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MASSLESS RARITA-SCHWINGER PARTICLES AND LEPTONIC DECAYS

(Presented by H. Keres)

The effects of possible nonmaximal helicities of spin 3/2 massless particles in weak decay processes are considered. It is shown that in some processes these effects are quite strong.

In his pioneering work [1] E. P. Wigner gives a method for the classification of irreducible representations known as the little group method. Using this method, he came to the conclusion that the irreducible representations characterizing massless particles can be determined only by one (maximal) helicity. Afterwards S. Weinberg established the well-known result [2]: when the irreducible representation (k, l) of the homogeneous Lorentz group is used to describe massless states, one can have only $\lambda=\pm(k-l)$ helicities. This result prevents a massless particle from having more than one helicity but it allows nonmaximal helicities. In [3] it was shown that when the finitedimensional representations of the homogeneous Lorentz group are used, the classification of massless representations differs from the one given by Wigner. It appears that one can avoid also the Weinberg theorem and for a given representation all helicities are allowed.

The massless higher-spin particle with a full set of helicities when taking part in reactions may give quite different effects as compared to the conventional particles having only the maximal helicity. In [4,5] this problem was discussed and the effects of nonmaximal helicity states of zero mass spin 3/2 particle in some weak decay processes were found. In this paper, we continue investigating the effects of non-maximal helicities in weak decay processes. The decay processes with one or two spin 3/2 leptons among the decay products will be considered.

Let us take the leptonic decay of a polarized particle,

$$l_1\rightarrow l_2+l_3+\tilde{l}_4, \qquad (1)$$

in its rest frame as a local four-fermion current-current ([1,2]+[3,4]) process. All final particles are considered to be massless or to have a vanishing mass. We investigate the reactions with the following spin assignments of particles:

$$1/2\rightarrow 3/2+1/2+\tilde{1}/2, \qquad (2)$$

$$1/2\rightarrow 1/2+3/2+\tilde{3}/2. \qquad (3)$$

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To describe the massless spin 3/2 particle, the antisymmetric tensor (spin curl) formalism $u^{\alpha\beta}=p^{\alpha}u^{\beta}-p^{\beta}u^{\alpha}$ is used. Here $p^{\alpha}(p^{\beta})$ is the four-momentum of the particle, and $u^{\alpha}(u^{\beta})$ is the Rarita-Schwinger vector-spinor. We avoid the ordinary Rarita-Schwinger vector-spinor formalism to describe the massless spin 3/2 lepton. The difficulties in connection with using that formalism in our problem lie in the following. Let us take a massive spin 3/2 particle described by the Rarita-Schwinger vector-spinor. Suppose we want to proceed to the theory of massless spin 3/2 particle with a full set of helicities by using the mass zero limit process. However, by using this limit process we get infinite contributions. One can show that it is due to the longitudinal (helicity zero) vector contribution, $|1,0\rangle$, to the nonmaximal spin 3/2 states $|3/2,\pm1/2\rangle$. As $m\rightarrow0$, the state $|1,0\rangle$ has the components that blow up. The curl formalism removes these (infinite) longitudinal contributions [6]. In using the curl formalism we have no difference between the theory of zero mass particles with a full set of helicities and the one we get from the massive particle theory by using the mass limit process. However, there exist discontinuities between the quantities of the standard maximal helicity massless particle theory and those of the theory one gets by using the limit process mentioned above. In the following, our aim is to discuss these discontinuities in process (1). We find the energy-angular distribution of one of the final particles in process (1) with spin assignments (2) and (3) in two cases:

a) the massless spin 3/2 particles have only the maximal helicities ($\pm3/2$);

b) the massless spin 3/2 particles have all four helicity states ($\pm3/2$, $\pm1/2$).

We label the quantities in the first case by the index m (maximal helicities) and in the second case, by the index f (full set of helicities). We analyze the energy-angular distributions and compare the total decay rates and asymmetry parameters of both cases. First we consider case (2). We use the V, A currents, thus we take the decay amplitude

$$M=GJ^{\alpha}_{1}J^{+}_{2\alpha} \tag{4}$$

with

$$J^{\alpha}_{1}=\bar{u}_3\gamma^{\alpha}(I-\gamma_5)u_4 \tag{4a}$$

as an ordinary V—A current and

$$J_{2\alpha}=\frac{1}{m^2}\bar{u}_{2\alpha\beta}(p_2)(p_1-p_2)^{\beta}(I-a\gamma_5)u_1(p_1) \tag{4b}$$

as one of the simplest V, A currents with the spin 3/2 curl particle.

The simplest V, A current with the zero mass spin 3/2 curl particle, $\bar{u}_{\alpha\beta}\gamma^{\beta}(I-\gamma_5)u$, is equal to zero due to the supplementary condition

$$\gamma_{\alpha}u^{\alpha}=0. \tag{5}$$

Now, the general form of the one-particle energy-angular distribution in the decaying particle rest frame is

$$\frac{d\Gamma}{dx\,d(\cos\theta)}=B[f_0(x,a)+\vec{k}\vec{\xi}f_1(x,a)], \tag{6}$$

where $\vec{\xi}$ is the initial particle polarization vector; $\vec{k}$ — the unit vector along the direction of the motion of the distribution particle; θ — the angle between $\vec{k}$ and $\vec{\xi}$ and the parameter a is from Eq. (4b). We define the energy variable as $x=\frac{E}{m}$ (E — kinetic energy of the final particle, m — the mass of the initial particle). The constant B and the invariant distribution functions in case of the final particle l_3 in (6) are found for currents (4a) and (4b) to be

$$B=\frac{2G^2m^5}{(2\pi)^3}, \tag{7}$$

$$\begin{aligned} f_0^m(x,-1) &= \frac{x^4}{15}(5-4x), \\ f_1^m(x,-1) &= \frac{x^4}{15}(1+4x), \\ f_0^m(x,1) &= 4x^2\left(\frac{1}{8}-\frac{2}{3}x+\frac{7}{6}x^2-\frac{2}{3}x^3\right), \\ f_1^m(x,1) &= 4x^2\left(\frac{1}{24}-\frac{1}{3}x+\frac{5}{6}x^2-\frac{2}{3}x^3\right); \end{aligned} \tag{8}$$

$$\begin{aligned} f_0^f(x,-1) &= f_0^m(x,-1)+\frac{1}{3}f_0^m(x,1), \\ f_1^f(x,-1) &= f_1^m(x,-1)+\frac{1}{3}f_1^m(x,1), \\ f_0^f(x,1) &= f_0^m(x,1)+\frac{1}{3}f_0^m(x,-1), \\ f_1^f(x,1) &= f_1^m(x,1)+\frac{1}{3}f_1^m(x,-1). \end{aligned} \tag{9}$$

One can show that due to the spin conservation the decay process with spin 3/2 maximal helicity particles has to be suppressed near the origin of l_3 energy spectrum if $a=-1$ and near the endpoint if $a=1$. The decay process with the spin 3/2 particle with all four helicity states does not share these features of distribution. One can see it very well in Figs. 1 and 2. The total decay rates in two different cases are related as

$$\frac{\Gamma_f}{\Gamma_m}=\frac{4}{3}. \tag{10}$$

If we define the asymmetry as

$$A(\theta)=\frac{\int\limits_0^{1/2} dx\left[\frac{d\Gamma}{dx\,d(\cos\theta)}(\uparrow)-\frac{d\Gamma}{dx\,d(\cos\theta)}(\downarrow)\right]}{\int\limits_0^{1/2} dx\left[\frac{d\Gamma}{dx\,d(\cos\theta)}(\uparrow)+\frac{d\Gamma}{dx\,d(\cos\theta)}(\downarrow)\right]}, \tag{11}$$

where ($\uparrow$) indicates a spin-up initial particle and ($\downarrow$), a spin-down initial particle, then in our cases

$$A_m(\theta, a=-1)=\frac{4}{5}\cos\theta, \tag{12}$$

$$A_f(\theta, a=-1)=\frac{3}{5}\cos\theta, \tag{13}$$

$$A_m(\theta, a=1)=0, \tag{14}$$

$$A_f(\theta, a=1)=\frac{1}{5}\cos\theta. \tag{15}$$

The fact that asymmetry is equal to zero in Eq. (14) does not indicate the parity conservation in the decay process. One can see in Fig. 2 that parity is nonconserved, except in one point in which $f_1^m(x, 1)=0$. Looking at Fig. 2, it becomes clear why the global asymmetry is equal to zero.

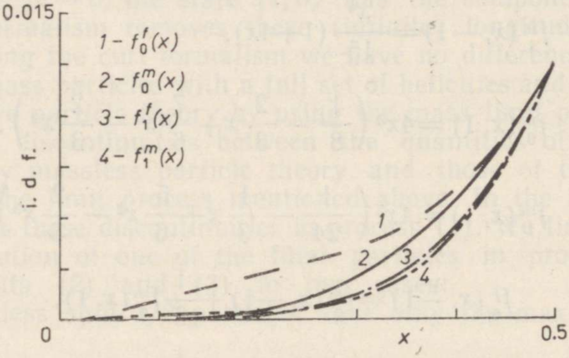

Fig. 1. Invariant distribution functions Eqs. (8), (9) in the case of $a=-1$.

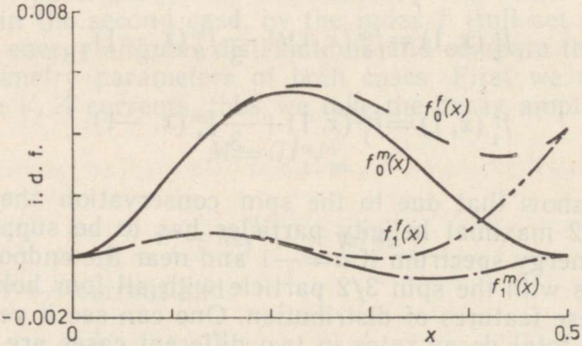

Fig. 2. Invariant distribution functions Eqs. (8), (9) in the case of $a=1$.

Now let us investigate case (3). Here we choose first decay process amplitude (5) with the currents

$$J_1^{\lambda}=\frac{1}{m^2}\,\bar{u}_3^{\alpha\beta}\gamma^{\lambda}(I-\gamma_5)\,u_{4\alpha\beta}, \tag{16}$$

$$J_{2\lambda}^{+}=\bar{u}_2\gamma_{\lambda}(I-\gamma_5)\,u_1. \tag{17}$$

Current (16) with spin 3/2 massless curl particles is equal to zero if the particles have only maximal helicities [7]. So, it is only the case of a full set of helicities that gives nonzero decay rate of process (2) with this current. In this case the calculations give for l_3 particle distribution (6) with

$$B=\frac{8G^2m^5}{3(2\pi)^3} \tag{18}$$

and the invariant functions connected with the ones in Eq. (8) as

$$f_0^f(x) = f_0^m(x, -1),$$
$$f_1^f(x) = -f_1^m(x, -1). \tag{19}$$

As a last example we consider process (2) with scalar-pseudoscalar currents. We take

$$M = GJ_1 J_2^+ \tag{20}$$

with

$$J_1 = \frac{1}{m^2} \bar{u}_{3\alpha\beta}(I \pm \gamma_5) u_4^{\alpha\beta}, \tag{21}$$

$$J_2^+ = \bar{u}_2(I \mp \gamma_5) u_1 \tag{22}$$

which gives distribution (6) for the l_3 final lepton with

$$B = \frac{1}{5} \frac{G^2 m^5}{(2\pi)^3}, \tag{23}$$

$$f_0(x) = f_0^m(x) = \frac{9}{10} f_0^f(x) = x^4(5 - 8x), \tag{24}$$

$$f_1(x) = f_1^m(x) = \frac{9}{10} f_1^f(x) = \pm x^4(3 - 8x), \tag{25}$$

where the upper and lower signs are taken in accordance with the ones in (21) and (22). Hence, the distributions of l_3 particles in two cases are identical here. The only difference lies in the total decay rates. As one can find from Eqs. (24) and (25),

$$\frac{\Gamma_f}{\Gamma_m} = \frac{10}{9}. \tag{26}$$

The asymmetry given by invariant distribution functions (24) and (25) is found to be

$$A(\theta) = \mp \frac{1}{5} \cos\theta. \tag{27}$$

Naturally, the distribution of any other final particles (l_2 or $\tilde{l}_4$) is also identical in the two cases. The calculations give for l_2 energy-angular distribution

$$\frac{d\Gamma}{dx\, d(\cos\theta)} = C \frac{G^2 m^5}{(2\pi)^3} x^2(1 - 6x + 12x^2 - 8x^3)(1 \pm \vec{k}\vec{\xi}) \tag{28}$$

with $C = 1$ in the case of maximal helicity spin 3/2 particles and $C = \frac{10}{9}$ in the case of spin 3/2 particles with all possible helicities. Since $f_0(x) = \pm f_1(x)$, it follows immediately from Eq. (11) that

$$A(\theta) = \pm \cos\theta. \tag{29}$$

In conclusion one can say that in the weak decay processes considered above the effects of nonmaximal helicity states of spin 3/2 leptons are quite strong. Besides, there are theoretical and aesthetical motivations to justify a systematic development of the theories of massless higher-spin particles with a full set of helicities and the search for their consequences.

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MASSITUD RARITA-SCHWINGERI OSAKESED JA LEPTONLAGUNEMISED

On vaadeldud kõigi võimalike spiraalsustega massituid Rarita-Schwingeri osakesi nõrkades leptonlagunemise protsessides. On uuritud efekte, mis tulenevad mittemaksimaalsete spiraalsuste olemasolust, ja näidatud, et mõnedes protsessides on need efektid üsna suured.

Ильмар ОТС, Рейн-Карл ЛОЙДЕ, Рейн СААР

БЕЗМАССОВЫЕ ЧАСТИЦЫ РАРИТЫ-ШВИНГЕРА И ЛЕПТОННЫЕ РАСПАДЫ

Обсужден вопрос об участии безмассовых частиц со спином 3/2 и со всеми спиральностями в процессах слабого лептонного распада. Исследованы эффекты немаксимальных спиральностей в этих распадах и показано, что в некоторых случаях эти эффекты довольно ощутимы.