Proc. Estonian Acad. Sci. Phys. Math., 1991, 40, N 4, 258—265
https://doi.org/10.3176/phys.math.1991.4.02

УДК 519.632.8

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# ON THE IDENTIFICATION OF THE COEFFICIENT OF PERMEABILITY BY MEANS OF METHOD OF CHARACTERISTICS

*(Presented by G. Vainikko)*

The method of characteristics for determining the coefficient of permeability in the steady state model of groundwater flow is discussed. Conditions are obtained for the asymptotic stability of the solutions of the equations that arise in the application of the method. A survey of experience obtained in the course of practical solution is given.

**1. Formulation of the problem and description of the method.** We consider the equation governing the steady state flow of groundwater in domain $\Omega \subset \mathbf{R}^n$. It can be written as

$$\sum_{i=1}^{n} (k(x) h_{x_i}(x))_{x_i} = -Q(x), \quad x \in \Omega, \tag{1}$$

where $h$ is the piezometric head, $k$ is the coefficient of permeability, and $Q$ is the function of sources (see [1]). Suppose that $h(x)$, $Q(x)$, $x \in \Omega$, are given and formulate the following inverse problem: find $k(x)$, $x \in \Omega$, such that (1) holds. Some indirect methods that lead to iterative algorithms have been proposed for this problem [2–5]. To apply any of such method initial values for $k$ everywhere in $\Omega$ are given ahead, and an improvement of $k$ by successive implementation of an algorithm will be obtained. Our objective in the current note is to study the method of characteristics which is referred to as a direct method. A single implementation gives the final variant of $k$. In Section 2 we shall state and prove stability theorems for the method of characteristics. Section 3 contains a survey of experiences obtained in a practical application.

We proceed now to the description of the method. Assume that $k \in C^1(\Omega)$, $h \in C^2(\overline{\Omega})$, and rewrite (1) in the form

$$\sum_{i=1}^{n} k_{x_i}(x) h_{x_i}(x) + k(x) \Delta h(x) = -Q(x), \quad x \in \Omega. \tag{2}$$

Consider the following system of equations:

$$\frac{d}{dt} \varphi^i_{x^0}(t) = -h_{x_i}(\varphi_{x^0}(t)), \quad 1 \leqslant i \leqslant n, \quad t \in \mathbf{R}, \quad \varphi_{x^0}(0) = x^0, \tag{3}$$

where $x^0 \in \Omega$. Every solution $\varphi_{x^0}(t)$ is a curve passing through $x^0$. Varying $x^0$ over $\Omega$ we obtain a family of curves $\varphi_{x^0}$, $x^0 \in \Omega$. They are called the characteristics of Eq. (2) The characteristics do not intersect one another: if $\varphi_{x^0}(t) = \varphi_{x^1}(\bar{t})$ is valid for some $t$, $\bar{t}$, then $\varphi_{x^0} \equiv \varphi_{x^1}$. If $\Omega$ is bounded, open and $|\nabla h| \neq 0$ everywhere in $\Omega$, then the characteristics go through $\Omega$: for each $\varphi_{x^0}$, $x^0 \in \Omega$, there exist $t_1$, $t_2$ such that $t_1 \neq t_2$

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and $\varphi_{x^0}(t_1)$, $\varphi_{x^0}(t_2) \in \partial\Omega$. Note that the characteristics and flow lines coincide. Indeed, it follows from (3) that a tangent vector of a characteristic and $-\nabla h$ have the same direction in each point. The Darcy's law says that $v = -k \cdot \nabla h$, where $v$ denotes the flow velocity. Therefore, a characteristic and a flow line have common direction in each point and, thus, they coincide.

Now let us fix some characteristic $\varphi_{x^0}(t)$, $t \in \mathbf{R}$, and simplify Eq. (2). Since

$$\sum_{i=1}^{n} k_{x_i}(x) h_{x_i}(x) \,|_{x=\varphi_{x^0}(t)} =$$

$$= -\sum_{1=i}^{n} k_{x_i}(\varphi_{x^0}(t)) \cdot \frac{d}{dt} \varphi^i_{x^0}(t) = -\frac{d}{dt} k(\varphi_{x^0}(t)),$$

we have

$$\frac{d}{dt} k(\varphi_{x^0}(t)) - \Delta h(\varphi_{x^0}(t)) k(\varphi_{x^0}(t)) = Q(\varphi_{x^0}(t)), \qquad t \in \mathbf{R}. \tag{4}$$

This is an ordinary differential equation with respect to $k$ on the characteristic $\varphi_{x^0}$. To solve this equation we need an initial condition. So, a value of $k$ at some point of the characteristic must be known. Therefore, to apply the method of characteristics to the inverse problem as a whole, we need a value of $k$ at some point on each characteristic.

The order of solving the posed inverse problem is as follows.
1° solve the systems (3) to obtain the characteristics of the equation (2),
2° give the value of $k$ at some point on each characteristic and solve the corresponding equations (4) to obtain an extension of the solution onto the whole domain.

**2. On the stability of the method of characteristics.** There are three important aspects in the study of problems in mathematical physics. They are the existence, the uniqueness and the stability of the solution with respect to small perturbations of the initial data. The existence and uniqueness theorems for the posed inverse problem have been proved under various assumptions. We can refer to detailed works [6–8]. In [7] the stability topic was treated too. It was shown that the error of the solution trends to zero as the error of reflux tends to zero, provided that $\Omega$ is bounded and $\max\limits_{x \in \overline{\Omega}} \{|\nabla h(x)|, \Delta h(x)\} > 0$. In the current note we consider the stability of the solution in a more strict sence. In addition to the convergence of the error of the solution, a condition that error must not increase along the characteristics is imposed. This is particularly important from the practical point of view because the real domains of interest (natural aquifers) are usually large, and one has to solve Eqs. (3), (4) on large intervals of parameter $t$.

Firstly, we shall study stability of the system (3).

Theorem 1. *Let the systems*

$$\frac{d}{dt} \varphi^i(t) = -h_{x_i}(\varphi(t)), \qquad 1 \leqslant i \leqslant n, \qquad t \in \mathbf{R}, \quad \varphi(0) = x^0, \tag{5}$$

$$\frac{d}{dt} \varphi^i_\delta(t) = -h^\delta_{x_i}(\varphi_\delta(t)), \qquad 1 \leqslant i \leqslant n, \qquad t \in \mathbf{R}, \quad \varphi_\delta(0) = x^0, \tag{6}$$

*be given. Here* $h \in C^2(\bar{\Omega})$, $h^\delta \in C^2(\bar{\Omega})$, *and* $\max_{x \in \bar{\Omega}} |h^\delta_{x_i}(x) - h_{x_i}(x)| \leqslant \delta_i$, $1 \leqslant i \leqslant n$. *Let the function h be everywhere strictly convex:*

$$h_{\nu\nu}(x) \geqslant \gamma > 0, \quad \forall x \in \bar{\Omega}, \quad \forall \nu \in \mathrm{R}^n: \quad |\nu| = 1, \tag{7}$$

*or everywhere strictly concave:*

$$h_{\nu\nu}(x) \leqslant \gamma < 0, \quad \forall x \in \bar{\Omega}, \quad \forall \nu \in \mathrm{R}^n: \quad |\nu| = 1. \tag{8}$$

*Then the difference of the solutions* $\varphi$, $\varphi_\delta$ *on the semiaxis* $\{t \mid \gamma t > 0\}$ *is estimated as follows:*

$$|\varphi(t) - \varphi_\delta(t)| \leqslant \sum_{i=1}^{n} \delta_i \frac{1}{|\gamma|} (1 - e^{-\gamma t}). \tag{9}$$

Proof. Let us substract Eqs. (5), (6):

$$\frac{d}{dt} (\varphi^i(t) - \varphi^i_\delta(t)) = (h^\delta_{x_i} - h_{x_i})(\varphi_\delta(t)) + h_{x_i}(\varphi_\delta(t)) - h_{x_i}(\varphi(t)). \tag{10}$$

The derivative of $|\varphi(t) - \varphi_\delta(t)|$ can be expressed as

$$\frac{d}{dt} |\varphi(t) - \varphi_\delta(t)| =$$

$$= |\varphi(t) - \varphi_\delta(t)|^{-1} \sum_{i=1}^{n} (\varphi^i(t) - \varphi^i_\delta(t)) \frac{d}{dt} (\varphi^i(t) - \varphi^i_\delta(t)).$$

Using (10) we have

$$\frac{d}{dt} |\varphi(t) - \varphi_\delta(t)| = \sum_{i=1}^{n} (h^\delta_{x_i} - h_{x_i})(\varphi_\delta(t)) \frac{\varphi^i(t) - \varphi^i_\delta(t)}{|\varphi(t) - \varphi_\delta(t)|} +$$

$$+ \sum_{i=1}^{n} (h_{x_i}(\varphi_\delta(t)) - h_{x_i}(\varphi(t))) \frac{\varphi^i(t) - \varphi^i_\delta(t)}{|\varphi(t) - \varphi_\delta(t)|}.$$

Define $\nu_t$ as the unit vector of the direction $\varphi_\delta(t) - \varphi(t)$:

$$\nu^i_t = \frac{\varphi^i_\delta(t) - \varphi^i(t)}{|\varphi(t) - \varphi_\delta(t)|}, \qquad 1 \leqslant i \leqslant n.$$

It follows from the mean value theorem that

$$\sum_{i=1}^{n} (h_{x_i}(\varphi_\delta(t)) - h_{x_i}(\varphi(t))) \frac{\varphi^i(t) - \varphi^i_\delta(t)}{|\varphi(t) - \varphi_\delta(t)|} =$$

$$= -(h_{\nu_t}(\varphi_\delta(t)) - h_{\nu_t}(\varphi(t))) = -h_{\nu_t \nu_t}(P_t) |\varphi(t) - \varphi_\delta(t)|,$$

where $P_t$ lies on the interval between $\varphi(t)$, $\varphi_\delta(t)$. Thus, the formula for the derivative is reduced to the ordinary differential equation

$$\frac{d}{dt} |\varphi(t) - \varphi_\delta(t)| + h_{\nu_t \nu_t}(P_t) |\varphi(t) - \varphi_\delta(t)| =$$

$$= \sum_{i=1}^{n} (h^\delta_{x_i} - h_{x_i})(\varphi_\delta(t)) \frac{\varphi^i(t) - \varphi^i_\delta(t)}{|\varphi(t) - \varphi_\delta(t)|}.$$

Let us add the initial condition $|\varphi(0)-\varphi_\delta(0)|=0$, and solve the equation. We obtain

$$|\varphi(t)-\varphi_\delta(t)|=\int_0^t \sum_{i=1}^n (h^\delta_{x_i}-h_{x_i})(\varphi_\delta(s))\frac{\varphi^i(s)-\varphi^i_\delta(s)}{|\varphi(s)-\varphi_\delta(s)|}e^{-\int_s^t h_{v_\tau v_\tau}(\tau)d\tau}\,ds.$$

Considering the assumptions and the condition $\gamma t>0$, we have the estimate

$$|\varphi(t)-\varphi_\delta(t)|\leqslant\sum_{i=1}^n \delta_i\Big|\int_0^t e^{-\gamma(t-s)}\,ds\Big|=\sum_{i=1}^n \delta_i\frac{1}{|\gamma|}(1-e^{-\gamma t}),$$

which completes the proof.

The estimate (9) shows that the difference of the solutions of the systems (5) and (6) remains bounded on the semiaxis $\gamma t>0$ as $|t|\to\infty$. Of course, we are also interested in the behaviour of these solutions on the semiaxis $\gamma t<0$. It appears that the points $\varphi_\delta(t)$ evaluated at large absolute values of $t$ may depart from the curve $\varphi$ exponentially. Let us illustrate this fact by a simple example. Take $h(x)=\gamma(x_1^2+\ldots+x_n^2)$. Then $h_{vv}(x)\equiv\frac{1}{2}\gamma$. Let $x^0=(x_0,\ldots,x_0)$ and $h^\delta_{x_i}(x)=\gamma x_i+\delta_i$, $\sum_{i=1}^n\delta_i=0$.

We have

$$\varphi^i(t)=x_0e^{-\gamma t},\quad \varphi^i_\delta(t)=x_0e^{-\gamma t}+\frac{\delta_i}{\gamma}(1-e^{-\gamma t}).$$

Define the function

$$F_t(\tau)=|\varphi_\delta(t)-\varphi(\tau)|^2=\sum_{i=1}^n\Big[x_0(e^{-\gamma t}-e^{-\gamma\tau})+\frac{\delta_i}{\gamma}(1-e^{-\gamma t})\Big]^2,$$

and search for its global minimum over $\tau$. To this end we solve the equation $\frac{d}{d\tau}F_t(\tau)=0$. The only stationary point is $\tau_*=t$. Since $F_t(\tau)\geqslant0$, $\lim_{\gamma\tau\to-\infty}F_t(\tau)=+\infty$, the point $\tau_*$ is the global minimum. Hence,

$$\min_{\tau\in R}|\varphi_\delta(t)-\varphi(\tau)|=|\varphi_\delta(t)-\varphi(t)|=\frac{1}{|\gamma|}\Big(\sum_{i=1}^n\delta_i^2\Big)^{\frac{1}{2}}\times$$

$$\times|1-e^{-\gamma t}|=\frac{1}{|\gamma|}\Big(\sum_{i=1}^n\delta_i^2\Big)^{\frac{1}{2}}(e^{-\gamma t}-1),\quad \gamma t<0.$$

We see that points $\varphi_\delta(t)$ evaluated at large absolute values of $t$ depart from the curve $\varphi$ exponentially in this example.

Let us make a short summary. When approximate data is available, the reconstruction of a characteristic (flow line) passing through a given point is trustworthily possible:

a) in the positive direction of parameter $t$ (downstream), if $h$ is strictly convex (condition (7)),

b) in the negative direction of parameter $t$ (upstream), if $h$ is strictly concave (condition (8)).

Let us proceed now to Eq. (4). Suppose that a value $k(x^0)$ of the coefficient of permeability in some point $x^0$ is known. Solving Eq. (4) with the initial condition $k(\varphi(0))=k(x^0)$, we obtain values of the coefficient on the characteristic $\varphi$ passing through $x^0$. The problem of trustworthiness of the results obtained on the basis of the approximate data arise here too. Let approximates $h^\delta$ and $Q^\delta$ be known instead of exact $h$ and $Q$. Instead of Eq. (4) we solve

$$\frac{d}{dt} k^\delta(\varphi_\delta(t)) - \Delta h^\delta(\varphi_\delta(t)) k^\delta(\varphi_\delta(t)) = Q^\delta(\varphi_\delta(t)),$$

$$t \in \mathbf{R}, \quad k^\delta(\varphi_\delta(0)) = k(x^0), \tag{11}$$

where $\varphi_\delta$ is the characteristic passing through $x^0$ evaluated on the basis of $h^\delta$ (the solution of system (6)). We are not interested in the behaviour of the difference of the functions $k(\varphi(t))$ and $k^\delta(\varphi_\delta(t))$, where $k(\varphi(t))$ is the solution of Eq. (4) satisfying the initial condition $k(\varphi(0)) = k(x^0)$. Indeed, the function $k^\delta(\varphi_\delta(t))$ is interpreted as an approximate solution of the inverse problem on the previously evaluated characteristic $\varphi_\delta(t)$. For that reason we are interested primarily in the behaviour of the difference of the approximate and the exact solutions on the characteristic $\varphi_\delta$, i.e. in the behaviour of $|k^\delta(\varphi_\delta(t)) - k(\varphi_\delta(t))|$. The following theorem gives sufficient condition for the stability.

Theorem 2. *Let* $h, h^\delta \in C^2(\overline{\Omega})$, $Q, Q^\delta \in C(\overline{\Omega})$, *where* $\max_{x\in\overline{\Omega}} |h_{x_i}(x) - h^\delta_{x_i}(x)| \leqslant \delta_i$, $\max_{x\in\overline{\Omega}} |\Delta h(x) - \Delta h^\delta(x)| \leqslant \delta_{\Delta h}$, $\max_{x\in\overline{\Omega}} |Q(x) - Q^\delta(x)| \leqslant \delta_Q$.

*Let there exist a solution* $k \in C^1(\overline{\Omega})$ *of the posed inverse problem. Let* $k^\delta(\varphi_\delta(t))$ *be the solution of* (11), *where* $\varphi_\delta$ *is the solution of the system* (6). *Assume that the condition*

$$\Delta h^\delta(x) \geqslant \gamma > 0, \quad \forall x \in \overline{\Omega}, \tag{12}$$

*or the condition*

$$\Delta h^\delta(x) \leqslant \gamma < 0, \quad \forall x \in \overline{\Omega}, \tag{13}$$

*holds. Then the following estimate*

$$|k^\delta(\varphi_\delta(t)) - k(\varphi_\delta(t))| \leqslant [\delta_Q + \|k\|_{C^1} \cdot \max\{\delta_{\Delta h}, \delta_1, \ldots, \delta_n\}] \times \\ \times \frac{1}{|\gamma|}(1 - e^{\gamma t}) \tag{14}$$

*is valid in the domain* $\{t \mid \gamma t < 0\}$.

Proof. The solution of (11) can be expressed in the form

$$k^\delta(\varphi_\delta(t)) = k(x^0) e^{\int_0^t \Delta h^\delta(\varphi_\delta(\tau))d\tau} + \int_0^t Q^\delta(\varphi_\delta(s)) e^{\int_s^t \Delta h^\delta(\varphi_\delta(\tau))d\tau} ds.$$

Let us define

$$\overline{Q}(x) := -\sum_{i=1}^n (k(x) h^\delta_{x_i}(x))_{x_i} = -\sum_{i=1}^n k_{x_i}(x) h^\delta_{x_i}(x) - k(x)\Delta h^\delta(x), \tag{15}$$

and treat (15) as if it were an equation with respect to $k$. The characteristic of Eq. (15) passing through $x^0$ is $\varphi_\delta$. Therefore, the solution $k$ of Eq. (15) is expressed on $\varphi_\delta$ as the solution of the equation

$$\frac{d}{dt} k(\varphi_\delta(t)) - \Delta h^\delta(\varphi_\delta(t)) k(\varphi_\delta(t)) = \overline{Q}(\varphi_\delta(t)), \quad t \in \mathbf{R}, \quad k(\varphi_\delta(0)) = k(x^0).$$

Hence, we obtain

$$k(\varphi_\delta(t))=k(x^0)e^{\int\limits_0^t \Delta h^\delta(\varphi_\delta(\tau))d\tau}+\int\limits_0^t \overline{Q}(\varphi_\delta(s))e^{\int\limits_s^t \Delta h^\delta(\varphi_\delta(\tau))d\tau}ds$$

and

$$k^\delta(\varphi_\delta(t))-k(\varphi_\delta(t))=\int\limits_0^t (Q^\delta-\overline{Q})(\varphi_\delta(s))e^{\int\limits_s^t \Delta h^\delta(\varphi_\delta(\tau))d\tau}ds. \qquad (16)$$

Let us estimate:

$$|(Q^\delta-\overline{Q})(x)|\leqslant|Q^\delta(x)-Q(x)|+|Q(x)-\overline{Q}(x)|\leqslant\delta_Q+$$

$$+\left|\sum_{i=1}^n (k(x)(h_{x_i}(x)-h^\delta_{x_i}(x)))_{x_i}\right|\leqslant\delta_Q+\|k\|_{C^1}\max\{\delta_{\Delta h},\ \delta_1,\ \ldots,\ \delta_n\}.$$

Therefore, from (16) we obtain

$$|k^\delta(\varphi_\delta(t))-k(\varphi_\delta(t))|\leqslant[\delta_Q+\|k\|_{C^1}\max\{\delta_{\Delta h},\ \delta_1,\ \ldots,\ \delta_n\}]\times$$

$$\times\left|\int\limits_0^t e^{\int\limits_s \gamma d\tau}ds\right|\leqslant[\delta_Q+\|k\|_{C^1}\max\{\delta_{\Delta h},\ \delta_1,\ \ldots,\ \delta_n\}]\frac{1}{|\gamma|}(1-e^{\gamma t}),$$

if $\gamma t<0$. Theorem has been proved.

Consequently, the sufficient condition for stability is $\gamma t<0$. The error is bounded when $|t|\to\infty$. It is possible to construct simple examples showing exponential increase $|k^\delta(\varphi_\delta(t))-k(\varphi_\delta(t))|$ if $\gamma t>0$, $|t|\to\infty$. We shall not represent these here.

Let us sum it up. In the case of available approximate data the reconstruction of a solution of the inverse problem along a characteristic is trustworthily possible

a) in the positive direction of the parameter $t$ (downstream) if $\Delta h^\delta$ is negative (condition (13)),

b) in the negative direction of the parameter $t$ (upstream) if $\Delta h^\delta$ is positive (condition (12)).

Stability conditions for the solution of the inverse problem contradict those for the evaluation of the characteristic. Indeed, if $\delta_{\Delta h}$ is small, then (7) implies (12) and (8) implies (13). If we follow the presented stability conditions for the solution of the inverse problem, then a divergence of exact and approximate characteristics may occur. Of course, such fact is not particularly important, because the purpose is not to reconstruct the characteristics but to identify the coefficient of permeability.

**3. On the experience obtained in a practical application of the method. Advantages and disadvantages of the method.** The method described has been used to study the deepest aquifer in the basic rock of Estonia. Our objective here is not to present the obtained numerical results. We want to distribute experience obtained in the course of practical solution and to give a survey of difficulties that occurred in this connection.

The initial data for the problem contained values of $h$ and $Q$ in certain irregularly located points $P_j$ (results of measurement in wells) and values of $k$ in certain points of boundary (obtained by means of the Darcy's law on the basis of measured flux). Unfortunately, we had to correct these data of $k$ in some points in order to attain a better harmony of results in the large domain studied.

The most difficult task confronted us when the discretization prepared was the interpolation of $h$. Suitable interpolation should enable to evaluate the 1st- and 2nd-order partial derivatives of $h$. Since the problem was quite labour-consuming, the interpolation had to provide these derivatives with simple formulae, spending the processing time as little as possible. To this end the domain to quadrangles with verteces $P_j$ was distributed and obtained quadrangles were joined in pairs. In this way we reached a hexagonal distribution of considered domain. The interpolant was determined as a quadratic polynomial on each hexagon, and set equal to $h$ at the vertecis $P_j$. Although such interpolant is not smooth on the sides of hexagon, it satisfies quite well the demands imposed above. It guarantees 2nd-order accuracy for $h_{x_i}$ and first-order accuracy for $\Delta h$.

Derivatives with respect to $t$ in the systems (3), (4) were replaced through the first-order differences, and an explicit scheme was obtained. In the choice of the direction of solution, the stability condition (sign of $\Delta h$) was followed. For that reason we could not always use a point of boundary and *a priori* known value of $k$ in this point as initial data of the scheme. Often we had to give some value to $k$ in the interior of the domain, and move along the characteristic up to the boundary. In the case of obtained unsuitable boundary value of $k$ we changed the initial value and repeated the procedure. Somewhere the coefficient of permeability computed on the basis of its boundary value became too large. In this case we had to correct the initial data (values of $k$ on the boundary). Since the stability condition for $k$ was followed, the characteristics moved apart in some areas (for reasons of this phenomenon see end of the previous section). We had to change many times the set of initial points in order to attain sufficiently uniform density of characteristics in such areas.

Mentioned nuances made a programming of the whole procedure of solving impossible. Because of large domain and great quantity of characteristics supplementary testing procedures would expand the processing time too much. The choice of suitable initial values as well as the determination of necessary density of characteristics in various subdomains were carried out greatly by intuition.

Let us give a summary of advantages and disadvantages of the method. Since the solution is performed along the flow lines (characteristics), the proposed method has a good accordance with the physical nature of the problem. Therefore, the use of information and, hence, attainable accuracy are better than in some other cases. Disadvantages of the method are its labourconsuming procedures.

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Received
Feb. 12, 1991

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## FILTRATSIOONIKOEFITSIENDI IDENTIFITSEERIMISEST KARAKTERISTIKUTE MEETODIL

On uuritud karakteristikute meetodit filtratsioonikoefitsiendi $k$ määramiseks võrrandist (1) teadaolevate funktsioonide $h$ ja $Q$ põhjal. On esitatud piisavad tingimused meetodi rakendamisel tekkivate süsteemide (3), (4) lahendite stabiilsuse tagamiseks parameetri $t$ suurte intervallide korral (teoreemid 1, 2). On kirjeldatud meetodi praktilisel rakendamisel tekkivaid raskusi ning võetud kokku meetodi eelised ja puudused.

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## ОБ ИДЕНТИФИКАЦИИ КОЭФФИЦИЕНТА ФИЛЬТРАЦИИ МЕТОДОМ ХАРАКТЕРИСТИК

Исследуется метод характеристик, используемый для определения коэффициента фильтрации $k$ из уравнения (1). Функции $h$, $Q$ заданы. Выводятся достаточные условия асимптотической устойчивости решений систем (3), (4), полученных с применением названного метода. Описываются трудности, возникающие при практическом применении метода, и дается обзор преимуществ и недостатков метода.