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THE WITTEN AND THE NILSSON-KALLOSH CONSTRAINTS IN $N=1$, $D=10$ SUPERGRAVITY

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Пирет КУУСК. СВЯЗИ ВИТТЕНА И НИЛССОНА—КАЛЛОШ В $N=1$, $D=10$ СУПЕРГРАВИТАЦИИ

(*Presented by H. Keres*)

On-shell $N=1$, $D=10$ supergravity [1] can be presented in a geometrical form by means of a torsion two-form T^A of a 10-dimensional superspace * and a closed three-form G given in this superspace [2]. Since the torsion two-form contains much more superfields than are necessary for the physical supergravity theory, kinematic constraints can be imposed on some components of T^A. These constraints restrict the geometry of the superspace, but do not imply the equations of motion for the x-space fields. If, however, the constraints are imposed also on the three-form G, then from the Bianchi identities

$$DT^A=-E^B R_B{}^A, \qquad dG=0$$

the equations of motion will follow.

A possible form of the constraints was presented by E. Witten [3]

$$T_{\alpha\beta}{}^a=2\Gamma^a_{\alpha\beta}, \qquad T_{\alpha\beta}{}^\gamma=0, \qquad G_{\alpha\beta\gamma}=0.$$

$$T_{\alpha a}{}^b=0, \qquad T_{a\alpha}{}^\beta=(\Gamma_a\cdot\psi)_\alpha{}^\beta.$$

In [4] the corresponding solution of the Bianchi identities for T^A and G has been found:

$$T_{ab}{}^c=T_{abd}\eta^{dc}, \qquad \psi^{\alpha\beta}=-\frac{1}{24}T_{abc}(\Gamma^{abc})^{\alpha\beta},$$

$$T_{ab}{}^\alpha=J_{ab}{}^\alpha+2J_{\beta[a}\Gamma_{b]}{}^{\beta\alpha},$$

$$J_{\gamma a}=\frac{1}{56\cdot 24}D_\beta[\Gamma_a{}^{\beta\tau}(\Gamma^{dbc})_{\tau\gamma}T_{dbc}],$$

$$J_{ab}{}^\delta=\frac{1}{16}D_\beta[T_{jk[a}(\Gamma_{b]}{}^{jk})^{\beta\delta}]+\frac{1}{8}D_\beta T_{abk}\Gamma^{k\beta\delta};$$

$$G_{d\alpha\beta}=2e^{-\Phi}\Gamma_{d\alpha\beta}, \qquad D_\beta(\Gamma^{abc})^{\beta\gamma}T_{abc}=0,$$

$$G_{db\alpha}=-e^{-\Phi}(\Gamma_{db})_\alpha{}^\gamma D_\gamma\Phi,$$

$$G_{abd}=-3e^{-\Phi}T_{abd}.$$

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* The 10-dimensional curved superspace has coordinates $z^M=(x^m, \theta^\alpha)$, $m=1, \ldots, 10$, $\alpha=1, \ldots, 16$, the basis one-form E^A and the connection one-form $\omega_A{}^B$. The symmetric Dirac matrices $\Gamma^{\alpha\beta}_a$, $\Gamma^a_{\alpha\beta}$ satisfy

$$\Gamma_{a\,\alpha\beta}\Gamma_b{}^{\beta\gamma}+\Gamma_{b\,\alpha\beta}\Gamma_a{}^{\beta\gamma}=-2\eta_{ab}\delta^\gamma_\alpha, \qquad \eta_{ab}=\mathrm{diag}(-1, +1, \ldots, +1).$$

Here D_A denotes the covariant derivative, $T_{abc}(z)$ is an arbitrary antisymmetric superfield and $\Phi(z)$ is a scalar superfield (the dilaton field).

Another form of the constraints has been proposed by R. E. Kallosh and B. E. W. Nilsson [5]:

$$T'_{\alpha\beta}{}^{a}=2\Gamma^{a}_{\alpha\beta},\qquad T'_{\alpha\beta}{}^{\gamma}=\delta^{\gamma}_{(\alpha}D'_{\beta)}\tilde{\Phi},\qquad G'_{\alpha\beta\gamma}=0,$$

$$T'_{\alpha a}{}^{b}=\delta^{b}_{a}D'_{\alpha}\tilde{\Phi},\qquad T'_{ab}{}^{d}=0;$$

The corresponding solution of the Bianchi identities has been given in [6]:

$$T'_{a\beta}{}^{\gamma}=-\frac{1}{2}(\Gamma^{b}\cdot\Gamma_{a})_{\beta}{}^{\gamma}D'_{\beta}\tilde{\Phi}+(\Gamma_{a}{}^{bcd}-6\delta^{b}_{a}\Gamma^{cd})_{\beta}{}^{\gamma}Y_{bcd},$$

$$T'_{ab}{}^{\gamma}=H^{\gamma}{}_{ab}+2H_{\delta[a}\Gamma_{b]}{}^{\delta\gamma},$$

$$H_{\delta a}=-\frac{3}{28}(\hat{D}'_{\beta}Y_{abc})(\Gamma^{bc})^{\beta}{}_{\delta},$$

$$H^{\delta}{}_{ab}=3\left[(\hat{D}'_{\beta}Y_{abc})\Gamma^{c\beta\delta}+\frac{1}{2}(\hat{D}'_{\beta}Y_{cd[a})(\Gamma_{b]}{}^{cd})^{\beta\delta}\right],$$

$$G'_{d\alpha\beta}=2\Gamma_{d\alpha\beta},\qquad \hat{D}'_{\beta}=D'_{\beta}+D'_{\beta}\tilde{\Phi},$$

$$G'_{db\alpha}=2(\Gamma_{db})_{\alpha}{}^{\gamma}D'_{\gamma}\tilde{\Phi},$$

$$G'_{abd}=72Y_{abd}.$$

Here $Y_{abd}(z)$ is an arbitrary antisymmetric superfield and $\tilde{\Phi}(z)$ is the dilaton superfield.

From the very beginning it was conjectured that both sets of constraints are equivalent up to some transformations and field redefinitions. A conformal transformation connecting the constraints on the torsion components was indicated in [7]. Here we present the full set of transformations/redefinitions by means of which the full solution of the Bianchi identities for T^A and G with the Nilsson-Kallosh constraints can be obtained from the solution with the Witten constraints.

1. A conformal transformation

$$E'^{A}=E^{B}\Lambda_{B}{}^{A},$$

$$T'_{BC}{}^{D}=(-1)^{b(c+a)}\tilde{\Lambda}_{C}{}^{A}\tilde{\Lambda}_{B}{}^{E}[T_{EA}{}^{F}\Lambda_{F}{}^{D}+\partial_{E}\Lambda_{A}{}^{D}-(-1)^{ae}\partial_{A}\Lambda_{E}{}^{D}],$$

$$\Lambda_{A}{}^{B}\tilde{\Lambda}_{B}{}^{D}=\delta^{D}_{A},\qquad \partial_{E}=E_{E}{}^{M}\partial_{M}$$

with

$$\Lambda_{b}{}^{a}=\exp\left(-\frac{\Phi}{2}\right)\delta^{a}_{b},\qquad \Lambda_{\beta}{}^{a}=0.$$

$$\Lambda_{\beta}{}^{\alpha}=-\exp\left(-\frac{\Phi}{4}\right)\delta^{\alpha}_{\beta},\qquad \Lambda_{\beta}{}^{a}=0.$$

2. A redefinition that connects superfields T_{abc}, Φ and Y_{abc}, $\tilde{\Phi}$:

$$\Phi=-2\tilde{\Phi},\qquad T_{abc}=-24e^{\tilde{\Phi}}Y_{abc}.$$

3. A redefinition of the connection $\omega_{AB}{}^{D}$ of the superspace:

$$\omega'_{AB}{}^{D}=\omega_{AB}{}^{D}+\Delta_{AB}{}^{D},$$

$$T'_{AB}{}^{D}=T_{AB}{}^{D}-(\Delta_{AB}{}^{D}-(-1)^{ab}\Delta_{BA}{}^{D}),$$

$$\Delta_{ab}{}^{d}=\left(\frac{1}{2}e^{-\tilde{\Phi}}T_{abc}+\partial'_{c}\tilde{\Phi}\eta_{ab}-\partial'_{b}\tilde{\Phi}\eta_{ac}\right)\eta^{cd},\qquad \Delta_{\alpha b}{}^{d}=0.$$

Here we have assumed that the connection one-form $\omega_A{}^B$ and the three-form G do not transform under the conformal transformation. This implies the covariant transformation rules for the components $\omega_{EA}{}^B$, G_{ABD}.

These direct calculations confirm once more that if the Maurer-Cartan structure equations hold, then the redefinition of the connection of the superspace converts a solution of the Bianchi identities for T^A and G into a new solution [8]. They also confirm another result that can be obtained by a direct substitution — any conformal transformation $E'^A = E^B \Lambda_B{}^A$, $\Lambda_B{}^A \sim \delta^A_B$ transforms a solution of the Bianchi identities for T^A and G into a new solution.

The most general transformations preserving the constraint $T_{\alpha\beta}{}^a = 2\Gamma^a_{\alpha\beta}$ were recently investigated in detail in [9]. These transformations also contain the above-considered conformal transformations and redefinitions of the connection of the superspace. The analysis given in [9] clarifies the general structure of possible constraints on the torsion components $T_{AB}{}^D$ and the corresponding solutions of the Bianchi identities for T^A.

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