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HIGHER RANK REPRESENTATIONS AND CAUSALITY

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P.-K. ЛОЙДЕ. ПРЕДСТАВЛЕНИЯ ВЫСШЕГО РАНГА И ПРИЧИННОСТЬ

(Presented by A. Humal)

In a recent paper [1] there were given two new causal $s=0$ wave equations, using higher rank representations. It was also stated that equations with nilpotent β-matrices need not be inconsistent. It is well known that the acausality in minimal electromagnetic coupling is necessarily connected with nilpotency of β-matrices. The examples of causal equations with nilpotent β-matrices [2] showed that in this case the nilpotency is not essential but is caused by barnacles (ballast components). Then the equation is equivalent to a simpler equation with diagonalizable β-matrices which is known to be causal [3].

In this note we demonstrate that the causal $s=0$ equations with higher-rank representations given in [1], are barnacled equations equivalent to the Kemmer-Duffin $s=0$ equation. Therefore these equations are not physically interesting, and indicate that the introduction of higher-rank representations is not useful.

Here we treat only the equation with representation $(1,1)\oplus 2(1/2,1/2)\oplus \oplus(0,0)$, since the other example with representation $(1,0)\oplus 2(1/2,1/2)\oplus \oplus(0,1)\oplus(0,0)$ may be treated similarly. In the notations of the paper [1], the equation is

$$S_{\mu\nu}=b_1\{\partial_\mu A_\nu\}_S+b_3\{\partial_\mu B_\nu\}_S,$$

$$A_\mu=\tilde b_1\partial^\nu S_{\mu\nu}+b_3\partial_\mu\Phi,$$

$$B_\mu=\tilde b_2\partial^\nu S_{\mu\nu}+b_4\partial_\mu\Phi, \tag{1}$$

$$\Phi=\tilde b_3\partial^\mu A_\mu+\tilde b_4\partial^\mu B_\mu.$$

Here $\{\partial_\mu A_\nu\}_S=1/2(\partial_\mu A_\nu+\partial_\nu A_\mu-1/2g_{\mu\nu}\partial^\rho A_\rho)$. There are imposed the following conditions on the coefficients

$$\tilde b_1 b_1+\tilde b_2 b_2=0, \tag{2}$$

$$b_1 b_3+b_2 b_4=0 \quad \text{or} \quad \tilde b_1\tilde b_3+\tilde b_2\tilde b_4=0, \tag{3}$$

$$\tilde b_3 b_3+\tilde b_4 b_4<0. \tag{4}$$

The condition (2) guarantees the nilpotency of $s=1$ block, and leaves only $s=0$ states. The condition (3) gives a unique $s=0$ state, and (4) gives the mass term.

Now we, using (2) and (3), demonstrate the barnacle structure of the equation (1). Depending on the choice of the condition (3), we get different types of barnacles.

1 case. $b_1 b_3+b_2 b_4=0$.
Let us introduce new vector fields C_μ and D_μ

$$C_\mu = b_1 A_\mu + b_2 B_\mu,$$

$$D_\mu = \alpha A_\mu + \beta B_\mu, \tag{5}$$

where α and β satisfy

$$d = b_1 \beta - b_2 \alpha \neq 0. \tag{6}$$

Using (1), (2), (5), and (6), we get a new system for $S_{\mu\nu}$, C_μ, D_μ, and Φ

$$S_{\mu\nu} = \{\partial_\mu C_\nu\}_S,$$

$$C_\mu = 0,$$

$$D_\mu = (\alpha b_3 + \beta b_4) \partial_\mu \Phi + (\alpha \tilde{b}_1 + \beta \tilde{b}_2) \partial^\nu S_{\mu\nu}, \tag{7}$$

$$\Phi = d^{-1}(\tilde{b}_4 b_1 - \tilde{b}_3 b_2) \partial^\mu D_\mu + d^{-1}(\tilde{b}_3 \beta - \tilde{b}_4 \alpha) \partial^\mu C_\mu.$$

From (7) we can see that $C_\mu = 0$, and $S_{\mu\nu} = 0$. Therefore for D_μ and Φ we have a separate equation

$$D_\mu = (\alpha b_3 + \beta b_4) \partial_\mu \Phi,$$

$$\Phi = d^{-1}(\tilde{b}_4 b_1 - \tilde{b}_3 b_2) \partial^\mu D_\mu. \tag{8}$$

The equation (8) is equivalent to the well-known Kemmer-Duffin $s = 0$ equation.

As we can see, $S_{\mu\nu}$ and C_μ are the ballast components (barnacles) [4] and are not playing an essential role. Introducing the minimal electromagnetic coupling by substitution $\partial_\mu \to \partial_\mu - ie\Phi_\mu$, where Φ_μ is the electromagnetic potential, we do not change the general structure of the equation (7), and, since $C_\mu = S_{\mu\nu} = 0$, we have the minimally coupled Kemmer-Duffin equation, which is causal. In [4] it is mentioned that the ballast components can play an essential role only in such couplings which change the structure of a given equation, therefore the coupling must be nonminimal.

The coefficients α and β are arbitrary except for the condition (6). Since one of the coefficients, b_1 or b_2, must be nonzero, the condition (6) may be easily satisfied.

2. c a s e. $\tilde{b}_1 \tilde{b}_3 + \tilde{b}_2 \tilde{b}_4 = 0$.

Now we introduce the following fields — C_μ and D_μ

$$C_\mu = \alpha A_\mu + \beta B_\mu,$$

$$D_\mu = \tilde{b}_3 A_\mu + \tilde{b}_4 B_\mu, \tag{9}$$

where α and β satisfy $d = \tilde{b}_3 \beta - \tilde{b}_4 \alpha \neq 0$.

For $S_{\mu\nu}$, C_μ, D_μ, and Φ we get the following system —

$$S_{\mu\nu} = d^{-1}(b_1 \beta - b_2 \alpha)\{\partial_\mu D_\nu\}_S + d^{-1}(b_2 \tilde{b}_3 - b_1 \tilde{b}_4)\{\partial_\mu C_\nu\}_S,$$

$$C_\mu = (b_1 b_3 + b_2 b_4) \partial_\mu \Phi + (\alpha \tilde{b}_1 + \beta \tilde{b}_2) \partial^\nu S_{\mu\nu},$$

$$D_\mu = (\tilde{b}_3 b_3 + \tilde{b}_4 b_4) \partial_\mu \Phi, \tag{10}$$

$$\Phi = \partial^\mu D_\mu.$$

From (10) we have a different barnacle structure. For D_μ and Φ we have the Kemmer-Duffin equation. Now, $S_{\mu\nu}$ and C_μ do not vanish identically. Using the nilpotency condition (2) which eliminates $s = 1$, it is possible to verify that $S_{\mu\nu}$ and C_μ are uniquely determined by the

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solution of Kemmer-Duffin equation for D_μ and Φ (most easily it may be verified choosing $\alpha = b_1$ and $\beta = b_2$). Therefore $S_{\mu\nu}$ and C_μ are ballast components which do not play an independent role.

The minimal substitution $\partial_\mu \to \partial_\mu - ie\Phi_\mu$ does not change the general structure of (11), and therefore (10) is causal.

In this note we have shown that the inessential nilpotency (caused by barnacles) does not lead to the acausality. The essential nilpotency, as in $s = 0$ equation with the representation $(1,1) \oplus (1/2, 1/2) \oplus (0, 0) \oplus \oplus (1, 0) \oplus (0, 1)$, gives the acausal equation, as has also been demonstrated in [1]. So we can see that the usage of higher rank representations does not help us to deal with causality problems. Higher-rank representations always introduced additional spins which had to be removed by nilpotency. Therefore the higher-rank representations allowed low-spin equations with nilpotent β-matrices. If the nilpotency is essential, we have acausality; otherwise we have barnacled equations which are equivalent to the simpler equations without higher-rank representations. In conclusion, the introduction of higher-rank representations is not useful.

In our case the barnacles are caused by the repeated representations (two vectors A_μ and B_μ). As it was first demonstrated in [5], in the case of repeated representations one has an additional freedom to mix them by unitary transformations and eliminate some of the linkages (see, for example, the Amar-Dozzio equation in [4]). This freedom means that using repeated representations one, in general, gets the barnacled equation (contrary to the proposition [5] stating that we get independent equations).

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