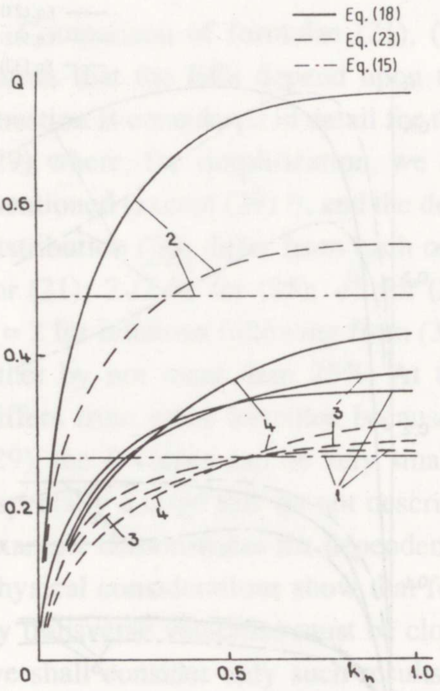

Fig. 5. The coefficient in the BC for equation of mass conservation: *1*, *2* – Favre's averaging; *3*, *4* – averaging by time; *1*, *3*, *4* – $\chi = 0.5$; 2 – $\chi = 0.25$; *3* – $k_\tau = 0$; *4* – $k_\tau = -1$.

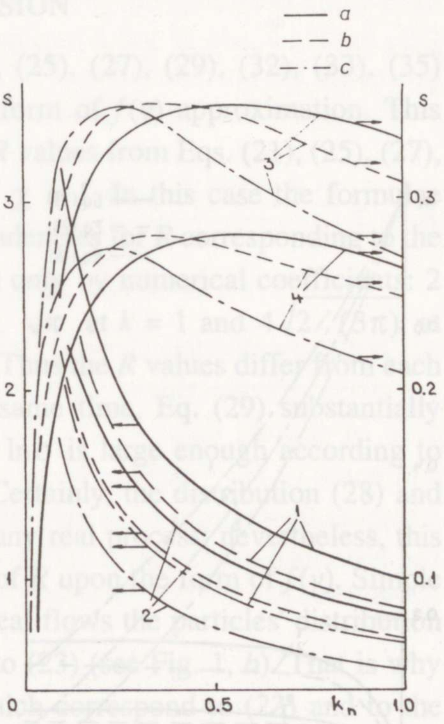

Fig. 6. The variation of S with k_n: *1*, *3* – approximation (18); *2*, *4* – approximation (23); *1*, *2* – $k_q = 1$; *3*, *4* – $k_q = 0.5$; *a* – Favre's averaging; *b* – averaging by time ($k_\tau = 0$); *c* – averaging by time.

$\left(\overline{v'^2}\right)_w = 0$, and the particles' flow rate J_{nw} cannot be connected with the fluctuation transverse velocity (see Eq. (22)). At the same time, more realistic approximation (23) provides the opportunity to consider this case, because $N^0 = k_n^{-1}(1 - 2/\pi) \neq 0$. From (25) it can be seen that the particles' density near the wall differs from zero for $\chi = 0$; this fact confirms considerations [7]. In Fig. 6 the values of the coefficient in the BC for the energy equation are presented. At $k_q = 1$ the function $S(k_n)$ decreases with the growth of k_n, and at $k_q = 0.5$ it has a maximum.

8. CONCLUSIONS

The BC for the equations of mass, momentum and energy conservation of the disperse phase have been constructed on the basis of approach [8], using various averaging procedures and various approximations for particles' distribution by transverse velocities. In a general case the BCs depend upon four coefficients: k_n, k_τ, k_q and χ. The formulae obtained have been compared with the known results. The influence of the averaging method, the form of the function $f(v)$ and physical properties of the particles and the wall on the coefficients in the formulae for BCs has been studied.

It should be noted that our model of particle-to-wall interaction is not unique. Other models accounting for the possibility of sliding are described in [13, 14].

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OSAKESTE MASSI, IMPULSI JA ENERGIA ÜLEKANDEVÕRRANDITE PIIRTINGIMUSED TURBULENTSES KAHEFAASILISES VOOLUSES

Aleksander ŠRAIBER, Vladimir NAUMOV

On esitatud range järjestikune meetod osakeste massi, impulsi ja energia ülekandevõrrandite piirtingimuste püstitamiseks turbulentses kahefaasilises vooluses. On analüüsitud keskendamise ja osakeste jaotuse aproksimeerimise meetodeid kahe erisuunalise osakeste ansambli puhul.

ГРАНИЧНЫЕ УСЛОВИЯ ДЛЯ УРАВНЕНИЙ ПЕРЕНОСА МАССЫ ЧАСТИЦ, ИМПУЛЬСА И ЭНЕРГИИ В ТУРБУЛЕНТНЫХ ДВУХФАЗНЫХ ПОТОКАХ

Александр ШРАЙБЕР, Владимир НАУМОВ

Развивается строгий и последовательный метод построения граничных условий к уравнениям переноса массы, импульса и энергии частиц в турбулентном двухфазном потоке. Рассматриваются два ансамбля частиц, движущихся к стенке и от нее. Анализируются различные методы осреднения, аппроксимации распределения частиц по поперечным скоростям и свойства взаимодействующих поверхностей.