

## OPTIMIZATION OF ENERGY FLOW IN BALANCED THREE- TO SINGLE-PHASE POWER CONVERSION CIRCUITS

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**Abstract.** Performance characteristics of power-electronic converters depend on energy conversion processes in corresponding circuits. Therefore, the principles of rational energy exchange and transfer processes create possibilities for synthesizing new conversion circuits with better characteristics. In the case of three- to single-phase converters, consideration of the conservation properties of power components allows both simplification of the power circuit and improvement of the power factor. For the realization of optimum-energy-flow conditions, the zero-reactive-power operation mode of the energy-storing element has to be used. The proposed design principle of balanced three- to single-phase power conversion circuits is illustrated by a new six-pulse circuit containing a switch-controlled inductor and an interphase autotransformer.

**Key words:** three- to single-phase converter, interphase time-variable transformer, power conservation.

### INTRODUCTION

Symmetrization and smoothing of supply currents guarantee electromagnetic compatibility of single-phase voltage switch-mode controllers with three-phase supply circuit. In an ideal case the instantaneous input power  $p_{in} = \text{const}$ , i. e. the input is balanced. For a single-phase resistive load and rectangular output voltage the instantaneous output power  $p_o = P_o = p_{in} = \text{const}$ . That enables, in principle, the conversion to be carried out using only a properly controlled time-variable transformer. In a more general case with a complex load and/or sinusoidal output voltage the instantaneous output power  $p_o = \text{var}$ . Therefore, to balance the input, at least one controlled energy-storing element is urgently needed. In the known balanced three- to single-phase conversion circuits, for obtaining the unity power factor, simultaneous use of both inductive and capacitive balancing elements is required.

Below a new efficient design principle of three- to single-phase converters, characterized by the unity maximum power factor and by the use of the only inductive balancing element, is analysed and illustrated. The principle is based on the optimization of energy flow and on the validity of the law of conservation for separate components of power.

## CONSERVATION PROPERTIES OF THE POWER COMPONENTS

Consider a one-port with the nonsinusoidal periodic voltage  $v = \sum_{k=0}^{\infty} v_{(k)}$  and current  $i = \sum_{n=0}^{\infty} i_{(n)}$ . Each of the current harmonics can be decomposed into the active and reactive components:  $i_{(n)} = i_{(n)p} + i_{(n)q}$ . The instantaneous power  $s$  can be described as the sum of three components:

$$s = vi = \sum_{k=0}^{\infty} v_{(k)} \sum_{n=0}^{\infty} i_{(n)} = \sum_{k=0}^{\infty} v_{(k)} i_{(k)p} + \sum_{k=1}^{\infty} v_{(k)} i_{(k)q} + \sum_{\substack{k, n=0 \\ k \neq n}}^{\infty} v_{(k)} i_{(n)} = p + q + d,$$

which are denominated the instantaneous active, reactive and distortion powers, respectively [1-4]. The sum  $(p+q)$  is called the instantaneous active—reactive power, which can be decomposed into the direct ( $P$ ) and alternating ( $p \sim +q$ ) components.

The conservation law of the instantaneous power states that the algebraic sum of instantaneous powers of all elements in an arbitrary circuit is zero. From all the components of instantaneous power the law of conservation is valid for the direct component of the instantaneous power ( $P$ ), for the alternating component of the instantaneous active—reactive power ( $p \sim +q$ ), and for the instantaneous distortion power ( $d$ ). In general the conservation property of the instantaneous power does not hold for the instantaneous active power ( $p$ ) and the instantaneous reactive power ( $q$ ) separately. We have proved it in [5, 6] by means of the Tellegen's theorem, which confirms that Kirchhoff's laws are sufficient for proving the conservation of power and energy in electrical circuits independently of the character of the circuit elements. In a general case the active and reactive components of the instantaneous currents and/or voltages do not agree with Kirchhoff's laws.

To illustrate the above let us consider energy exchange and the balance of the powers in the simple ideal circuit in Fig. 1. The voltage, current and power waveforms of all the circuit elements are presented in Fig. 2, where  $v = V_m \sin \omega t$ ,  $V_m = 1$ ,  $R = 1$ ,  $x = \omega L = 1$ ,  $z = \sqrt{R^2 + x^2} = \sqrt{2}$ ,  $\varphi = \pi/4$ ,  $I_0 = 0.5$ . The instantaneous power  $s = vi = p + q + d = s_R + s_{I_0} + s_L$ . Since  $s_R = p_R = v_R i$ ,  $s_{I_0} = -v_L I_0 = d_{I_0}$ ,  $s_L = v_L (i + I_0) = q_L + d_L$ ,  $-d_{I_0} = d_L$ , then  $s = p_R + q_L = v_R i + v_L i$ . Decomposing the current into the active and reactive components  $i = i_p + i_q$ , we get  $s = v i_p + v i_q$ . Fig. 2 shows that  $v_R i \neq v i_p$  and  $v_L i \neq v i_q$ , i.e. the balance principle does not hold for the instantaneous active and reactive powers separately. It holds for the instantaneous active—reactive power ( $v_R i + v_L i = v i_p + v i_q$ ), its direct and alternating components, and the instantaneous distortion power ( $-d_{I_0} = d_L$ ).

From the point of view of physics the invalidity of the conservation law for both the active and reactive components of the instantaneous power can be explained with the fact that in a general case the energy generated by the reactive elements does not return into the power supply directly.

Concerning the integral representation of powers, it is known that the conservation property holds only for the active and reactive powers; for the apparent and distortion powers it does not hold [7, 8]. It means that the sum of the active powers of all elements in an arbitrary circuit is equal to the input active power. The same is valid for the reactive

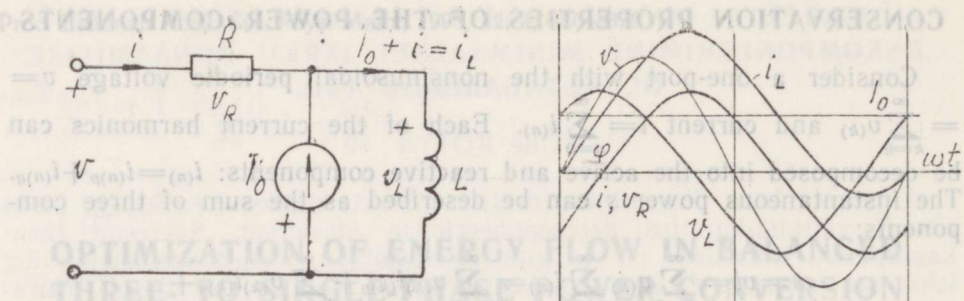


Fig. 1. A simple circuit to illustrate conversion properties of the power components.

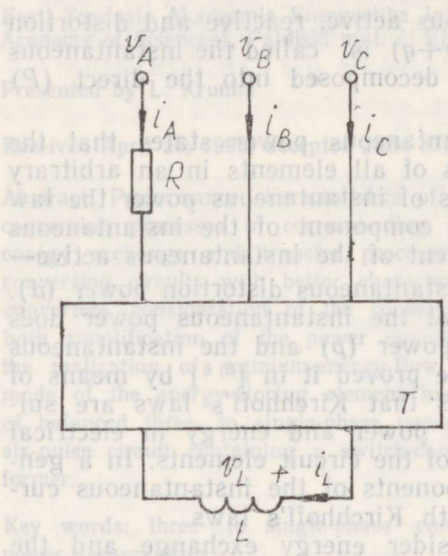


Fig. 3. Ideal three- to single-phase converter.

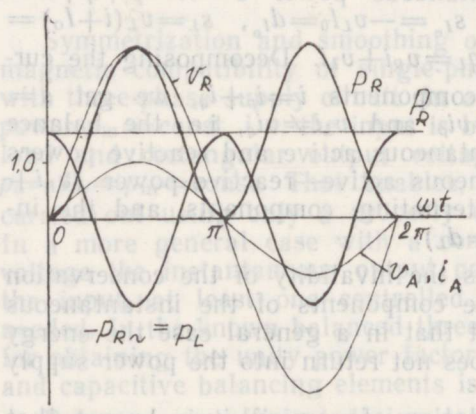


Fig. 4. Ideal voltage, current and power waveforms in the circuit in Fig. 3.

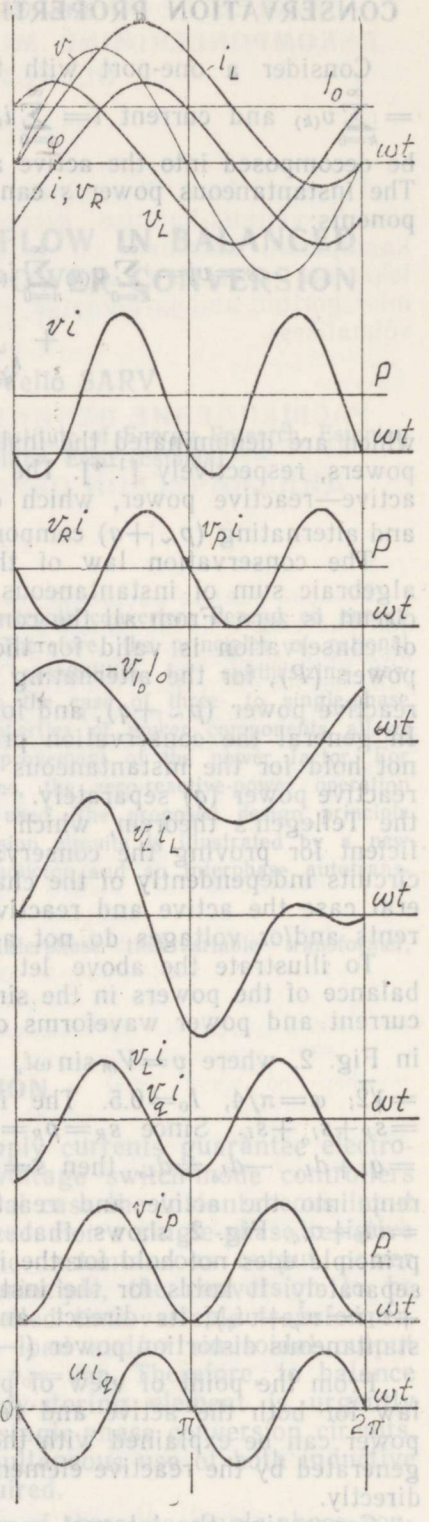


Fig. 2. Voltage, current and power waveforms in the circuit in Fig. 1.

power. The apparent and distortion powers cannot be summed by circuit elements and therefore they do not determine the input powers directly.

The illustrative calculations for the circuit in Fig. 1 and the above conditions confirm that  $S_{in} = VI = 1.4$ ;  $P_{in} = VI \cos \varphi = 0.99$ ;  $Q_{in} = VI \sin \varphi = 0.99$ ;  $D_{in} = 0$ ;

$$S_R = V_R I_R = P_R = 0.99; \quad S_L = V_L I_L = \sqrt{Q_L^2 + D_L^2} = 1.4$$

$$(Q_L = V_L I_{L\sim} = 0.99, \quad D_L = V_L I_L = 0.99); \quad S_{I_0} = V_{I_0} I_0 = D_{I_0} = 0.99;$$

$$P_{in} = P_R + P_{I_0} + P_L = P_R; \quad Q_{in} = Q_R + Q_{I_0} + Q_L = Q_L;$$

$$S_{in} = \sqrt{P_{in}^2 + Q_{in}^2 + D_{in}^2} \neq S_R + S_L + S_{I_0}; \quad D_{in} \neq D_{I_0} + D_L.$$

### BASIC PRINCIPLES

Consider a symmetrical sinusoidal system of supply voltages and currents where the instantaneous input power of conversion circuit  $p_{in} = \text{const}$ , i. e. the multiphase input is balanced. If simultaneously the instantaneous output (load) power

$$p_o = p_{in} = \text{const}, \quad (1)$$

then the conversion can be performed without energy storage. Such a conversion circuit can be modelled using an ideal time-variable transformer with multiphase input and single-phase output.

For a single-phase resistive load restriction (1) is satisfied when the load voltage has the rectangular waveform of any frequency. For a complex load and/or sinusoidal output voltage,

$$p_o = P_o + p_{o\sim} = \text{var}, \quad (2)$$

i. e. a controlled energy-storing element is needed, which compensates the pulsating component  $p_{o\sim}$  of the instantaneous output power  $p_o$  [9].

In the known circuits in case of a resistive load and zero displacement angle of input currents simultaneous use of both inductive and capacitive balancing elements is inevitable [9], since the instantaneous power of the energy-storing element ( $p_{st} = -p_{o\sim}$ ) is a product of the sinusoidal current  $i_{st(t)}$  and voltage  $v_{st(t)}$  of the input frequency with the phase shift of  $\pi/2$ .

Such a pair of  $i_{st(t)}$ ,  $v_{st(t)}$  results in the input-frequency reactive power component

$$Q_{st(t)} = |v_{st(t)} i_{st(t)}|_{\max} = V_{st(t)} I_{st(t)}, \quad (3)$$

where  $V_{st(t)}$  and  $I_{st(t)}$  are the rms values.

If in the known balancing circuits only a single inductive energy-storing element  $L$  is used, its inductive reactive power

$$Q_{L(t)} = |v_{L(t)} i_{L(t)}|_{\max} = V_{L(t)} I_{L(t)} \quad (4)$$

results in the input reactive power, i. e.)

$$Q_{in(t)} = Q_{L(t)}, \quad (5)$$

and, consequently, also in a lagging shift between the input voltages and currents even in the case of a purely resistive load.

For obtaining a zero displacement angle of input currents the use of another, capacitive energy-storing element  $C$  is required, which causes the capacitive reactive power  $Q_{C(t)} = -Q_{L(t)}$ . As the result,

$$Q_{in(t)} = Q_{L(t)} + Q_{C(t)} \equiv 0. \quad (6)$$

A zero displacement angle of input currents is obtainable by the use of only a single inductive balancing element which has the compensating instantaneous power

$$p_{st} = -p_{0\sim} = p_L = v_L i_L \quad (7)$$

and satisfies the condition  $Q_L \equiv 0$ .

For this we propose such a balancing three- to single-phase conversion circuit in which the instantaneous power of the compensating energy-storing element is the product of the current and voltage of different harmonic content. In that case requirement (7) can be satisfied while the product of  $V_L I_L$  can be interpreted not as the reactive power  $Q_L$ , but as either merely the distortion power  $D_L$  or a function of the distortion power and the active powers of the harmonics involved, while the sum of harmonic active powers equals zero. Conservation holds for the reactive and active powers but never for the distortion power. Therefore, the non-zero distortion power of a compensating energy-storing element causes neither the input reactive power nor inevitably the input distortion power.

A possible realization of the proposed principle is illustrated in Fig. 3 where the ideal balancing three- to single-phase converter contains an interphase transformer  $T$  and an energy-storing inductor  $L$ , both being time-variable elements, and the resistive load  $R$  is in the line  $A$ .

The corresponding current, voltage and power waveforms for the ideal mode of operation are shown in Fig. 4 for  $v_A = \sin \omega t$  and  $R=3$ . In this example the instantaneous compensating power

$$p_L = -p_{0\sim} = 1.5 \cos 2\omega t. \quad (8)$$

It can be obtained e.g. as the product of the following three current and voltage pairs with different harmonic content:

$$i_L = 1, \quad v_L = 1.5 \cos 2\omega t; \quad (9)$$

$$i_L = i_R = \sin \omega t, \quad v_L = 1.5 (\cos 2\omega t / \sin \omega t); \quad (10)$$

$$i_L = |i_R| = |\sin \omega t| = (\text{sign } \sin \omega t) \cdot \sin \omega t, \quad (11)$$

$$v_L = 1.5 (\text{sign } \sin \omega t) (\cos 2\omega t / \sin \omega t).$$

A possible version of the proposed principle for the current-voltage pair (11) is shown in Fig. 5. Here the time-variable inductor  $L(t)$  with controlled turn number has to form the inductor current

$$i_L = |i_R| = |I_m \sin \omega t|, \quad (12)$$

the commutator  $VS1 \dots VS4$  with the switching function

$$F(t) = \text{sign } \sin \omega t \quad (13)$$

has to guarantee the load current

$$i_R = i_A = F(t) i_L = I_m \sin \omega t, \quad (14)$$

and the time-variable transformer  $T$  has to divide the current  $i_A$  into two components ( $-i_B$  and  $-i_C$ ) so that the three line currents  $i_A$ ,  $i_B$  and  $i_C$  form a symmetrical sinusoidal system, i.e. the time-variable transformation ratio  $n_B(t)/n_C(t)$  has to be as follows:

$$\frac{n_B(t)}{n_C(t)} = \frac{\sin(\omega t + 2\pi/3)}{\sin(\omega t - 2\pi/3)} = \frac{i_C}{i_B} \quad (15)$$

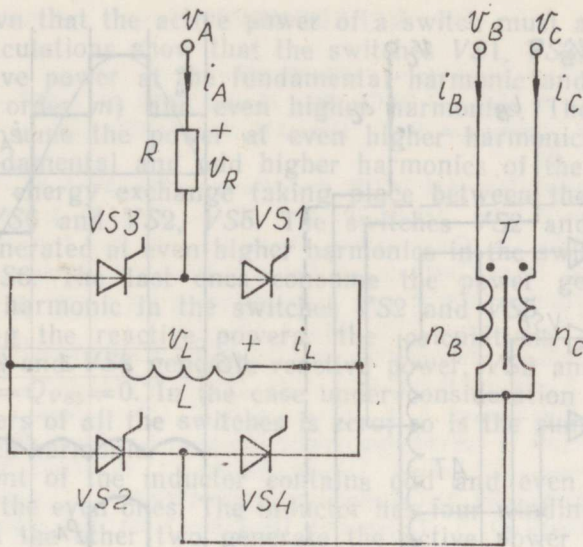


Fig. 5. Basic structure of the proposed three- to single-phase converter.

To implement the ideal relations (12) and (15) a continuous variation of the turns of both the inductor  $L$  and transformer  $T$  is needed.

In practice, instead of the ideal continuous variations indicated in (12) and (15), only a stepped variation of both the turns and the turns ratio that leads to some deviation from the ideal sinusoidal current waveform will be possible. Nevertheless, a suitable stepped variation of the turns, the turns ratio, and, as a result, line current ratios allows to implement the balanced  $p$ -pulse mode of operation and thus to eliminate the lower harmonics up to  $p - 1$  [10-12].

### EXAMPLE OF REALIZATION

Fig. 6 presents a possible power circuit of the six-pulse three- to single-phase converter implementing the proposed principle.

For the circuit operation analysis it is assumed that the three-phase supply is symmetrical, the interphase autotransformer  $AT$ , the switch-controlled inductor  $L(t)$  and the switches  $VS1 \dots VS6$  are ideal elements, and the single-phase load  $R$  in the line  $A$  is purely resistive.

The circuit waveforms for the zero control angle and stepped line currents are illustrated in Fig. 7. The phase voltage  $v_A$ , the line current  $i_A = i_R$  and the instantaneous powers  $p_1 = p_A + p_B + p_C = p_R + p_L$  are shown for  $R = 2.92$ .

Since the circuit elements are assumed to be ideal, the input active power has to be equal to the load power ( $P_1 = P_R$ ). As the output voltage  $v_R$  contains higher-harmonic components, its fundamental harmonic amplitude is smaller than that of the sinusoidal load voltage even for the zero control angle. The input active power can be written as follows:  $P_1 = 3V_{Am}V_{R(1)m}/2R$ . The load power at the fundamental harmonic may be expressed as  $P_{R(1)} = V_{R(1)m}^2/2R$ , therefore  $P_1 > P_{R(1)}$ .

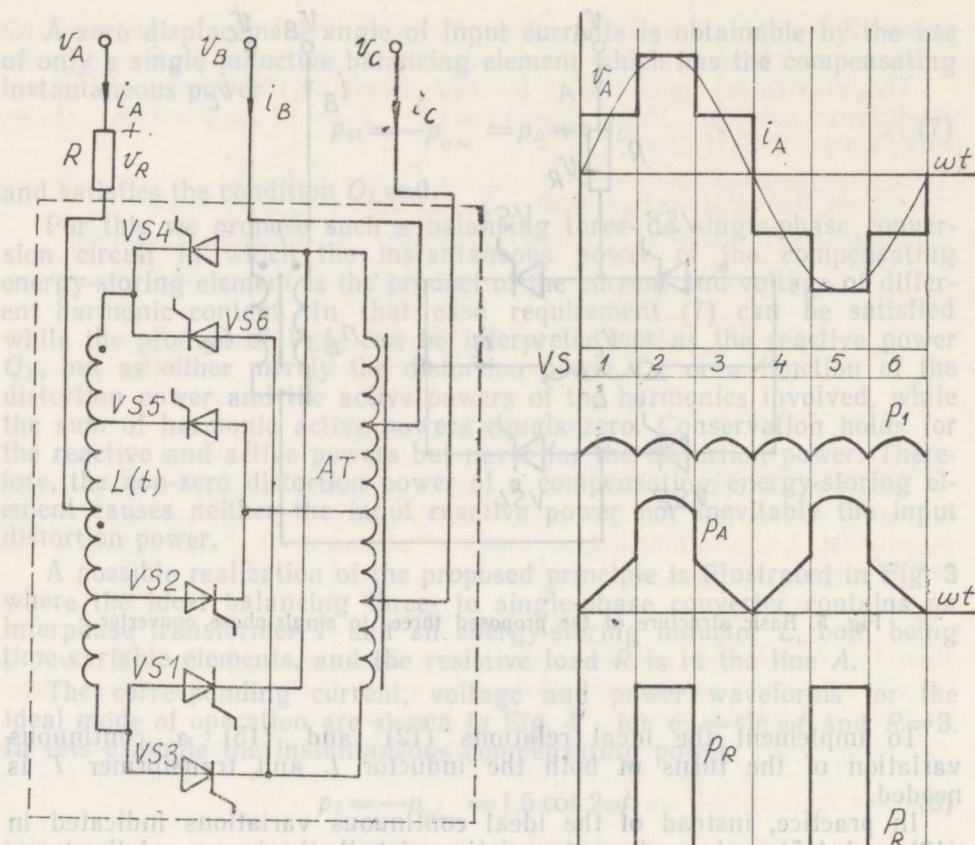


Fig. 6. Six-pulse three- to single-phase converter using time-variable inductor and interphase autotransformer.

Fig. 7. Voltage, current and power waveforms in the circuit in Fig. 6.

The input power is generated only at the fundamental harmonic, the load consumes power at the fundamental and odd higher harmonics of the order  $m=5, 7, 11, 13$ , etc. The power at the fundamental harmonic is delivered to the load by (symmetrical three-phase) alternating-voltage source, the power at the odd higher harmonics, by the switches. The current and voltage waveforms of the switches contain odd and even higher harmonics. The switches VS1...VS6 consume active power at the fundamental harmonic ( $P_1 - P_{R(1)}$ ) and generate it at the odd higher harmonics of the order  $m$ . The sum of the active powers consumed and generated by all switches in a circuit must be equal to zero. The active power generated in the switches at the odd higher harmonics is consumed in the load.

It is known that the active power of a switch must also be equal to zero. The calculations show that the switches VS1, VS3, VS4, and VS6 consume active power at the fundamental harmonic and generate it at odd (of the order  $m$ ) and even higher harmonics. The switches VS2 and VS5 consume the power at even higher harmonics and generate it at the fundamental and odd higher harmonics of the order  $m$ . It is explained by energy exchange taking place between the switches VS1, VS3, VS4, VS6 and VS2, VS5. The switches VS2 and VS5 consume the power generated at even higher harmonics in the switches VS1, VS3, VS4, and VS6. The last ones consume the power generated at the fundamental harmonic in the switches VS2 and VS5.

Concerning the reactive powers, the calculations show that the switches VS1 and VS6 generate reactive power, VS3 and VS4 consume it, and  $Q_{VS2} = Q_{VS5} = 0$ . In the case under consideration the sum of the reactive powers of all the switches is zero; so is the sum of the reactive powers of each harmonic.

The current of the inductor contains odd and even harmonics, the voltage only the even ones. The inductor has four windings, two of them consume and the other two generate the active power at even higher harmonics so that the sum of the powers for each of the harmonics is zero. The reactive power of the windings is also zero, therefore only distortion power exists in the inductor.

The proposed scheme was experimentally verified in laboratory under different load conditions.

## CONCLUSION

From the point of view of the energy transfer processes, the balancing of the three-phase to single-phase conversion circuits is based on the appropriate energy exchange between the supply phases. A straightforward effective technique for proper energy exchange control is the use of a time-variable interphase transformer. In the case of a single-phase resistive load and rectangular output voltage, the three- to single-phase conversion requires the application of a properly controlled time-variable transformer. For a complex load and/or sinusoidal output voltage, in addition to the time-variable transformer, at least one energy-storing element is needed to compensate the difference between the constant instantaneous input power and the inevitably pulsating load power. To obtain the unity displacement factor in the case of a single energy-storing element, e.g. the inductor, its voltage and current waveforms must be such that the product of the corresponding rms values is interpreted not as the reactive power, but as the distortion power. Such a single energy-storing element in the zero-reactive-power operation mode is essential for balanced supply for both the time-invariant and time-variable load.

For the ideal undistorted three- to single-phase conversion, a continuous variation of the interphase transformation ratios and inductivity is needed. In practice, instead of an ideal continuous turn number variation, only a stepped variation can be realized. Nevertheless, a suitable stepped variation of turn numbers enables the implementation of the  $p$ -pulse balanced operation mode and thus the elimination of the lower harmonics up to  $p - 1$  in the input current.



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## ENERGIAÜLEKANDE OPTIMEERIMINE KOLMEFAASISE TOITE JA ÜHEFAASISE KOORMUSEGA TASAKAALUSTATUD MUUNDUS- AHELATES

Maire OJAVEER, Vello SARV

On tõestatud, et käsitletud faasimuundusahelate klassi energiavahe-  
tusprotsesside optimeerimiseks ning tehniliste näitajate parandamiseks  
on otstarbekas kasutada reaktiivvoimsusvabas režiimis töötavat ener-  
giasalvestit. Illustratsiooniks on esitatud uus täiustatud kuuepulsiline  
sümmeetrimisskeem, milles kasutatakse astmeliselt juhivat paispooli  
ja autotrafot.

## ОПТИМИЗАЦИЯ ПЕРЕДАЧИ ЭНЕРГИИ В УРАВНОВЕШЕННОЙ ПРЕОБРАЗОВАТЕЛЬНОЙ ЦЕПИ С ТРЕХФАЗНЫМ ПИТАНИЕМ И ОДНОФАЗНОЙ НАГРУЗКОЙ

Майре ОЯВЕЭР, Велло САРВ

Показано, что для оптимизации процессов энергообмена и техни-  
ческого совершенствования рассмотренного класса цепей преобразова-  
ния числа фаз целесообразно применять энергонакопитель в режиме  
с нулевой реактивной мощностью. Для иллюстрации названного принципа  
синтеза предложена новая усовершенствованная шестипульсная схема  
симметрирования с применением ступенчатоуправляемого дросселя и  
автотрансформатора.