Proc. Estonian Acad. Sci. Phys. Math., 1990, 39, N 4, 344–352
https://doi.org/10.3176/phys.math.1990.4.04

УДК 530.12 : 531.51

Väino UNT

ON INTERACTING GAUGE FIELDS

(Presented by A. Sapar)

It is demonstrated that a traditional approach tó interacting gauge fields may need some modification when applied to sources moving either at the velocity of light or at ultrarelativistic velocities. In a traditional approach, solutions belonging to the non-Hilbert space are used with the implicit assumption that non-Hilbert space solutions are physically equivalent to Hilbert space solutions, which is shown not to be true.

1. Introduction

Examples of gauge fields are electromagnetic and Yang-Mills fields. Special features of free gauge fields which are due to the calibration invariance are well understood. It is believed that these special features are not influenced by spinor or scalar fields with which the gauge fields interact [1]. In this paper we argue that in the case of light-like sources, i. e. sources which move at the velocity of light (and, probably, in the case of sources which move at ultrarelativistic velocities to be defined below) the character of the interaction of gauge fields with sources is changed. This "high-energy" phenomenon occurs already within the framework of classical theories. As we shall see, light-like sources, if properly described, do not emit waves but "drag" them along, the sources remaining at wave-fronts all the time. This peculiar feature of interaction needs a more subtle treatment than one can find in the existing quantization schemes, where the emission and absorption of real and fictitious quantized waves occurs which is described by the inverse of the wave operator $\Box^{-1}$ (or, in the momentum representation, by $(k^2+i\varepsilon)^{-1}$). In fact, the existing quantization schemes make use of the non-Hilbert space of solutions, distort the correct physical picture of the interaction of light-like and ultrarelativistic sources and give an incorrect result for the interaction energy between the sources, and, probably, an incorrect (gauge-dependent) value for the S-matrix.

Gravitational fields may also be viewed as gauge fields. A peculiar feature of these fields is the following: a gravitational wave always carries its radiative self-energy which, in its turn, is a light-like source "dragging" along the higher order field. This means that covariant gravity quantization schemes may also need some modification. As we shall see, modification is needed already in the case of Abelian gauge fields. Therefore we begin our analysis with the scalar electrodynamics of massless fields. This simplest case, which probably has a didactical value only, provides us with insights useful in more complicated realistic cases. This paper, the first in a series, is dedicated to classical fields. In the second section the main equations of scalar electrodynamics are summarized, in the third section electromagnetic fields and potentials of light-like sources are found. The fourth section treats massive ultrarelativistic sources; the fifth section the gauge dependence of the interaction energy. In the sixth section "unusual" properties of the interacting electromagnetic field are extended to the case of self-interacting gravitational radiation.

Notation. Greek indices take values 0, 1, 2, 3 and denote the components of a vector- or tensor-field in rectilinear (quasi-) Cartesian coordinates even in case the arguments are written as polar coordinates (r, ϑ, φ). Three-dimensional vectors k^i are also used: $k^\alpha = (k^0, k^i)$. Index r always refers to a radial component. The Einstein summation convention is used. Indices are raised and lowered with the help of the Minkowski metric $\eta^{\mu\nu} = \eta_{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1)$;

$$f_{,\mu} \equiv \partial_\mu f \equiv \frac{\partial f}{\partial x^\mu}, \quad f_{,}{}^{\mu} \equiv \eta^{\mu\alpha} f_{,\alpha}, \quad \Box f \equiv \eta^{\alpha\beta} f_{,\alpha\beta}.$$

Symbol "$\equiv$" denotes equality by definition, "$\sim$" denotes proportionality. $F = O(r^{-n})$ means $F < Mr^{-n}$ where M is a constant of finite value. We use the units in which the Newton gravitational constant γ and the velocity of light c are equal to 1.

2. Scalar electrodynamics

Let the potential of the electromagnetic field be denoted by $A_\mu(x)$, the scalar field by $\Phi(x)$ and the complex conjugate of any function f by f^*. A gauge transformation is described by a scalar function $\Lambda(x)$:

$$A'_\mu = A_\mu + \Lambda_{,\mu}, \quad \Phi' = \Phi e^{-iQ\Lambda(x)}.$$

The invariant Lagrangian is

$$L_{inv} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} + (D_\mu \Phi)^* D^\mu \Phi - m^2 \Phi \Phi^*, \tag{1}$$

$$F_{\mu\nu} \equiv A_{\nu,\mu} - A_{\mu,\nu},$$

$$D_\mu \equiv \partial_\mu + iQA_\mu.$$

Here m is the mass and Q is the charge of a scalaron. We also have

$$D_\mu \Phi (D^\mu \Phi)^* = \partial_\mu \Phi \partial^\mu \Phi^* - J^\mu A_\mu,$$

$$J^\mu \equiv iQ[\Phi^* D^\mu \Phi - \Phi (D^\mu \Phi)^*].$$

From the Lagrangian (1) the field equations follow

$$D^\mu D_\mu \Phi + m^2 \Phi = 0, \quad (D^\mu D_\mu \Phi)^* + m^2 \Phi^* = 0, \qquad \Box A^\mu - A^\alpha{}_{,\alpha}{}^\mu = 4\pi J^\mu. \tag{2}$$

By expanding Φ, A^μ and J^μ in powers of Q we have in the lowest non-vanishing approximation

$$\Box \Phi_{(0)} + m^2 \Phi_{(0)} = 0, \quad \Box \Phi^*_{(0)} + m^2 \Phi^*_{(0)} = 0, \tag{3}$$

$$A^\mu_{(1),\alpha}{}^\alpha - A^\alpha_{(1),\alpha}{}^\mu = 4\pi J^\mu_{(1)}, \tag{4}$$

$$J^\mu_{(1)} = i(\Phi^*_{(0)} \partial^\mu \Phi_{(0)} - \Phi_{(0)} \partial^\mu \Phi^*_{(0)}).$$

Further we are interested only in the lowest-order non-vanishing fields and omit indices (0) and (1) denoting in expansions the coefficients of powers of Q. (The well-understood free electromagnetic field $A^\mu_{(0)}$ has been left out of consideration.)

3. Electromagnetic fields and potentials of light-like sources

Let us assume that $m=0$ and consider a spherical pulse of radiation $\left(u\equiv t-r,\ \dot{\Phi}\equiv\frac{\partial\Phi}{\partial u}\right)$

$$\Phi=\frac{b(u,\vartheta,\varphi)}{r}+O(r^{-2}),\qquad \text{if}\qquad u_1<u<u_2,$$

$$\Phi^*=\frac{b^*(u,\vartheta,\varphi)}{r}+O(r^{-2}),\quad \text{if}\qquad u_1<u<u_2,$$

$$\dot{\Phi}=\dot{\Phi}^*=0,\qquad\qquad \text{if}\quad u<u_1 \text{ or } u>u_2.$$

This spherical pulse of scalar radiation creates a spherical pulse of the light-like current

$$J^\mu=\frac{\sigma(u,\vartheta,\varphi)\,k^\mu}{r^2}+O(r^{-3}),$$

$$\sigma(u,\vartheta,\varphi)\equiv iQ(b^*\dot{b}-\dot{b}^*b). \tag{5}$$

Here $k_\mu\equiv u_{,\mu}$ is the wave vector.

In order to avoid a misinterpretation of the interaction of an electromagnetic field with sources due to the choice of gauge, we first find electric and magnetic fields by integrating the non-homogeneous Maxwell equation with sources (5). The electric field $\mathbf{E}$ is described by the constraint equation

$$\operatorname{div}\mathbf{E}=4\pi J^0. \tag{6}$$

In terms of the angular momentum operator $\mathbf{L}=\frac{1}{i}(\mathbf{r}\times\operatorname{grad})$

$$\operatorname{div}\mathbf{E}=\frac{1}{r^2}\frac{\partial}{\partial r}(r^2E_r)-\frac{i}{r^2}(\mathbf{r}\times\mathbf{L})\cdot\mathbf{E}. \tag{7}$$

Let us divide $\mathbf{E}$ into longitudinal and transverse parts

$$\mathbf{E}(u,r,\vartheta,\varphi)=E_r(u,r)\frac{\mathbf{r}}{r}+\frac{i}{r^2}(\mathbf{r}\times\mathbf{L})U(u,\vartheta,\varphi)+\ldots \tag{8}$$

and make an expansion

$$\sigma(u,\vartheta,\varphi)=\sigma_0(u)+\sum_{l=1}^{\infty}\sum_{m=-l}^{l}\sigma_{lm}(u)Y_{lm}(\vartheta,\varphi).$$

By inserting (5), (7) and (8) into equation (6) we obtain for any fixed t

$$E_r(u,r)=\frac{\bar{Q}-q(u)}{r^2},\qquad q(u)\equiv4\pi\int_{u_1}^{u}\sigma_0(u')\,du',\qquad \bar{Q}\equiv q(u_2), \tag{9}$$

$$L^2U=4\pi[\sigma(u,\vartheta,\varphi)-\sigma_0(u)]. \tag{10}$$

From quantum mechanics we know that $-L^2$ is the Laplace operator on a sphere: $L^2Y_{lm}(\vartheta,\varphi)=l(l+1)Y_{lm}(\vartheta,\varphi)$. Now, equations (8) to (10) give $\left(\mathbf{n}\equiv\frac{\mathbf{r}}{r}\right)$

$$\mathbf{E}=\frac{\bar{Q}-q(u)}{r^2}\mathbf{n}-\frac{4\pi}{r}\sum_{l,m}\frac{\sigma_{lm}(u)}{\sqrt{l(l+1)}}\mathbf{Y}^{(1)}_{lm}(\vartheta,\varphi), \tag{11}$$

$$i\mathbf{Y}^{(1)}_{lm}(\vartheta,\varphi)\equiv\mathbf{n}\times\mathbf{Y}^{(0)}_{lm}(\vartheta,\varphi),\quad \mathbf{Y}^{(0)}_{lm}(\vartheta,\varphi)\equiv\frac{1}{\sqrt{l(l+1)}}\mathbf{L}Y_{lm}(\vartheta,\varphi).$$

Here we have used definitions of the transverse vector spherical harmonics as given in [2]. The remaining Maxwell equations determine the asymptotic magnetic field

$$\mathbf{H}=-\frac{4\pi i}{r}\sum_{l,m}\frac{\sigma_{lm}(u)}{\sqrt{l(l+1)}}\mathbf{Y}^{(0)}_{lm}(\vartheta,\varphi). \tag{12}$$

Solutions (11) and (12) consist of a radial Coulomb field and a transverse field. There are important analogies between our transverse waves and the TEM waves in wave guides and around electric transmission lines [3]: both obey elliptic equations and have their sources on the wave-front; one could say that waves are "dragged" by sources. Note that our waves have no independent degrees of freedom in addition to those of scalar fields. Nevertheless, they are not fictitious fields.

Next, let us see how it is possible to represent the above properties of the field in terms of potentials. In the Lorentz gauge, where $A^{\alpha}{}_{,\alpha}=0$ equations (4) are reduced to four wave-equations

$$\Box A^{\mu}=4\pi\frac{\sigma(u,\vartheta,\varphi)}{r^2}k^{\mu}+O(r^{-3}). \tag{13}$$

Instead of the "dragging" of the field by sources, as described by equation (10), we have equations (13) which express the emission of waves. The validity of this physical picture may be argued. The solution of equations (13) is the following [4]

$$\begin{cases} A^0=\dfrac{\bar{Q}}{r}+\dfrac{\ln r}{2r}\varepsilon(u,\vartheta,\varphi)+O(r^{-2}),\\[2ex] A^r=\dfrac{\ln r}{2r}\varepsilon(u,\vartheta,\varphi)+O(r^{-2}\ln r),\\ \cdots\cdots\cdots\cdots \end{cases} \tag{14}$$

$$\varepsilon(u,\vartheta,\varphi)\equiv4\pi\int_{u_1}^{u}\sigma(u',\vartheta,\varphi)\,du'. \tag{15}$$

The distortion of the physical content of solutions (11) and (12) of equation (10) by wave equations (13) is related to the abandonment of a mathematically reasonable class of solutions. We shall demonstrate that in the case of sources of type (5) operator $\Box^{-1}$ takes us out of the Hilbert space and spoils the reasonable asymptotical behaviour of solutions. It could be said that in the case of sources (5) operator $\Box^{-1}$ is defined neither within the Hilbert space of solutions nor within the Sobolev space of solutions H^s which is the basic space of the existence of solutions for non-linear hyperbolic equations.

Let us now give a proof that (14) does not belong to the Hilbert space of solutions $L_2(\Omega)$, i.e. the space of square integrable functions in the region $\Omega=V\Delta t$, where Δt is some finite interval of time and V will be defined below via an explicit integration. Let $\varepsilon=O(1)$, then at any fixed moment of time

$$\int_V (A^r)^2\,dV=\int_{r_1-L}^{r_1}\int_0^{\pi}\int_0^{2\pi}(A^r)^2r^2\sin\vartheta\,d\vartheta\,d\varphi\sim L\ln^2 r_1+\ldots\,.$$

Here r_1 denotes some value of r on the light-cone $u=u_1$, and L is the length of a wave pulse; $L \ll r_1$. Let us denote values of t on the light cone $u=u_1$ by t_1. Now, with increasing time t_1 our pulse of radiation moves farther and farther off, and for the finite $u_1=t_1-r_1$ we have $r_1 \to \infty$, $\ln^2 r_1 \to \infty$ if $t_1 \to \infty$. Hence, we are out of the Hilbert space and out of the Sobolev space H^s. (A function F belongs to H^s if F and its derivatives up to the order s belong to $L_2(\Omega)$.)

Next we find the Hilbert space solution assuming

$$A^\mu = \frac{a^\mu(u, \vartheta, \varphi)}{r} + O(r^{-2}), \quad \text{if} \quad u_1 < u < u_2. \tag{16}$$

By inserting (16) into (4) and making use of definitions (15) and (9) we obtain $[\Box A^\mu = O(r^{-3})!]$

$$-A^\mu{}_{,\mu}{}^0 = \frac{4\pi\sigma(u, \vartheta, \varphi)}{r^2} + O(r^{-3}),$$

$$A^\mu{}_{,\mu} = -\frac{\varepsilon(u, \vartheta, \varphi)}{r^2} + O(r^{-3}), \tag{17}$$

$$\dot{a}^0 = \dot{a}^r, \tag{18}$$

$$A^0 = \frac{\bar{Q} - q(u)}{r}, \tag{19}$$

$$\mathbf{A} = \frac{\bar{Q} - q(u)}{r}\,\mathbf{n} - \frac{1}{r}\sum_{l,m} \frac{\varepsilon_{lm}(u)}{\sqrt{l(l+1)}}\,\mathbf{Y}^{(1)}_{lm}(\vartheta, \varphi), \tag{20}$$

$$\varepsilon(u, \vartheta, \varphi) = q(u) + \sum_{l=1}^{\infty} \sum_{m=-l}^{l} \varepsilon_{lm}(u)\, Y_{lm}(\vartheta, \varphi).$$

The potential of the electromagnetic field A^μ, which is constrained to follow its sources, satisfies a peculiar scalar field equation (17), where $A^\mu{}_{,\mu}$ does not describe the spin-0 field: the substitution $A^\mu = a_{,}{}^\mu$ would compel us once more to abandon the Hilbert space. Theories in which no definite spin can be ascribed to interacting fields have been classified as theories of class B [5].

Instead of a currently accepted physical picture with real spin-1 photons and fictitious spin-0 photons, both satisfying ordinary wave equations, we have arrived at a different physical picture, contained in (17), (19) and (20), which coincides with the one we obtained for electric and magnetic vectors. The change of the physical picture is related to the breaking of gauge invariance. We note that within the non-Hilbert space of solutions (14) a gauge-invariant theory can be developed on the basis of the gauge-invariant Lagrangian (1), if we allow phase transformations of the form

$$\Phi \to \Phi e^{-iQ\Lambda(x)}, \quad \Lambda \sim \ln r \quad \text{or} \quad \Lambda \sim \ln r/r.$$

In this paper we shall not examine this possibility. If we stay in the Hilbert space of solutions and assume that the phase factor has the asymptotic behaviour $\Lambda(x) = O\left(\frac{1}{r}\right)$, the breaking of the gauge symmetry will occur.

Next we extend our result to include the high-frequency solutions of massive complex scalar fields. We consider a very simple special case to demonstrate that the gauge fields generated by massive ultrarelativistic sources are similar to the fields generated by light-like sources.

4. On the electrodynamics of massive scalar fields

Let

$$\Box\,\Phi + m^2\Phi = 0, \tag{21}$$

$$\Phi = \frac{f(r)}{r}\, e^{-i\omega t}. \tag{22}$$

By inserting (22) into equation (21) we have

$$\lambda^2 f^2 - f_{,rr} = 0, \quad \lambda \equiv \pm\sqrt{m^2-\omega^2}, \quad f = Ce^{\lambda r},$$

where C is a constant. As we are interested neither in an exponentially decreasing nor increasing potential, we shall consider only the case where $\omega > m$, $f = e^{i\kappa r}$, $\kappa^2 \equiv \omega^2 - m^2$. Suppose that at some moment of time f is defined as the Fourier series between two concentric spheres with radii r_2 and $r_1 = r_2 + L$, $L \equiv 2\pi$, and consider the wave packet

$$\Phi = \frac{1}{r}\sum_{\kappa} C_\kappa \exp\left[-i\varkappa_\alpha(\kappa)\, x^\alpha\right], \tag{23}$$

$$\varkappa_\alpha(\kappa) \equiv \left(\sqrt{\kappa^2+m^2},\ -\kappa\frac{x^\kappa}{r}\right), \quad \varkappa_\alpha \varkappa^\alpha = m^2.$$

The conserved current is

$$J^\mu = \frac{Q}{r^2}\sum_{\kappa=1}^{\infty} C_\kappa C_\kappa^* \varkappa^\mu(\kappa) +$$

$$+\frac{Q}{r^2}\sum_{\kappa=1}^{\infty}\ \sum_{l=\kappa+1}^{\infty}\left\{C_\kappa^* C_l\left[\varkappa^\mu(\kappa)+\varkappa^\mu(l)\right]\exp\left[iK_\alpha(l,\kappa)\,x^\alpha\right]\right\} + \text{c. c.}, \tag{24}$$

$$K_\alpha(l,\kappa) \equiv \varkappa_\alpha(l) - \varkappa_\alpha(\kappa).$$

If $\varkappa^r(\kappa) = \kappa \gg 1$, and $C_l \neq 0$ only for $|\kappa - l| \ll \kappa$ we can define the radial group velocity v_{gr} of the wave packet as follows

$$v_{gr} = \frac{\varkappa^r(\kappa)+\varkappa^r(l)}{\varkappa^0(\kappa)+\varkappa^0(l)} \approx \frac{\kappa}{\varkappa^0(\kappa)}.$$

(This definition coincides with the standard definition used in the case of the continuously changing frequency $\omega(\kappa) = \sqrt{m^2+\kappa^2}$, $v_{gr} = \frac{\partial\omega}{\partial\kappa} = \frac{\kappa}{\omega}$.) If $\kappa^2 \gg m^2$, then $v_{gr} = 1 - \frac{m^2}{2\kappa^2} + \ldots$ is close to the velocity of light, and we shall call such sources ultrarelativistic. In the case of ultrarelativistic sources we shall neglect in v_{gr} the terms $\sim \frac{m^4}{\kappa^4}$.

Next we shall integrate the Maxwell equations with ultrarelativistic sources, considering only the first term in the current (24). One could say that we leave the oscillating part of the current out of consideration or, equivalently, that we use the averaged value of the current. Suppose we have a wave packet with $v_{gr} = \frac{\kappa}{\varkappa_0(\kappa)}$, $\kappa \gg 1$. We also suppose that the length of our wave pulse L is small as compared to its distance from the origin: $L \ll r_2$.

Assume that our wave packet carries the total charge Q:

$$\int J^0\, dV = 4\pi \int_{r_2}^{r_2+L} 2Q\, dr \sum C_\kappa C_\kappa^* \varkappa^0(\kappa) = Q.$$

Hence, between our concentric expanding spheres

$$J^0 = \frac{Q}{4\pi L r^2}, \quad J^r = \frac{Q v_{gr}}{4\pi L r^2}. \tag{25}$$

By inserting these currents into the Maxwell equations we have for $u_1 < \varkappa_\alpha(\kappa) x^\alpha < u_2$, $u_A \equiv \varkappa_\alpha(\kappa) x^\alpha_A$ the following solution

$$\begin{cases} A^r = -[L \cdot \varkappa_0(\kappa) \cdot r]^{-1} Q \varkappa_\alpha (x^\alpha - x^\alpha_2) + \dots, \\ A^0 = v_{gr} A^r + \dots, \end{cases} \tag{26}$$

$$A^\alpha{}_{,\alpha} = -[L \cdot \varkappa_0(\kappa) \cdot r]^{-1} Q \left[v_{gr} + \frac{1}{r} \varkappa_\alpha (x^\alpha - x^\alpha_2) \right] + \dots .$$

5. On interaction energies

The gauge invariance of a theory means that the physical properties of a system do not depend on the gauge used. Here we demonstrate that in the case of current (5) the interaction energy $A_\alpha J^\alpha$ depends on the space of functions in which A^α is evaluated, the Hilbert space and non-Hilbert space solutions give different values for the interaction energy. Let $A^\alpha = (A^0, \mathbf{A})$.

From (5), (19) and (20) follows that $A_\alpha J^\alpha = 0$; i. e. light-like charges do not interact. Next, let us evaluate the interaction energy of light-like sources in the Lorentz gauge. From (5) and (14) it follows that $A_\alpha J^\alpha = = \frac{Q}{r} J^0 + J_r \cdot \frac{1}{2} \frac{\ln r}{r^2} \int\limits_{u_1}^{u} \varepsilon(u') du'$. Here the second term has been evaluated in a spherically symmetric case. The same calculations can be formally performed in the momentum space. By denoting the Fourier transform of the vector J_μ with $\mathcal{J}_\mu$, we can write the interaction term in the Lorentz gauge as follows

$$\mathcal{J}'_\mu \mathcal{J}^\mu (\omega^2 - \kappa^2)^{-1}, \quad \omega \equiv k_0, \quad -\kappa^2 \equiv k_i k^i.$$

This represents the law of electromagnetic interaction expressed by means of an interchange of a virtual photon. The conservation of the charge $k^\mu \mathcal{J}_\mu = 0$ becomes $k^0 \mathcal{J}_0 + k^r \mathcal{J}_r = 0$. Next we follow the argumentation applied by Feynman [6] to the case of gravitational radiation and replace the terms involving $\mathcal{J}^r$ components by $\mathcal{J}^0$ components: $\mathcal{J}^\mu \mathcal{J}'_\mu (\omega^2 - \kappa^2)^{-1} = = (\omega^2 - \kappa^2)^{-1} \cdot \left(\mathcal{J}^0 \mathcal{J}'_0 - \frac{\omega^2}{\kappa^2} \mathcal{J}^0 \mathcal{J}'_0 \right) = -\frac{1}{\kappa^2} \mathcal{J}^0 \mathcal{J}'_0$. Now, $\frac{1}{\kappa^2}$ means $\frac{1}{r}$ in space; so there is an instantaneous interaction between charges described by Coulomb's law. This contradicts the noninteraction found in the Hilbert space of solutions. An identical result follows in the Lorentz gauge for ultrarelativistic massive sources. If we neglect higher order terms in the law of electromagnetic interaction $\{k_0^2 - \kappa^2\}^{-1} \cdot \mathcal{J}^\mu \mathcal{J}'_\mu$ we shall find Coulomb's interaction energy. At the same time the Hilbert space solution (26) and current (25) give us $J_\alpha A^\alpha = 0$.

What is wrong with the generally accepted proof of the invariance of $\int A^\mu J_\mu d^4x$? Solution (14) differs from solution (19), (20) by the gauge transformation $A^\mu \to A^\mu + \Lambda_,{}^\mu$, but $\int \Lambda_{,\mu} J^\mu d^4x = \int (\Lambda J^\mu)_{,\mu} d^4x - \int \Lambda J^\mu{}_{,\mu} d^4x \neq \neq 0$, for in our case $\int (\Lambda J^\mu)_{,\mu} d^4x \neq 0$. Due to the bad asymptotics of Λ and the non-insular-like character of J^μ, $\int (\Lambda J^\mu)_{,\mu} d^4x$ cannot be transformed into the vanishing 3-surface integral $\int \Lambda J^i d\Sigma_i$.

6. Gravitational potentials of light-like electromagnetic and gravitational sources

Let us give a few sketchy proofs that gravitational potentials generated by light-like sources in general relativity are similar to the corresponding potentials in the Maxwell theory.

The Einstein equations are

$$-2\sqrt{-g}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)=16\pi\sqrt{-g}\,T^{\mu\nu}, \tag{27}$$

where $g^{\mu\nu}$ is the metric tensor, $g=\det g_{\mu\nu}$, $R^{\mu\nu}$ is the Ricci tensor, and $T^{\mu\nu}$ is the matter tensor. The linearized form of equation (27) is [7]

$$\Box\,\psi^{\mu\nu}-\psi^{\mu\alpha}{}_{,\alpha}{}^{\nu}-\psi^{\nu\alpha}{}_{,\alpha}{}^{\mu}+\eta^{\mu\nu}\psi^{\alpha\beta}{}_{,\alpha\beta}=16\pi T^{\mu\nu}, \tag{28}$$

$$\psi^{\mu\nu}\equiv\sqrt{-g}\,g^{\mu\nu}-\eta^{\mu\nu}.$$

In the case of electromagnetic radiation from insular sources

$$T^{\mu\nu}=\frac{w(u,\vartheta,\varphi)}{r^2}k^{\mu}k^{\nu}+O(r^{-3}). \tag{29}$$

We assume that $T^{\mu\nu}$ has a pulse character similar to that of J^{μ} in (5). In the harmonic gauge $\psi^{\alpha\beta}{}_{,\beta}=0$ and $\psi^{\mu\nu}=16\pi\,\Box^{-1}T^{\mu\nu}$ has the same type of "non-Hilbert behaviour" as A^{μ} in solution (14). Following Trautman [8], we assume

$$\psi^{\alpha\beta}=\frac{a^{\alpha\beta}(u,\vartheta,\varphi)}{r}+O(r^{-2}). \tag{30}$$

By inserting (30) into equation (28) and equating respectively the coefficients of r^{-1} and of r^{-2} we have

$$\dot{a}^{\alpha\beta}k_{\alpha}=\mathrm{const}=0, \tag{31}$$

$$-\dot{\psi}^{\mu\alpha}{}_{,\alpha}k^{\nu}-\dot{\psi}^{\nu\alpha}{}_{,\alpha}k^{\mu}=\frac{16\pi w(u,\vartheta,\varphi)}{r^2}k^{\mu}k^{\nu}+O(r^{-3}),$$

or

$$\dot{\psi}^{\alpha\mu}{}_{,\mu}=-\frac{8\pi w(u,\vartheta,\varphi)}{r^2}k^{\alpha}+O(r^{-3}). \tag{32}$$

By taking $\alpha=0$ and defining

$$\varepsilon(u,\vartheta,\varphi)\equiv 8\pi\int_{u_1}^{u}w(u',\vartheta,\varphi)\,du',\quad \psi^{00}\equiv A^{0},\quad \psi^{0i}\equiv A^{i},\quad i=1,2,3,$$

we find the solution of equation (32) in the form of (19) and (20). By taking into account condition (31) we obtain

$$\psi^{00}=\psi^{0r}=\psi^{rr}=\frac{2[m-M(u)]}{r}. \tag{33}$$

Instead of $\bar{Q}$ and $q(u)$ we have written $2m$ and $2M(u)$. The other components of $\psi^{\mu\nu}$ can be determined as well.

The full Einstein theory is given by the equation

$$\Box\,\psi^{\mu\nu}-\psi^{\mu\alpha}{}_{,\alpha}{}^{\nu}-\psi^{\nu\alpha}{}_{,\alpha}{}^{\mu}+\eta^{\mu\nu}\psi^{\alpha\beta}{}_{,\alpha\beta}=16\pi t^{\mu\nu}, \tag{34}$$

where $t^{\mu\nu}$ denotes non-linear terms in tensor density $\frac{\sqrt{-g}}{8\pi}\left(R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R\right)$. Using (30) and the formulae given by Goldberg [7] we find

$$t^{\mu\nu}=\frac{\left(\dot{a}^{\beta}_{\alpha}\dot{a}^{\alpha}_{\beta}-\frac{1}{2}\dot{a}^{\alpha}_{\alpha}\dot{a}^{\beta}_{\beta}\right)}{32\pi r^2}k^{\mu}k^{\nu}+O(r^{-3})\equiv\frac{w_g(u,\vartheta,\varphi)}{r^2}k^{\mu}k^{\nu}+O(r^{-3}), \quad (35)$$

$$\dot{\psi}^{\alpha\beta}{}_{,\beta}=-\frac{8\pi w_g(u,\vartheta,\varphi)}{r^2}k^{\alpha}+O(r^{-3}). \quad (36)$$

We can see that the self-interaction of gravitational radiation is of the same form as the interaction of the gravitational field with electromagnetic radiation.

A rigorous method of integrating the Einstein equations by making use of the Bondi coordinates, spin spherical harmonics and expansions in powers of $1/r$ is to be found in an earlier paper by the present author [9]. We shall not reproduce the results since our aim here is to argue that current views about gauge fields and gravitational radiation may need some revision.

REFERENCES

1. *Славнов А. А., Фаддеев Л. Д.* Введение в квантовую теорию калибровочных полей. М., Наука, 1988.
2. *Ахиезер А. И., Берестецкий В. Б.* Квантовая электродинамика. М., Наука, 1981.
3. *Джексон Дж.* Классическая электродинамика. М., Мир, 1965.
4. *Фок В. А.* Теория пространства, времени и тяготения, § 87. М., ГИФ-МЛ, 1961.
5. *Ogievetskij, V. I., Polubarinov, I. V.* Ann. Phys., 1963, **25**, № 3, 358—386.
6. *Feynman, R. P.* Acta Phys. Polon., 1963, **54**, Fasc 6(12), 697—722.
7. *Goldberg, J.* In Gravitation (ed. L. Witten), New York; London, J. Wiley & Sons, Inc., 1962.
8. *Trautman, A.* Bull. Acad. Polon. Sci. Math. Astr. Phys., 1958, **6**, № 5, 407—412.
9. *Kuusk, P., Unt, V.* Gen. Rel. Grav., 1976, **7**, 399—417.

Estonian Academy of Sciences,
Institute of Astrophysics and
Atmospheric Physics

Received
May 14, 1990

Väino UNT

INTERAKTEERUVATEST KALIBRATSIOONIVÄLJADEST

On tõestatud, et traditsioonilist interakteeruvate kalibratsiooniväljade käsitlust tuleb muuta, kui seda tahetakse rakendada valguse kiirusega liikuvate allikate või ultrarelativistlike allikate tekitatud väljade korral. Traditsioonilistes käsitlustes eeldatakse mitteilmutatud kujul, et Hilberti ruumi ja mitte-Hilberti ruumi kuuluvad lahendid on ekvivalentsed. Siinses töös on näidatud viimase väite paikapidamatust.

Вяйно УНТ

О ВЗАИМОДЕЙСТВУЮЩИХ КАЛИБРОВОЧНЫХ ПОЛЯХ

Традиционный подход к взаимодействующим гравитационным полям неприменим в случае источников, движущихся со скоростью света или с ультрарелятивистскими скоростями. В самых популярных калибровках появляются решения, которые не принадлежат к пространству Гильберта и не являются физически эквивалентными решениям гильбертова пространства.