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EQUIVALENT INPUT-OUTPUT DESCRIPTION
 FOR BUCY CANONICAL STATE-SPACE REPRESENTATION

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ЮЛЛЕ КОТТА. ЭКВИВАЛЕНТНЫЕ ВХОД-ВЫХОД-ПРЕДСТАВЛЕНИЯ УРАВНЕНИИ СОСТОЯ-
 НИЯ В КАНОНИЧЕСКОЙ ФОРМЕ БУСИ

(Presented by N. Alumäe)

1. Introduction. Consider the linear multivariable finite-dimensional discrete-time time-invariant dynamic system. Such systems can be modelled by state-space equations

$$x(t+1) = Fx(t) + Gu(t), \quad y(t) = Hx(t), \quad (1)$$

where $x(t)$, $u(t)$, $y(t)$ are the $n \times 1$ state vector, $m \times 1$ input vector and $p \times 1$ output vector, respectively, F , G , H are constant matrices of compatible dimensions or by ARMA forms such as

$$A(z)y(t) = B(z)u(t). \quad (2)$$

In (2), $A(z)$, $B(z)$ are matrices whose elements are polynomials in the forward shift operator z . Our purpose here will be to find the representation (2) from the state-space form (1) and vice versa, under the assumption that the state representation is in Bucy canonical form. The analogical relations for Luenberger canonical form are given in [1].

2. Canonical state-space representation. It is known [2] that in representation (1) the couple (F, H) of a completely observable system can be transformed to the following (Bucy) canonical structure:

$$F = [F_{ij}], \quad H = [H_{ij}], \quad i, j = 1, \dots, p,$$

$$F_{ii} = \begin{bmatrix} 0 & I_{n_i-1} \\ a_1^{ii} & \dots & a_{n_i}^{ii} \end{bmatrix}, \quad F_{ij} = \begin{bmatrix} 0 \\ a_1^{ij} & \dots & a_{n_j}^{ij} \end{bmatrix}, \quad j < i,$$

$$F_{ij} = 0, \quad j > i, \quad H_{ii} = [1 \ 0 \ \dots \ 0], \quad H_{ij} = 0, \quad i \neq j.$$

3. Input-output structure. The input-output difference equation of the type (2) will now be deduced from the triple (F, G, H) . Because of the canonical structure of the matrices F and H , it is easy to derive from (1) the following expression

$$x(t) = V(z)y(t) - WZ(z)u(t), \quad (3)$$

where

$$V(z) = V_{ij}(z), \quad i, j = 1, \dots, p,$$

$$V_{ii}(z) = [1 z \dots z^{n_i-1}]^T, \quad V_{ij}(z) = 0, \quad i \neq j,$$

$$\begin{matrix} (n_i \times 1) \\ (n_i \times 1) \end{matrix}$$

$$W = [W_{ij}], \quad i=1, \dots, p, \quad j=1, \dots, n_M, \quad n_M = \max_i n_i,$$

$$W_{ij} = \begin{matrix} \begin{matrix} 0 \\ \vdots \\ 0 \\ g_{n_i+\dots+n_i-i+1} \\ \vdots \\ g_{n_i+\dots+n_i-j} \end{matrix} \end{matrix}, \quad G = \begin{matrix} \begin{matrix} g_1 \\ \vdots \\ g_n \end{matrix} \end{matrix}, \quad Z(z) = \begin{matrix} \begin{matrix} I_m \\ zI_m \\ \vdots \\ z^{n_M-1}I_m \end{matrix} \end{matrix}.$$

The substitution of (3) in (1) leads to the input-output description

$$\{(zI - F)V(z)\}y(t) = \{(zI - F)WZ(z) + B\}u(t). \quad (4)$$

In representation (4), only the n_1 th, (n_1+n_2) th, ..., n th equations are significant; the remaining ones are simple identities. The significant equations in (4) can be written in the form

$$A(z)y(t) = B(z)u(t),$$

where

$$A(z) = [a_{ij}(z)], \quad i, j=1, \dots, p; \quad B(z) = [b_{ij}(z)], \quad \begin{matrix} i=1, \dots, p, \\ j=1, \dots, m. \end{matrix}$$

The polynomials of $A(z)$ can be immediately obtained by computing the left side of expression (4); it follows that

$$a_{ii}(z) = z^{n_i} - a_{n_i}^{ii} z^{n_i-1} - \dots - a_1^{ii},$$

$$a_{ij}(z) = a_{n_i}^{ij} z^{n_i-1} - \dots - a_1^{ij}, \quad j < i,$$

$$a_{ij}(z) = 0, \quad j > i.$$

The polynomials of $B(z)$ are obtained by computing the right side of expression (4); it follows that

$$b_{ij}(z) = b_{v_i}^{ij} z^{v_i-1} + \dots + b_1^{ij},$$

where the coefficients b_h^{ij} are the elements of the matrix

$$B = MG = \begin{matrix} \begin{matrix} b_1^{11} & \dots & b_1^{1m} \\ \vdots & & \vdots \\ b_{v_1}^{11} & \dots & b_{v_1}^{1m} \\ \vdots & & \vdots \\ b_1^{p1} & \dots & b_1^{pm} \\ \vdots & & \vdots \\ b_{v_p}^{p1} & \dots & b_{v_p}^{pm} \end{matrix} \end{matrix}, \quad v_i = \max\{(n_1-1), (n_2-1), \dots, n_i\}$$

and the matrix M is given by

$$M = [M_{ij}], \quad i, j = 1, \dots, p,$$

$$M_{ii} = \begin{pmatrix} -a_2^{ii} & -a_3^{ii} & \dots & -a_{n_i}^{ii} & 1 \\ -a_3^{ii} & & & & 1 \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ -a_{n_i}^{ii} & 1 & & & \\ 1 & & & & \\ 0 & & & & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & & & & \end{pmatrix},$$

$$M_{ij} = \begin{pmatrix} -a_2^{ij} & -a_3^{ij} & \dots & -a_{n_j}^{ij} & 0 \\ -a_3^{ij} & & & & 0 \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ -a_{n_j}^{ij} & 0 & & & \\ 0 & & & & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & & & & \end{pmatrix}, \quad j < i,$$

$$M_{ij} = 0, \quad j > i.$$

The equivalence between the Bucy canonical state-space representation and the input-output representation has thus been established.

The structural indices n_1, \dots, n_p can be immediately deduced by inspection, indifferently from the knowledge of F or of $A(z)$; also from the parametric standpoint, F and $A(z)$ are equivalent. Matrix H can be directly written if $n_1 \dots n_p$ are known.

To obtain the matrix $B(z)$ from the knowledge of the couple (F, G) , it is necessary to construct the matrix M (by direct inspection of elements of F), the elements of $B = MG$ are then the coefficients of the polynomials $b_{ij}(z)$. Note that not always the elements of B are independent, it is only if for every i $n_i + 1 \geq \max(n_1, \dots, n_i)$.

To obtain the matrix G from the knowledge of $A(z)$, $B(z)$ it is first necessary to construct by direct inspection the matrices M and B , then find the nonsingular submatrix M_1 of order n from M ; matrix G is then given by $G = M_1^{-1}B$. Note that it is always possible to find M_1 because of the structure of M , in fact, we can always choose the following rows of M :

$$1, 2, \dots, n_1 + n_2, v_1 + v_2 + 1, \dots, v_1 + v_2 + n_3, \dots,$$

$$1 + \sum_{i=1}^{p-1} v_i, \dots, n_p + \sum_{i=1}^{p-1} v_i.$$

4. Example. Consider the canonical triple

$$F = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ a_1^{11} & a_2^{11} & a_3^{11} & 0 \\ a_1^{21} & a_2^{21} & a_3^{21} & a_1^{22} \end{pmatrix}, \quad G = \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{pmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In this case $n_1=3$, $n_2=1$ and it follows that

$$B = \begin{pmatrix} b_1^{11} \\ b_2^{11} \\ b_3^{11} \\ b_1^{21} \\ b_2^{21} \end{pmatrix} = \begin{pmatrix} -a_2^{11} & -a_3^{11} & 1 & 0 \\ -a_3^{11} & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -a_2^{21} & -a_3^{21} & 0 & 1 \\ -a_3^{21} & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{pmatrix}.$$

Therefore the input-output representation is

$$\begin{bmatrix} z^3 & -a_3^{11} z^2 & -a_2^{11} z & -a_1^{11} & 0 \\ -a_3^{21} z^2 & -a_2^{21} z & -a_1^{21} & z - a_1^{22} & 0 \end{bmatrix} y(t) = \begin{bmatrix} g_1 z^2 + (g_2 - a_3^{11} g_1) z + (g_3 - a_3^{11} g_2 - a_2^{11} g_1) \\ -a_3^{21} g_1 z + (g_4 - a_2^{21} g_1 - a_3^{12} g_2) \end{bmatrix} u(t).$$

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Received
Apr. 27, 1981