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Naum VEKSLER *

FREQUENCIES OF RESONANCE AMPLIFICATION OF THE PERIPHERAL WAVES AT SCATTERING OF AN OBLIQUELY INCIDENT PLANE ACOUSTIC WAVE BY A FINITE LENGTH CIRCULAR-CYLINDRICAL SHELL

(Presented by J. Engelbrecht)

The resonance frequencies $x_n(\alpha)$ of the peripheral waves generated in an empty shell of infinite length by an obliquely incident plane acoustic wave are found via the procedure of the resonance scattering theory. The frequencies of spatial amplification $x_m(\alpha)$ are found for the case of a finite length shell simply supported on the ends. The points of intersection of these two families of curves, $x_n(\alpha)$ and $x_m(\alpha)$, give the frequencies of the resonance amplification. At these frequencies the n-th angular resonance of the peripheral wave occurs, and exactly m wave half-lengths fit the length of the shell. With sufficient accuracy the frequencies of the resonance amplification x_{nm} can be found for every peripheral wave generated in the shell from a simple model problem on waves propagating in a plane elastic layer.

Let us consider a steady-state problem of scattering of an obliquely incident plane acoustic wave by a circular-cylindrical shell with finite length $2L$ embedded into a liquid. The shell is referred to the cylindrical coordinate system r, θ, ζ. The ζ axis coincides with the longitudinal axis of the shell. The ends of the shell are situated at $\zeta=\pm L$ and are simply supported. The shell is considered to be empty. The direction of propagation of the incident wave makes an angle α with the normal to the longitudinal axis of the shell. The frequencies of the resonance amplification should be found. We shall use the following notations: $x\equiv ka$ is the wave radius, k is the wave number in the ambient liquid, a is the outer radius of the shell, index n denotes the ordinal number of the angular resonance of the peripheral wave, and index m defines the number of half-waves of the peripheral wave on the full length $2L$ of the shell.

We propose to solve the problem in two stages. In the first stage the problem of resonance frequencies of an infinite shell should be considered while in the second stage the problem on frequencies of resonance amplification for the shell of finite length is to be solved. The resonance frequencies of the infinite shell have been investigated previously [1] and therefore here only the results of the computation are presented.

Resonance frequencies of a shell with infinite length

As it is known, the resonance frequencies of the peripheral (running) waves coincide with those of modal resonances. Here we have used the procedure of the resonance scattering theory to obtain the frequencies of modal resonances. The computation has been carried out for the case of

1 Eesti Teaduste Akadeemia Küberneetika Instituut (Institute of Cybernetics, Estonian Academy of Sciences), Akadeemia tee 21, EE0108 Tallinn, Estonia.

aluminium shell immersed in water with the following physical parameters

$$\varrho_1=2790\ \mathrm{kg/m^3},\quad c_l=6380\ \mathrm{m/s},\quad c_t=3100\ \mathrm{m/s},$$
$$\varrho=1000\ \mathrm{kg/m^3},\quad c=1470\ \mathrm{m/s},\quad h=1-b/a=1/32,\quad L/a=4. \tag{1}$$

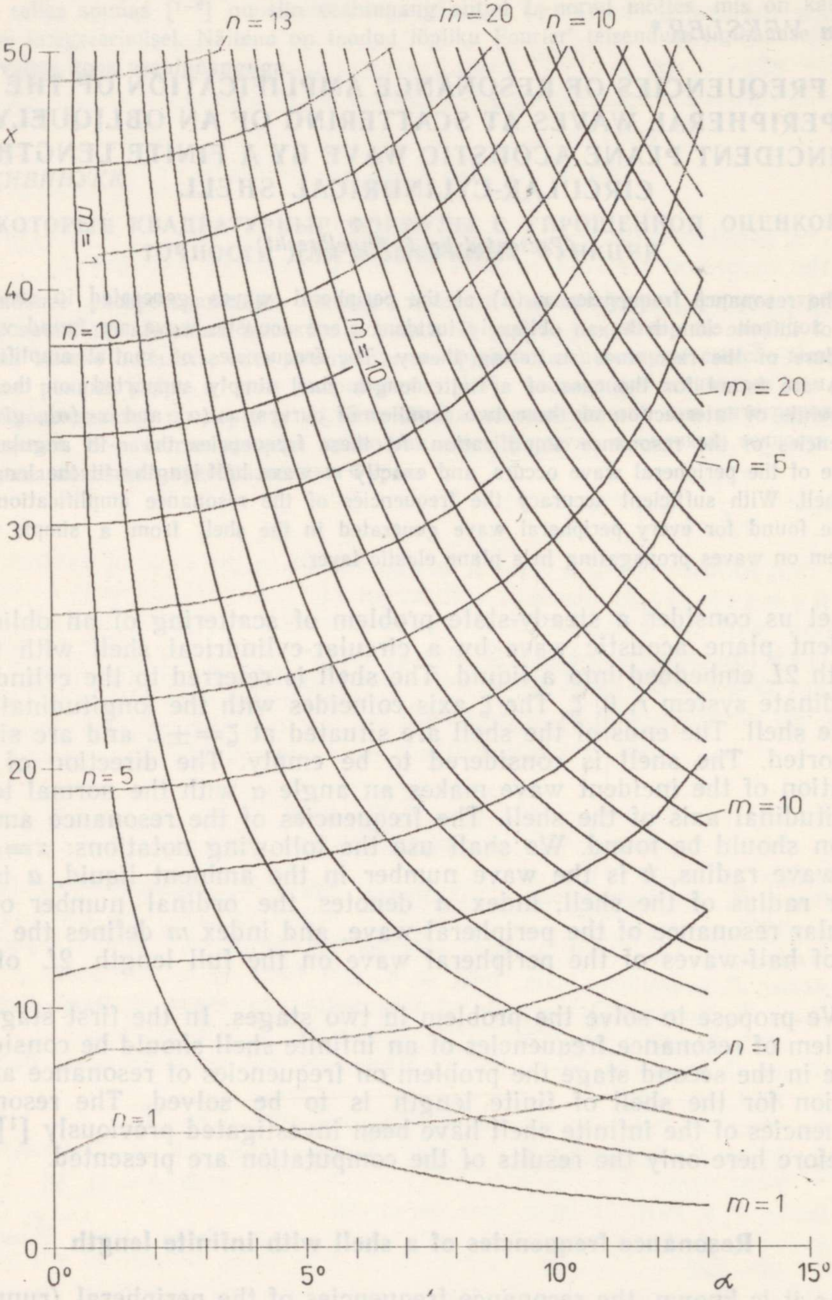

Fig. 1. **The frequencies of the resonance amplification of the Lamb-type peripheral wave S_0. The curves n=const (n=1 — 13) define the positions of the angular resonances $x_n(\alpha)$; the curves m=const (m=1 — 27) define the positions of the spatial amplification $x_m(\alpha)$; the points of intersection of both families of curves give the frequencies of the resonance amplification x_{nm}. The computation is carried out for the case of aluminium shell immersed in water (h=1/32, L/a=4).**

Here the following notations are used: ϱ_1, c_l, c_t are the density, velocities of longitudinal and transverse wave, respectively; ϱ, c are the density and sound velocity of the ambient liquid; $2L$, k, b are the full length of the shell, its relative thickness and inner radius, respectively. The frequency domain of computation is defined as

$$0 \leqslant x \leqslant 450, \quad x \equiv ka, \quad k = \omega/c, \tag{2}$$

where ω is the circular frequency. The computational step size is $l_x = 10/256$. The computations have been carried out with the same parameters as in paper [1]. In the considered frequency range the following

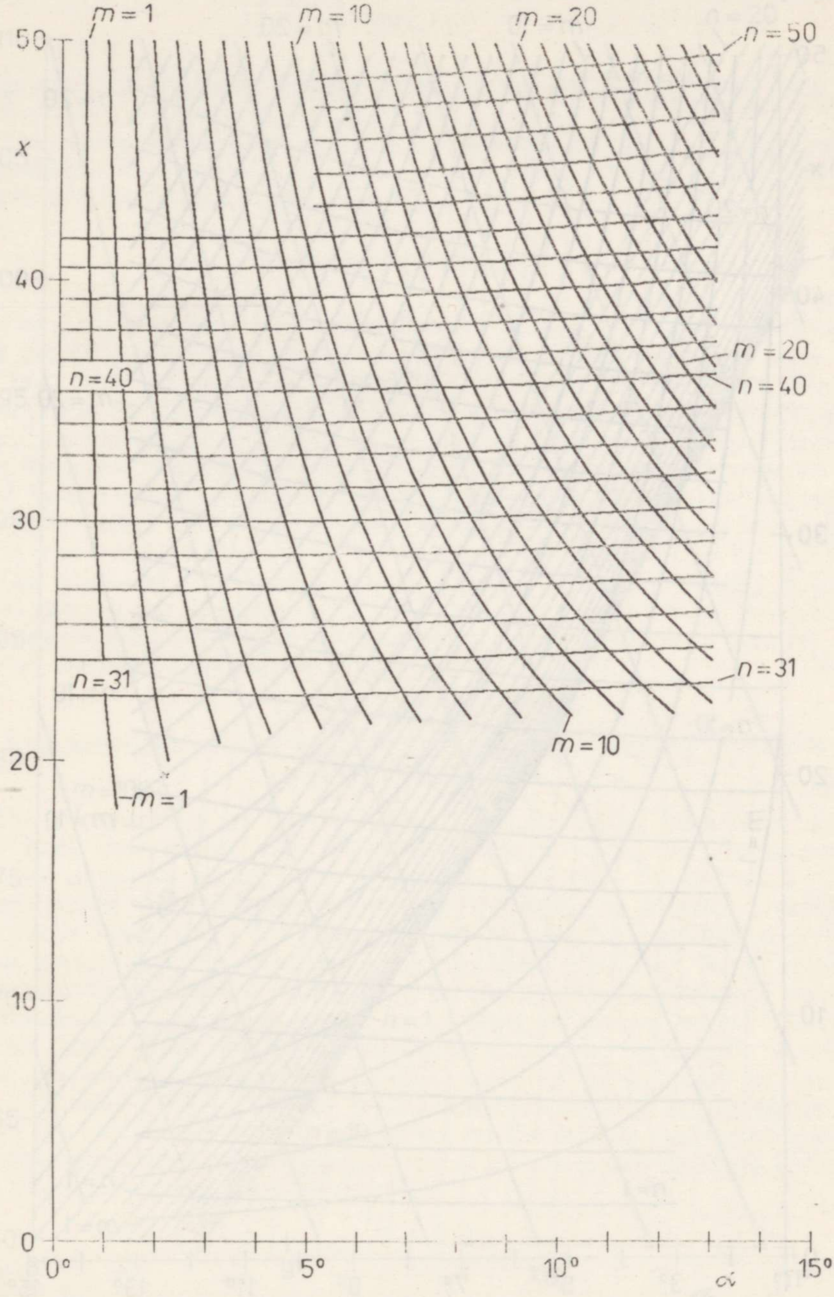

Fig. 2. The same as in Fig. 1 but for the Stoneley-type wave A ($n = 31 - 50$, $m = 1 - 27$).

peripheral waves are generated in the shell: S_0, A, A_1, T_0 and T_1. The typical results of computation are presented in Figs. 1—5 by the families of curves $x_n(\alpha)$. The computations have been carried out for such n values when the amplitudes of modal resonances of the peripheral waves are large. It should be noted that for the A wave the modal resonances with large amplitudes are generated only with $n<5$, and the amplitudes of modal resonances with $5<n<30$ are very small and the resonances may be neglected. The T_0 and T_1 waves cannot be generated for zero angle of incidence ($\alpha=0$), the relevant resonance frequencies being absent. As a rule, we have used acoustically rigid background with only one exception: with $71<n<90$ for the T_1 wave the acoustically soft

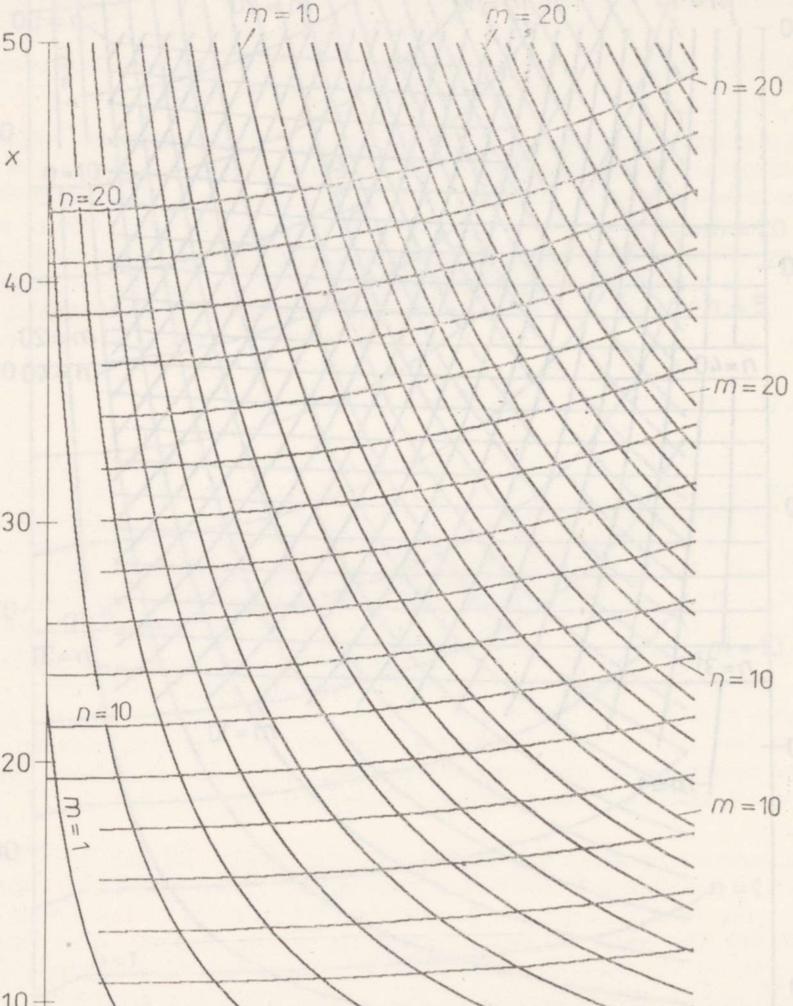

Fig. 3. The same as in Fig. 1 but for the shear wave $T_0(n=1-20,\ m=1-27)$.

background has been used. In this n domain the acoustically rigid background is not adequate: there is a minimum instead of a typical maximum on the resonance frequency.

With fixed n, we shall define the dependence of the resonance frequency on α as $x_n(\alpha)$. The resonance frequencies $x_n(\alpha)$ have been computed with one degree step on the angle of incidence. In order to obtain a continuous dependence $x_n(\alpha)$ we have used a cubic spline. A check computation of the dependence $x_n(\alpha)$ for the angles of incidence situated on half a degree values, has shown that the approximation of the curve by a spline inserts an error less than the step size l_x. Thus the error of the determination of the resonance frequency is never larger than

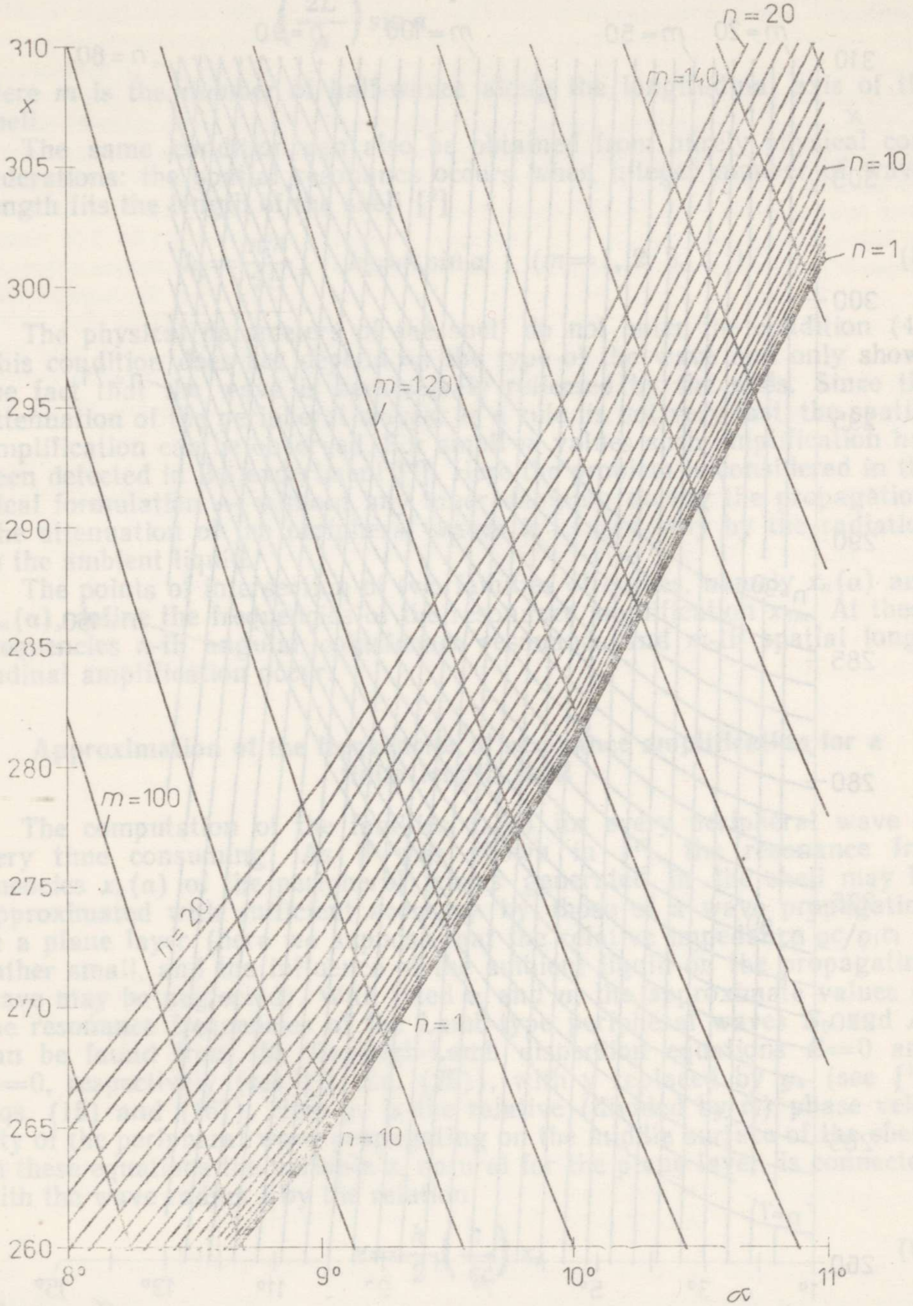

Fig. 4. The same as in Fig. 1 but for the Lamb-type wave A_1($n = 1 - 20$, $m = 95, 100, \ldots, 150$).

the step size l_x. As the presented figures only illustrate the applied procedure, we did not try to reduce the chosen l_x value.

The restriction $\alpha \leqslant 13^0$ is caused by the used algorithm of the computation. With $\alpha > \alpha_* = \arcsin(c/c_l)$ the argument of the cylindrical functions entering the elements of determinants from which the coefficient of the exact solution in the series form is found, becomes imaginary, and the used procedure just does not work. Certainly, by the modification of the algorithm, as it is proposed in [2], the computation can be continued for $\alpha > \alpha_*$. We did not use this opportunity, because with α increasing, the $x_n(\alpha)$ values grow rapidly and therefore they can fall into the x domain where the computations have not been carried out.

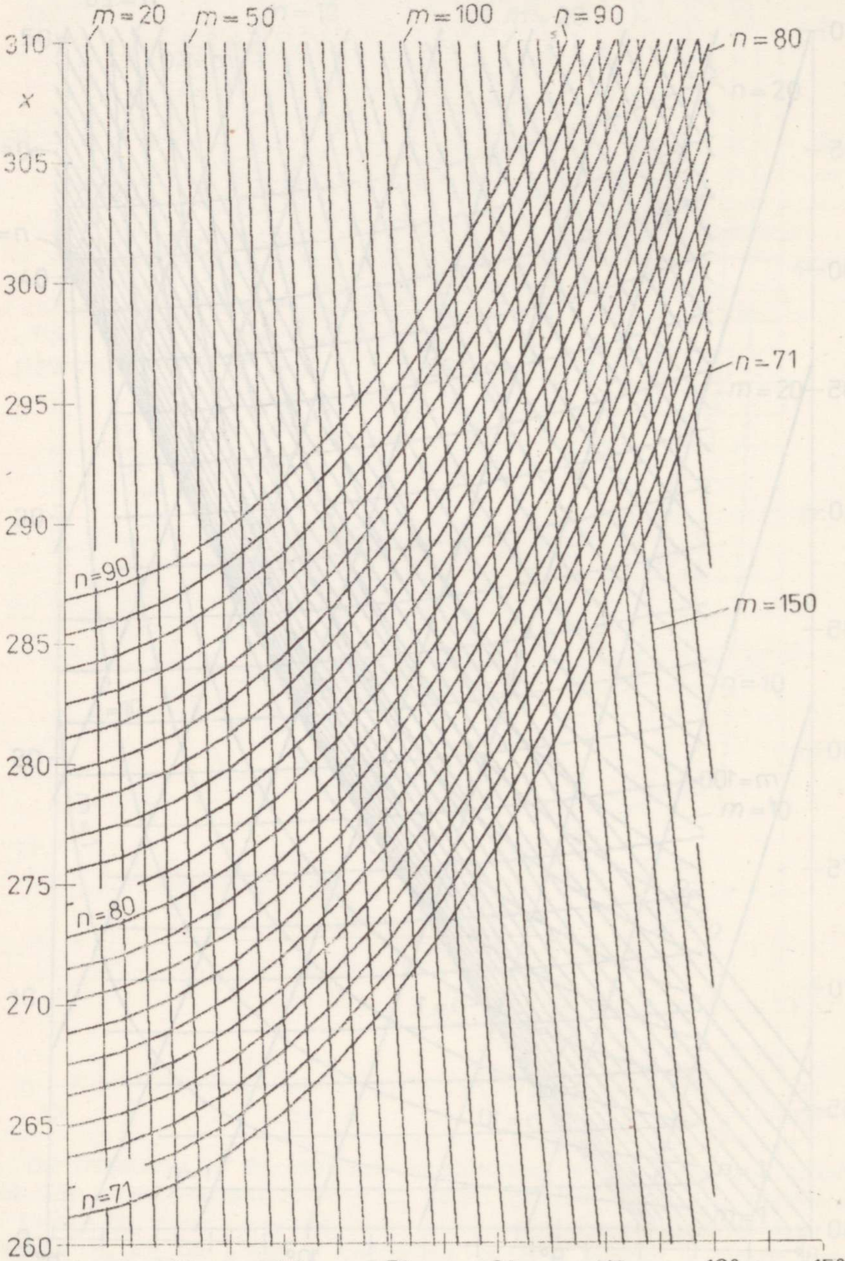

Fig. 5. The same as in Fig. 1 but for the shear wave T_1 ($n=71-90$, $m=20, 25, \ldots, 175$).

Frequencies of the resonance amplification for finite length shell

The peripheral waves generated in the shell reflect from its ends. For the case of simply supported ends, from the boundary conditions we obtain

$$\exp(i\xi k \sin\alpha)=0 \quad \text{at} \quad \zeta=\pm L. \tag{3}$$

By equating the imaginary part in Eq. (3) to zero, we obtain the condition of spatial amplification

$$x_m=\frac{\pi m}{\left(\frac{2L}{a}\right)\sin\alpha} \qquad (m=1, 2, 3, \ldots). \tag{4}$$

Here m is the number of half-waves along the longitudinal axis of the shell.

The same condition can also be obtained from purely physical considerations: the spatial resonance occurs when integer number of wavelength fits the length of the shell [2]

$$k_\xi=\frac{\pi m}{2L}, \quad k_\zeta=k\sin\alpha \qquad (m=1, 2, 3, \ldots). \tag{5}$$

The physical parameters of the shell do not enter the condition (4). This condition does not depend on the type of the wave and only shows the fact that the wave is successively reflected by the ends. Since the attenuation of the peripheral waves, as a rule, is not very fast, the spatial amplification can be observed. For small m values such amplification has been detected in the experiment [2]. Here the problem is considered in the ideal formulation — without any inner damping during the propagation. The attenuation of the peripheral waves is caused only by the radiation in the ambient liquid.

The points of intersection of two families of curves, namely $x_n(\alpha)$ and $x_m(\alpha)$, define the frequencies of the resonance amplification x_{nm}. At these frequencies n-th angular coincidence resonance and m-th spatial longitudinal amplification occur.

Approximation of the frequencies of resonance amplification for a finite length shell

The computation of the families $x_n(\alpha)$ for every peripheral wave is very time consuming. As it was shown in [1], the resonance frequencies $x_n(\alpha)$ of the peripheral waves generated in the shell may be approximated with sufficient accuracy by those of a wave propagating in a plane layer (here we suppose that the relative impedance $\varrho c/\varrho_1 c_l$ is rather small, and the influence of the ambient liquid on the propagating wave may be neglected). With fixed α and n, the approximate values of the resonance frequencies of the Lamb-type peripheral waves S_0 and A_1 can be found from the Rayleigh-Lamb dispersion equations $E=0$ and $F=0$, respectively (see [3], Eq. (29)), with y replaced by y_* (see [1], Eqs. (15) and (16)). Here y_* is the relative (divided by c_t) phase velocity of the peripheral wave propagating on the middle surface of the shell. In these equations the variable z, natural for the plane layer, is connected with the wave radius x by the relation

$$z=\frac{h}{2}\left(\frac{c}{c_t}\right)x, \tag{6}$$

where

$$z=k_t d, \quad k_t=\omega/c_t, \quad 2d=a-b. \tag{7}$$

Analogously, the approximate values of the resonance frequencies of the Stoneley-type wave A can be found from the equation $E(F+\psi)+F(E+\psi)=0$ (see [3], Eq. (28)) with the replacement y by y_*. In this case one should take $a_2=1$, which means that the phase velocity is found on the outer surface of the shell.

The approximate values of the resonance frequencies of shear waves T_0 and T_1 are defined in [3] (see Eqs. (18) and (19)).

For the waves S_0, T_0 and A, which, in principle, could be generated beginning from zero on x, the utilization of the model problem on waves propagating in plane layer inserts a significant error with small n values ($n<10$). Physically this can be explained by the fact that a long peripheral wave propagating on the shell "feels" the curvature of the shell, which is just absent in the model problem. With n increasing, the wave becomes shorter and the influence of the curvature on it gradually comes to naught.

The waves A_1 and T_1 can be generated only beginning with the cut-off frequency $z=\pi/2$. On these waves the influence of the curvature of the path is insignificant even for small n orders ($n\sim1$). Therefore for these waves the resonance frequencies obtained from the model problems on waves propagating in "dry" plane layer are very close to the exact values. The error of the approximation of the resonance frequencies of A_1 and T_1 waves diminishes with n increasing. For the T_1 wave with $1^0\leqslant\alpha\leqslant13^0$ and $71\leqslant n\leqslant90$ the error of the approximation of the resonance frequency found from the model problem is smaller than one-two step size l_x, i. e., the approximation gives the value of the resonance frequency with the accuracy of four significant digits, and therefore the approximate value practically does not differ from the exact one.

Let us denote the approximate value of the resonance frequency found from the model problem $\bar{x}_n(\alpha)$. It, as the exact one, can be computed for the angle of incidence with one—degree step. The cubic spline could be used for the intermediate values of the angle of incidence. We shall label the points of intersection of the curves $\bar{x}_n(\alpha)$ and $x_m(\alpha)$ as $\bar{x}_{nm}$. As the results of the computation have shown, for waves S_0, T_0, A_1 and T_1 and $n\geqslant10$ the difference between x_{nm} and $\bar{x}_{nm}$ does not exceed one-two step size l_x. For the A wave this difference is somewhat bigger.

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Naum VEKSLER

RINGSILINDRILISE LÕPLIKU PIKKUSEGA KOORIKU PERIFEERSETE LAINETE RESONANTSSAGEDUSTE VÕIMENDUSED KALDU LANGEVA AKUSTILISE TASANDLAINE HAJUMISEL

On kirjeldatud akustilise tasandlaine poolt genereeritud perifeersete lainete resonantssageduste võimenduse ligikaudse leidmise algoritmi. Näitena on toodud vette asetatud alumiiniumist kooriku perifeersete lainete resonantssageduste arvutus.

Наум ВЕКСЛЕР

ЧАСТОТЫ УСИЛЕНИЯ РЕЗОНАНСОВ ПЕРИФЕРИЧЕСКИХ ВОЛН ПРИ РАССЕЯНИИ НАКЛОННО ПАДАЮЩЕЙ ПЛОСКОЙ АКУСТИЧЕСКОЙ ВОЛНЫ КРУГОВОЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКОЙ КОНЕЧНОЙ ДЛИНЫ

Изложен способ отыскания частот усиления резонансов периферических волн, генерированных в оболочке плоской акустической волной. Он состоит из двух этапов. На первом этапе рассчитываются резонансные частоты бесконечно длинной оболочки. Для их отыскания используется процедура резонансной теории рассеяния. Для каждой периферической волны строится зависимость $x_n(\alpha)$ (здесь $x \equiv ka$, k — волновое число в жидкости, a — наружный радиус оболочки, α — угол падения, n — порядковый номер углового резонанса, $x_n(\alpha)$ — резонансная частота). На втором этапе рассматривается процесс переотражения периферических волн от свободно опертых торцов оболочки. Из краевых условий на торцах получается условие усиления резонансов. Физически оно соответствует ситуации, когда на полной длине оболочки укладывается целое число m полуволн каждой из периферических волн. Для оболочки конечной длины строится зависимость $x_m(\alpha)$. Точки пересечения двух семейств линий ($x_n(\alpha)$ и $x_m(\alpha)$) определяют положения резонансных частот усиления резонансов x_{nm}. На рис. 1—5 показаны эти частоты для периферических волн, возбужденных в алюминиевой оболочке, погруженной в воду. Предложен способ приближенного определения частот x_{nm}.