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RETRIEVAL OF SCATTERING PHASE FUNCTION FROM SKY BRIGHTNESS DISTRIBUTION DATA

(Presented by G. Vainikko)

The system of linear equations is derived from the equation of radiation transfer of the atmosphere. The coefficients of the system of equations depend on sky brightness distribution and the solutions of this system appear to be the scattering phase function values at various scattering angles. The test results of this algorithm are presented.

1. Introduction

The form of the scattering phase function of the atmosphere (SPF), i.e. the angular distribution of scattered light, is influenced by atmospheric aerosols. The size distribution of them can be assessed according to Mie theory [1] by deriving the actual phase function from experimental data and taking advantage of some reasonable aerosol size distribution model.

Box, Deepak [2], Deepak, Adams [3] and Deepak et al. [4] as well as Thomalla, Quenzel [5] have successfully developed the solar aureole methods in order to determine the SPF (at scattering angles $\theta < 20^\circ$) and aerosol size distribution. The problem of taking into account the effect of multiple scattering (MS) has been one of the biggest in these methods. These techniques are based on the single scattering (SS) theory of the solar aureole. The MS on molecules is included as an empirical approximation.

The present paper is based on the earlier paper of [6]. We give a method of calculating the SPF values at scattering angles $\theta > 10^\circ$ from all-sky brightness distribution data, taking into account the effects of SS and MS. In our theory we take advantage of the plain-parallel model of atmosphere and the equation of radiation transfer (ERT). From this formula we derive a system of linear equations with coefficients depending on brightnesses of sky points. Our system contains the information about SS and MS effects as far as the ERT covers both of them. The solutions of this system of equations appear to be the values of SPF at different scattering angles.

We have got experimental sky brightness data by taking all-sky photos at various wavelengths and digitizing the film by means of microdensitometer.

We have tested our algorithm, using data obtained at Equatorial Atlantic and within the region of the Canary Islands.

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2. Theory

1. General expression. Consider the model of locally plain-parallel horizontally homogenous atmosphere which scatters the externally incident radiation without change of wavelength and contains no internal emitters of radiation. We use the well-known ERT in extended scattering media:

$$\mu \frac{\partial I(\tau, \Omega)}{\partial \tau} + I(\tau, \Omega) = \frac{\lambda}{4\pi} \int_S G(\Omega \cdot \Omega') I(\tau, \Omega') d\Omega' + \\ + \frac{\lambda}{4\pi} F e^{-\tau/\mu_0} G(\Omega \cdot \Omega_0), \quad (1)$$

where F is the solar constant, $I(\tau, \Omega)$ is the intensity of radiation at optical thickness τ and coming from direction $\Omega = (\mu, \varphi)$. Here μ is cosine of zenith angle and φ is azimuth. Integral is taken over unit sphere S . $\Omega_0 = (\mu_0, \varphi_0)$ is the direction of the sun, λ — the single scattering albedo. $G(\Omega \cdot \Omega')$ is the SPF with normalization condition

$$\frac{1}{4\pi} \int_S G(\Omega \cdot \Omega') d\Omega' = 1. \quad (2)$$

Sign \cdot denotes scalar multiplication, i. e.

$$\Omega \cdot \Omega' = \mu \mu' + [(1 - \mu^2)(1 - \mu'^2)]^{1/2} \cos(\varphi - \varphi'). \quad (3)$$

We also define $g(\Omega \cdot \Omega_0) = \lambda G(\Omega \cdot \Omega_0)$; $\lambda \approx 1$, if absorption is negligible. We may rewrite (1) in the form

$$\mu \frac{\partial I(\tau, \Omega)}{\partial \tau} + I(\tau, \Omega) = B(\tau, \Omega) + \frac{F}{4\pi} \exp(-\tau/\mu_0) g(\Omega \cdot \Omega_0), \quad (4)$$

where

$$B(\tau, \Omega) = \frac{1}{4\pi} \int_S g(\Omega \cdot \Omega') I(\tau, \Omega') d\Omega'. \quad (5)$$

The solution of Eq. (4) is

$$I(\tau, \Omega) = \frac{1}{\mu} \int_0^\tau \exp[(s - \tau)/\mu] B(s, \Omega) ds + \\ + \frac{F}{4\pi \mu} \int_0^\tau \exp[(s - \tau)/\mu] \exp(-s/\mu_0) g(\Omega \cdot \Omega_0) ds = \\ = I_\infty(\tau, \Omega) + I_1(\tau, \Omega), \quad (6)$$

where ∞ denotes the MS and 1 the SS part of solution.

Before rewriting I_∞ we take into account that we have got experimental data by taking all-sky photos. Therefore, the directions $\Omega(\mu, \varphi)$ of interest are confined into a part of the upper hemisphere $D = \{(\mu, \varphi) : \delta \leq \mu \leq 1; 0 \leq \varphi \leq 2\pi\}$. Here δ is determined by the field of view of photographic system. The MS part of radiation coming at direction $\Omega \in D$ is given by

$$B_D(\tau, \Omega) = \frac{1}{4\pi} \int_D g(\Omega \cdot \Omega') I(\tau, \Omega') d\Omega'. \quad (7)$$

We assume

$$I_\infty(\tau, \Omega) \approx \frac{1}{\mu} \int_0^\tau \exp[(s - \tau)/\mu] B_D(s, \Omega) ds. \quad (8)$$

As $B_D(0, \Omega) = 0$, we can define B_D being linear function of s : $B_D(s, \Omega) \approx ks$, $k = B_D(\tau)/\tau$ and $s \in [0, \tau]$. Now, the MS part of the expression (6) is given by

$$I_\infty(\tau, \Omega) \approx RB_D(\tau, \Omega), \quad (9)$$

where

$$R(\mu) = 1 - \frac{\mu}{\tau} [1 - \exp(-\tau/\mu)]. \quad (10)$$

The sky point brightness is now given by

$$\begin{aligned} I(\tau, \Omega) &\approx RB_D(\tau, \Omega) + \\ &+ \frac{F}{4\pi\mu} \int_0^\tau \exp[(s - \tau)/\mu] \exp(-s/\mu_0) g(\Omega \cdot \Omega_0) ds. \end{aligned} \quad (11)$$

We divide both sides of Eq. (11) by $\exp(-\tau/\mu_0)F$ and define a dimensionless quantity:

$$J(\tau, \Omega) = \frac{I(\tau, \Omega)}{\exp(-\tau/\mu_0)F}. \quad (12)$$

Thus $J(\tau, \Omega)$ is the observed sky point brightness normalized by direct sunlight irradiation. The both latter quantities can therefore be measured in relative units in order to get $J(\tau, \Omega)$. We can rewrite Eq. (6) in terms of J :

$$J(\tau, \Omega) \approx J_\infty(\tau, \Omega) + J_1(\tau, \Omega), \quad (13)$$

where

$$J_\infty(\tau, \Omega) = \frac{R}{4\pi} \int_D g(\Omega \cdot \Omega') J(\tau, \Omega') d\Omega'. \quad (14)$$

J_1 can be expressed in the form

$$J_1(\tau, \Omega) = \frac{C}{4\pi} g(\Omega \cdot \Omega_0), \quad (15)$$

where

$$C(\mu) = \begin{cases} \frac{\mu_0}{\mu_0 - \mu} \left[1 - \exp \left(\frac{\tau(\mu - \mu_0)}{\mu\mu_0} \right) \right], & \mu \neq \mu_0, \\ \frac{\tau}{\mu_0}, & \mu = \mu_0. \end{cases} \quad (16)$$

We have got our main expression by putting (13)–(16) together:

$$J(\tau, \Omega) \approx \frac{R}{4\pi} \int_D g(\Omega \cdot \Omega') J(\tau, \Omega') d\Omega' + \frac{C}{4\pi} g(\Omega \cdot \Omega_0), \quad (17)$$

$$\Omega \in D,$$

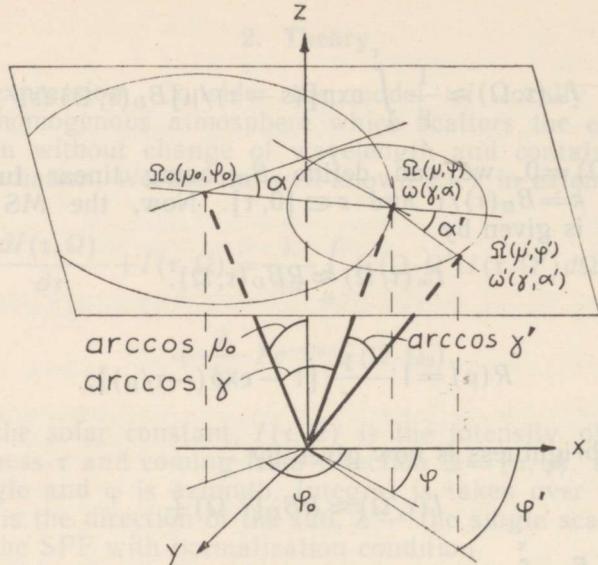


Fig. 1. Mutual geometry of the systems of coordinates (μ, φ) and (γ, α) .

2. Integral equation. Our purpose is to find SPF as a function of cosine of scattering angle with normalization condition

$$\frac{1}{2} \int_S g(\gamma) d\gamma = 1, \quad (18)$$

$$\gamma \in [\beta, 1], \quad \beta = \mu_0 \delta - [(1 - \mu_0^2)(1 - \delta^2)]^{1/2}.$$

Using the operator S , defined by Avaste, Knyazikhin [6], we change the system of coordinates as follows: $\Omega = S(\Omega_0)\omega$, $\Omega' = S(\Omega)\omega'$, where $\omega = (\gamma, \alpha)$ and $\omega' = (\gamma', \alpha')$ (see also Fig. 1). Now the main Eq. (17) is given by

$$f(\tau, \gamma, \alpha) = \frac{R}{4\pi} \int_{\beta}^1 K(\gamma, \gamma', \alpha, \tau) g(\gamma') d\gamma' + \frac{C}{4\pi} g(\gamma), \quad (19)$$

where

$$f(\tau, \gamma, \alpha) = J(\tau, S(\Omega_0)\omega) \quad (20)$$

and

$$K(\gamma, \gamma', \alpha, \tau) = \int_0^{2\pi} J(\tau, S(\Omega)\omega') d\alpha'. \quad (21)$$

We obtain our integral equation as integral of $f(\tau, \gamma, \alpha)$ over $\alpha \in [0, 2\pi]$:

$$y(\tau, \gamma) = \frac{1}{4\pi} \int_{\beta}^1 P(\gamma, \gamma') g(\gamma') d\gamma' + \frac{1}{4\pi} d(\gamma) g(\gamma), \quad (22)$$

where

$$y(\tau, \gamma) = \int_0^{2\pi} f(\tau, \gamma, \alpha) d\alpha, \quad (23)$$

$$P(\gamma, \gamma') = \int_0^{2\pi} K(\gamma, \gamma', \alpha, \tau) R(\mu) d\alpha \quad (24)$$

and

$$d(\gamma) = \int_{\psi_2}^{\psi_1} C(\mu) d\alpha, \quad (25)$$

where

$$\psi_1(\gamma) = \begin{cases} \arccos \left(\frac{\mu_0 \gamma - \delta}{[(1 - \mu_0^2)(1 - \gamma^2)]^{1/2}} \right) & \mu_0 \delta - [(1 - \mu_0^2)(1 - \delta^2)]^{1/2} \leq \gamma \leq \\ & \leq \mu_0 \delta + [(1 - \mu_0^2)(1 - \delta^2)]^{1/2}, \\ 0, & \mu_0 \delta + [(1 - \mu_0^2)(1 - \delta^2)]^{1/2} \leq \gamma \leq 1, \end{cases} \quad (26)$$

$$\psi_2(\gamma) = 2\pi - \psi_1,$$

and

$$\mu(\gamma, \alpha) = \mu_0 \gamma - [(1 - \mu_0^2)(1 - \gamma^2)]^{1/2} \cos \alpha.$$

3. The system of linear equations. In order to calculate the discrete SPF values $g_i = g(\gamma_i)$ ($i=1-n$) by computer, we use the method of discrete ordinates and rearrange the integral Equation (22) into the system of linear equations with g_i ($i=1-n$) being unknown quantities:

$$y(\tau, \gamma_i) = \frac{1}{4\pi} \sum_{j=1}^n P(\gamma_i, \gamma'_j) g_j \Delta \gamma'_j + d(\gamma_i) g_i, \quad (27)$$

$$i=1-n.$$

Here

$$y(\tau, \gamma_i) \approx \sum_{s=1}^{m_t} J(\tau, \alpha_s, \gamma_i) \Delta \alpha_i, \quad (28)$$

$$P(\gamma_i, \gamma'_j) = \sum_{s=1}^{m_t} R(\alpha_s, \gamma_i) \left(\sum_{k=1}^{m_j} J(\tau, \alpha'_k, \gamma'_j) \Delta \alpha'_j \right) \Delta \alpha_i, \quad (29)$$

$$d(\gamma_i) = \sum_{s=1}^{m_t} C(\alpha_s, \gamma_i) \Delta \alpha_i, \quad (30)$$

$$i, j = 1-n.$$

$\Delta \gamma'$, $\Delta \alpha$ and $\Delta \alpha'$ are the weights while γ' , α and α' are the nodes of quadrature formulas. Relation between coordinates (μ, φ) and (γ, α) is given in appendix.

Experimental data can be used to calculate $y(\tau, \gamma_i)$, $P(\gamma_i, \gamma'_j)$, $d(\gamma_i)$, and g_i ($i=1-n$) can be found from the system of algebraic equations (27). Note that in case $P(\gamma_i, \gamma'_j) = 0$ ($i, j=1-n$) (27) becomes the SS approximation.

We have calculated the integrals (28), (29), (30) using trapezoidal summation rule. Some efforts have been made to choose the best function $N=f(\gamma)$, where N is the number of nodes of the quadrature formula and γ is cosine of scattering angle. Using the results of Vainikko et al. [7], the sky brightnesses $I(\tau, \mu, \mu_0)$ in case $g(\gamma)=\text{const}$ were calculated. The optimal relation $N=f(\gamma)$ was chosen by minimizing the variance of $g(\gamma_i)$, $i=1-n$, computed by our method from generated $I(\tau, \mu, \mu_0)$ data. However, additional investigation of this minimizing technique is needed.

3. Experimental data

Experimental data have been obtained by means of photographic photometry techniques described by Deepak, Adams [3] and Avaste, Reinart [8].

The process consisted of several steps:

1) The all-sky photos at wavelengths, where absorption is negligible, were taken on the 35 mm black-and-white film through the systems of lenses PENTACON-auto and ALBINAR. The system had the angle of view 140° . Monochrome interference filters were mounted into the optical system to the place where the deviation of the rays from the optical axis did not exceed 12° . The occultation of the sun was done by neutral density filter on the support rod.

2) The optical system was tested in order to take into account its angular characteristics later at data processing.

3) Densitometry of the image on the film was performed by microdensitometer «Perkin Elmer», which provides a digital magnetic tape output. The data files consist of optical densities of 50×50 pixels per image digitalized with steps $\Delta x = \Delta y = 500 \mu\text{m}$ and spot $50 \times 400 \mu\text{m}$.

4) Each film was tested by exposing it through step density tablet in order to get the $\log E = f(D)$ plot, where D is optical density of the film, $E = BT$ is exposure, B is intensity of incident light, and T the exposure time.

5) The solar irradiation was obtained in the same manner as the sky-point brightness. The sun was photographed through the diffuse filter and through the same optical system used at step 1. The film was densitometried.

6) The normalized intensity for the i -th pixel J_i (i.e. our input data) was expressed in the form

$$J_i = \frac{K_i B_i T}{M B_s T} = \frac{K_i}{M} 10^{f(D_i) - f(D_s)},$$

where B_s is the intensity of the direct sunlight coming through the diffuse filter, B_i is the brightness of the i -th sky point, M is the extinction coefficient of the diffuse filter, D_i and D_s are the optical densities of the i -th sky image pixel and the image of the sun accordingly. The coefficients K_i were obtained at step 2.

4. Test results

We have used the data obtained at Equatorial Atlantic and near the Canary Islands. The former is the region with Saharan dust aerosol and the latter with marine aerosol prevailing in the atmosphere. We have tested the dependence of the computed SPF on wavelength (Fig. 2) and on atmosphere content (Fig. 3). The magnitude of MS at both regions can also be compared in Fig. 3.

The SPF-s in Fig. 2. were calculated using Equatorial Atlantic data. The amount of light scattered forward increases with increasing wavelength within the interval $\lambda = 475 - 625 \text{ nm}$.

The results in Fig. 3 correspond to integral ($\lambda = 350 - 600 \text{ nm}$) data in both regions. Curves 1a and 2a are obtained in case $P(\gamma_i, \gamma'_j) = 0$ (see section 3) i.e. without taking into account the effect of MS. Compare the curves 1a and 1b with 2a and 2b. It is evident that the process of MS is much more important at Equatorial Atlantic (curves 1a, 1b) than near the Canary Islands (curves 2a, 2b) and that the forward scattering ability of marine aerosols is bigger than that of dust aerosols.

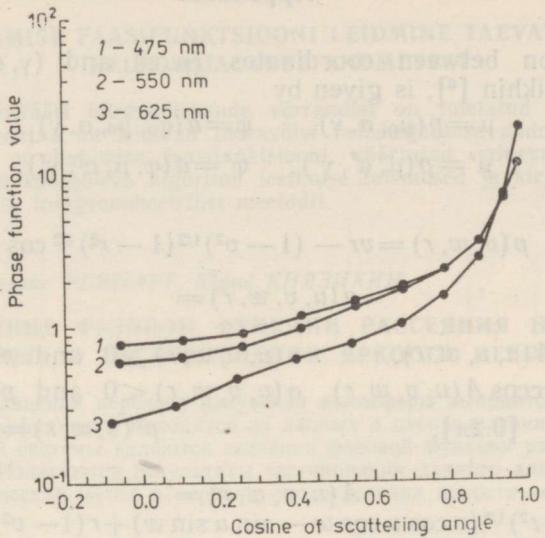


Fig. 2. Spectral scattering phase functions obtained at Equatorial Atlantic.

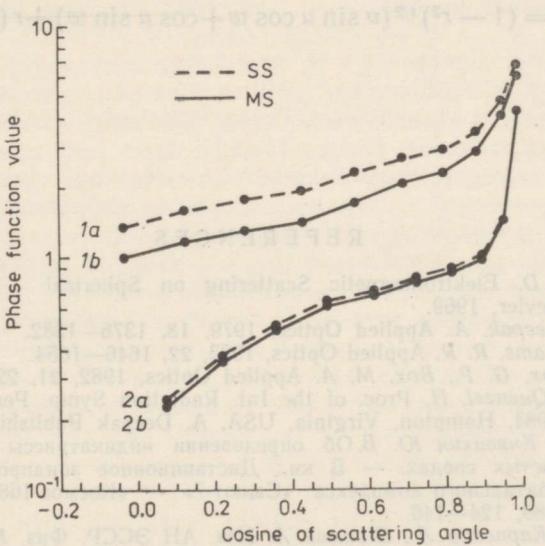


Fig. 3. Integral ($\lambda = 350 - 600$ nm) scattering phase functions obtained at Equatorial Atlantic (1a, 1b) and in the region of the Canary Islands (2a, 2b).

5. Concluding remarks

The results presented here show that the SPF of the atmosphere can be retrieved from sky brightness data by our all-sky technique.

However, a number of problems need to be solved prior to using the SPF-s calculated by this method:

a) Avaste, Knyazikhin [9] show that arbitrary SPF algorithm gives acceptable results $g(\gamma)$ inside some scattering angle interval $\arccos(\gamma_1) \leq \arccos(\gamma) \leq \arccos(\gamma_2)$. The SPF values may be of the same magnitude as measuring errors outside this interval.

b) While photographing the sky, the filter on the support rod, the clouds and other objects yield abrupts in the sky brightness data. This causes deviation of the SPF shape from that of the real one:

Appendix

The relation between coordinates (μ, φ) and (v, a) , according to Avaste, Knyazikhin [6], is given by

$$\mu = p(\mu_0, a, \gamma), \quad \varphi = a(\varphi_0, \mu_0, a, \gamma),$$

$$\mu' = p(\mu, a', \gamma'), \quad \varphi' = a(\varphi, \mu, a', \gamma'),$$

where

$$p(v, w, r) = vr - (1 - v^2)^{1/2}(1 - r^2)^{1/2} \cos w,$$

$$a(u, v, w, r) =$$

$$= \begin{cases} \arccos A(u, v, w, r), & \sigma(u, v, w, r) \geq 0 \text{ and } p^2(v, w, r) \neq 1, \\ 2\pi - \arccos A(u, v, w, r), & \sigma(u, v, w, r) < 0 \text{ and } p^2(v, w, r) \neq 1, \\ [0.2\pi], & p^2(v, w, r) = 1, \end{cases}$$

$$A(u, v, w, r) =$$

$$= \frac{(1 - r^2)^{1/2}(v \cos u \cos w - \sin u \sin w) + r(1 - v^2)^{1/2} \cos u}{(1 - p^2(v, w, r))^{1/2}}$$

$$\sigma(u, v, w, r) = (1 - r^2)^{1/2}(v \sin u \cos w + \cos u \sin w) + r(1 - v^2)^{1/2} \sin u.$$

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HAJUMISE FAASIFUNKTSIOONI LEIDMINE TAEVAVÖLVI HELEDUSJAOTUSE ANDMETEST

Lähtudes atmosfääri kiirgusülekande võrrandist on tuletatud lineaarne võrrandi-süsteem, mille kordajad on leitavad taevavölvि heledusjaotuse andmetest. Võrrandisüs-teemi lahenditeks on hajumise faasifunktsooni väärtsused erinevate hajumisnurkade korral. On esitatud kõnesoleva algoritmi testimise tulemused ja kirjeldatud taeva hele-dusjaotuse mõõtmise fotogrammeetristilist meetodit.

Маргус РОЛЛ, Мээлис РЕЙНАРТ, Юрий КНЯЗИХИН

ВЫЧИСЛЕНИЕ ФАЗОВОЙ ФУНКЦИИ РАССЕЯНИЯ НА ОСНОВЕ ДАННЫХ О РАСПРЕДЕЛЕНИИ ЯРКОСТИ НЕБОСВОДА

Исходя из уравнения переноса излучения атмосферы выводится линейная система уравнений. Ее коэффициенты находятся из данных о распределении яркости небосвода. Решениями данной системы являются значения фазовой функции рассеяния при разных углах рассеяния. Излагаются результаты тестирования данного алгоритма. Описывается фотограмметрический метод измерения распределения яркости небосвода.

Рис. 1. Двумерное сечение фазовых скоростей гиперболических волн — линий Френеля в конфокальной параллаксии. Видно, что для каждого из трех изображенных волн в конфокальной параллаксии имеется одна линия Френеля, соответствующая определенному значению фазовой скорости.