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# ON INTERPRETATION OF SUPERCONDUCTING ORDERS INDUCED BY INTERBAND REPULSION AND ATTRACTION

*(Presented by V. Hižnjakov)*

It is well known that the superconducting state arising as a result of effective attraction between electrons in the BCS theory can be treated as a ferromagnetic-type ordering of pseudospins in the transversal field which varies from pseudospin to pseudospin [1, 2]. This is not the only possible superconducting state. In [3, 4], a conception has been developed according to which in connection with the uncertainty of the multiplicity of pseudospins in the system described by the BCS Hamiltonian, besides the pseudoferromagnetic-type superconductivity caused by attractive interaction, a pseudoantiferrromagnetic-type superconductivity appears due to repulsion between electrons. The existence of superconducting states corresponding to different pseudospin orderings in many-valley semiconductors with intra- and intervalley interactions of both attractive and repulsive nature have been shown in [5].

In the two-band (multiband) models which originate from [6, 7], both the interband attraction and repulsion of electrons lead to a superconducting order [8]. This trend in the theory of superconductivity acquired additional actuality in the context of high-$T_C$ systems (see for review [9, 10]). The idea of interband interaction between the carriers of partly overlapping hole bands as a possible mechanism for superconductivity in metal-oxide compounds has been proposed and developed in [11–14]. Thereby it is the supposed repulsive nature of this interband coupling that gives an opportunity to get high values of $T_C$ [11] and enables one to describe the dependence of the transition temperature on the carrier concentration [12, 13].

The superconducting orders induced correspondingly by interband repulsion and attraction between electrons must evidently be different. A pseudospin representation gives a simple and figurative interpretation for this difference. The situation arising here is somewhat analogous to the one considered in [5].

The Hamiltonian of a two-band system of electrons with the interband interaction inducing superconductivity is the following [11]:

$$H=\sum_{\sigma,\vec{k},s}\tilde{\varepsilon}_\sigma(\vec{k})\,a^+_{\sigma\vec{k}s}\,a_{\sigma\vec{k}s}+2W\sum_{\sigma,\sigma'}{}'\sum_{\vec{k},\vec{k}'}a^+_{\sigma\vec{k}\uparrow}\,a^+_{\sigma-\vec{k}\downarrow}\,a_{\sigma'-\vec{k}'\downarrow}\,a_{\sigma'\vec{k}'\uparrow}\,. \qquad (1)$$

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Here $\tilde{\varepsilon}_\sigma(\vec{k})=\varepsilon_\sigma(\vec{k})-\mu$; $\varepsilon_\sigma(\vec{k})$ is the energy of the electron created by $a^+$ with the wave vector $\vec{k}$, and the spin $s$ in the band numbered by $\sigma=1,2$; $\mu$ is the chemical potential. The second term in Hamiltonian (1) corresponds to the scattering processes of $(\vec{k}\uparrow, -\vec{k}\downarrow)$ electron pairs from one band to another with the coupling constant $W$. For the interband repulsion, $W>0$, and for the interband attraction, $W<0$. Thereby, the regions of $\vec{k}$-space, where $W\neq 0$, are in principle different for the cases of repulsion and attraction due to the difference in the physical mechanisms of these types of pairing. The intraband couplings have been omitted in the term of interaction of Hamiltonian (1).

According to [1], we define the operators $\vec{S}_{\sigma\vec{k}}=(S_{\sigma\vec{k}x}, S_{\sigma\vec{k}y}, S_{\sigma\vec{k}z})$ for pseudospins 1/2 by means of relations

$$1-a^+_{\sigma\vec{k}\uparrow}a_{\sigma\vec{k}\uparrow}-a^+_{\sigma-\vec{k}\downarrow}a_{\sigma-\vec{k}\downarrow}=2S_{\sigma\vec{k}z},$$

$$a^+_{\sigma\vec{k}\uparrow}a^+_{\sigma-\vec{k}\downarrow}=S_{\sigma\vec{k}x}-iS_{\sigma\vec{k}y}, \tag{2}$$

$$a_{\sigma-\vec{k}\downarrow}a_{\sigma\vec{k}\uparrow}=S_{\sigma\vec{k}x}+iS_{\sigma\vec{k}y},$$

with an additional condition $a^+_{\sigma\vec{k}\uparrow}a_{\sigma\vec{k}\uparrow}=a^+_{\sigma-\vec{k}\downarrow}a_{\sigma-\vec{k}\downarrow}$, which restricts the space of the electron states to the subspace of electron pairs. Now Hamiltonian (1) can be represented in the form

$$H=\sum_{\sigma,\vec{k}}\tilde{\varepsilon}_\sigma(\vec{k})\left(1-2S_{\sigma\vec{k}z}\right)+4W\sum_{\vec{k},\vec{k}'}\left(S_{1\vec{k}x}S_{2\vec{k}'x}+S_{1\vec{k}y}S_{2\vec{k}'y}\right). \tag{3}$$

Introducing the effective field depending on $\sigma$ and $\vec{k}$,

$$\vec{h}_\sigma(\vec{k},\lambda_\sigma)=(h_{\sigma x},\ h_{\sigma y},\ h_{\sigma z})=(-4W\lambda_\sigma,\ 0,\ 2\varepsilon_\sigma(\vec{k})), \tag{4}$$

where $\lambda_{1,2}$ are order parameters and $2\varepsilon_\sigma(\vec{k})$ manifests itself as a transversal field, one can rewrite (3) as

$$H=H_0+H_1, \tag{5}$$

$$H_0=\sum_{\sigma,\vec{k}}\left[\frac{1}{2}h_{\sigma z}(\vec{k})-\vec{h}_\sigma(\vec{k},\lambda_\sigma)\vec{S}_{\sigma\vec{k}}\right], \tag{6}$$

$$H_1=\sum_{\sigma,\vec{k}}h_{\sigma x}(\lambda_\sigma)S_{\sigma\vec{k}x}+4W\sum_{\vec{k},\vec{k}'}\left(S_{1\vec{k}x}S_{2\vec{k}'x}+S_{1\vec{k}y}S_{2\vec{k}'y}\right). \tag{7}$$

With no generality restriction effective field (4) has been chosen to be in $xz$ plane.

As Hamiltonian (3) has been defined in the subspace of pairs, the temperature $T$ must be replaced with $2T$ in the thermodynamic functions calculated on the basis of this Hamiltonian [15]. Proceeding from the form (5)—(7) of the pseudospin Hamiltonian, the mean-field free energy can be expressed in general as [16]

$$F = 2k_BT \ln Z(H_0) + \langle H_1\rangle_{H_0}, \quad (8)$$

where $Z(H_0) = \mathrm{Sp}\, e^{-H_0/2k_BT}$ is the partition function and $\langle\ldots\rangle_{H_0} = Z^{-1}(H_0)\,\mathrm{Sp}\ldots e^{-H_0/2k_BT}$. Rotating the system of co-ordinates around $y$-axis so that the direction of $z$-axis coincides with the direction of $\vec{h}_\sigma(\vec{k}, \lambda_\sigma)$ for each $\sigma$ and $k$, one can find that the partition function and the necessary pseudospin averages equal to

$$Z(H_0) = 2\prod_\sigma \prod_{\vec{k}} e^{-h_{\sigma z}(\vec{k})/4k_BT}\, ch\frac{h_\sigma(\vec{k}, \lambda_\sigma)}{4k_BT}, \quad (9)$$

$$\langle S_{\sigma\vec{k}x}\rangle_{H_0} = \frac{h_{\sigma x}(\lambda_\sigma)}{2h_\sigma(\vec{k}, \lambda_\sigma)}\, th\frac{h_\sigma(\vec{k}, \lambda_\sigma)}{4k_BT}, \quad (10)$$

$$\langle S_{\sigma\vec{k}y}\rangle_{H_0} = 0. \quad (11)$$

Here $h_\sigma(\vec{k}, \lambda_\sigma) = |\vec{h}_\sigma(\vec{k}, \lambda_\sigma)|$. Taking into account (9)—(11), we get the following expression from (8) for the mean-field free energy as a function of two independent variables $\lambda_{1,2}$:

$$F(\lambda_1, \lambda_2) = \sum_\sigma \left\{ \sum_{\vec{k}} \left[ \frac{1}{2} h_{\sigma z}(\vec{k}) - 2k_BT \ln 2\, ch\frac{h_\sigma(\vec{k}, \lambda_\sigma)}{4k_BT} \right] + \right.$$

$$\left. + \frac{1}{2} h^2_{\sigma x}(\lambda_\sigma)\, \xi_\sigma(T, \lambda_\sigma) \right\} + W h_{1x}(\lambda_1)\, h_{2x}(\lambda_2)\, \xi_1(T, \lambda_1)\, \xi_2(T, \lambda_2), \quad (12)$$

where

$$\xi_\sigma(T, \lambda_\sigma) = \sum_{\vec{k}} h^{-1}_\sigma(\vec{k}, \lambda_\sigma)\, th\frac{h_\sigma(\vec{k}, \lambda_\sigma)}{4k_BT}. \quad (13)$$

The equations for the equilibrium values $\lambda_{1,2} = \bar{\lambda}_{1,2}$ of superconducting order parameters follow from the minimum conditions ** of free energy (12). Using definitions (4) one obtains

$$\bar{\lambda}_1 = -2W\bar{\lambda}_2 \xi_2(T, \bar{\lambda}_2),$$
$$\bar{\lambda}_2 = -2W\bar{\lambda}_1 \xi_1(T, \bar{\lambda}_1). \quad (14)$$

Hereby, in the superconducting phase ($T < T_C$ where the phase transition temperature $T_C$ has been determined by the equation $4W^2\xi_1(T_C, 0)\,\xi_2(T_C, 0) = 1$) only the solutions $\bar{\lambda}_{1,2} \neq 0$ and in the normal phase ($T > T_C$) only the solutions $\bar{\lambda}_{1,2} = 0$ correspond to the absolute minima of free energy (12).

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** For free energy (12) as a function of independent variables $\delta_1 = -2W\lambda_1$ and $\delta_2 = 2W\lambda_2$, the existence of the absolute minima corresponding to the superconducting and the normal phase has been proved in [14] where the case of the superconducting phase transition caused by interband repulsion is considered. Note, by the way, that this proof does not depend on the nature of coupling and is valid also for interband attraction.

It can be seen from (14) that in case of interband repulsion ($W>0$) the nonzero solutions of this system of equations must have opposite signs: $\operatorname{sgn} \bar{\lambda}_1 \neq \operatorname{sgn} \bar{\lambda}_2$. For the interband attraction ($W<0$), $\operatorname{sgn} \bar{\lambda}_1 = \operatorname{sgn} \bar{\lambda}_2$. These two classes of solutions, which both satisfy the minimum conditions of the free energy (12), correspond to different pseudospin configurations described by Hamiltonian (3). From (10) together with (4) at $\lambda_\sigma = \bar{\lambda}_\sigma$ it follows that in case of interband repulsion an antiferromagnetic-type arrangement of pseudospins takes place in the superconducting state, i. e. the ordered components of pseudospins ($\langle S_{\sigma \vec{k} x} \rangle_{H_0}$) in different bands are of opposite directions. On the other hand, the interband attraction causes a ferromagnetic-type ordering of pseudospins.

Consequently, the interband interaction between electrons leads to energetically stable superconducting states of different structures in dependence on the nature of this coupling. The interband repulsion induces the superconducting phase as a pseudoantiferromagnetic-type order and the interband attraction, as a pseudoferromagnetic-type order in the subspace of electron pairs.


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## TSOONIDEVAHELISE TÕUKUMISE JA TÕMBUMISE POOLT PÕHJUSTATUD ÜLIJUHTIVUSLIKE KORRASTUSTE INTERPRETATSIOONIST

Ülijuhtivuse tsoonidevaheline mudel on esitatud pseudospinnformalismis. On näidatud, et laengukandjate tsoonidevaheline vastastikmõju põhjustab sõltuvalt paardumise iseloomust erineva struktuuriga ülijuhtivuslikke seisundeid. Ülijuhtiv faas realiseerub tsoonidevahelise tõukumise korral elektronpaaride alamruumis pseudospinnide antiferromagnetilist tüüpi korrastusena, tsoonidevahelise tõmbumise juhul aga ferromagnetilist tüüpi korrastusena.

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## ОБ ИНТЕРПРЕТАЦИИ СВЕХПРОВОДЯЩИХ УПОРЯДОЧЕНИЙ, ИНДУЦИРОВАННЫХ МЕЖЗОННЫМ ПРИТЯЖЕНИЕМ И ОТТАЛКИВАНИЕМ

Межзонная модель сверхпроводимости представлена в псевдоспиновом формализме. Показано, что межзонное взаимодействие носителей приводит к сверхпроводящим состояниям различной структуры в зависимости от характера спаривания. В случае межзонного отталкивания сверхпроводящая фаза реализуется в виде антиферромагнитного типа, а в случае притяжения — в виде ферромагнитного типа упорядочения псевдоспинов в подпространстве электронных пар.