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EQUIVALENT INPUT-OUTPUT DESCRIPTION FOR BUCY CANONICAL STATE-SPACE REPRESENTATION

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ЮЛЛЕ КОТТА. ЭКВИВАЛЕНТНЫЕ ВХОД—ВЫХОД-ПРЕДСТАВЛЕНИЯ УРАВНЕНИЙ СОСТОЯНИЯ В КАНОНИЧЕСКОЙ ФОРМЕ БЮСИ

(Presented by N. Alumäe)

1. Introduction. Consider the linear multivariable finite-dimensional discrete-time time-invariant dynamic system. Such systems can be modelled by state-space equations

$$x(t+1)=Fx(t)+Gu(t), \quad y(t)=Hx(t), \tag{1}$$

where $x(t)$, $u(t)$, $y(t)$ are the $n\times1$ state vector, $m\times1$ input vector and $p\times1$ output vector, respectively, F, G, H are constant matrices of compatible dimensions or by ARMA forms such as

$$A(z)y(t)=B(z)u(t). \tag{2}$$

In (2), $A(z)$, $B(z)$ are matrices whose elements are polynomials in the forward shift operator z. Our purpose here will be to find the representation (2) from the state-space form (1) and vice versa, under the assumption that the state representation is in Bucy canonical form. The analogical relations for Luenberger canonical form are given in [1].

2. Canonical state-space representation. It is known [2] that in representation (1) the couple (F, H) of a completely observable system can be transformed to the following (Bucy) canonical structure:

$$F=[F_{ij}], \quad H=[H_{ij}], \quad i, j=1, \ldots, p,$$

$$\underset{(n_i\times n_i)}{F_{ii}}=\begin{bmatrix} 0 & I_{n_i-1} \\ a_1^{ii} & \ldots\ a_{n_i}^{ii} \end{bmatrix}, \quad \underset{(n_i\times n_j)}{F_{ij}}=\begin{bmatrix} & 0 & \\ a_1^{ij} & \ldots & a_{n_j}^{ij} \end{bmatrix}, \quad j<i,$$

$$\underset{(n_i\times n_j)}{F_{ij}}=0, \quad j>i, \quad \underset{(1\times n_i)}{H_{ii}}=[1\ 0\ \ldots\ 0], \quad \underset{(1\times n_j)}{H_{ij}}=0, \quad i\neq j.$$

3. Input-output structure. The input-output difference equation of the type (2) will now be deduced from the triple (F, G, H). Because of the canonical structure of the matrices F and H, it is easy to derive from (1) the following expression

$$x(t)=V(z)y(t)-WZ(z)u(t), \tag{3}$$

where

$$V(z)=V_{ij}(z), \quad i, j=1, \ldots, p,$$

$$\underset{(n_i\times 1)}{V_{ii}(z)}=[1\,z\ldots z^{n_i-1}]^{\mathrm{T}},\quad \underset{(n_i\times 1)}{V_{ij}(z)}=0,\quad i\neq j,$$

$$W=[W_{ij}],\quad i=1,\ldots,p,\quad j=1,\ldots,n_M,\quad n_M=\max_i n_i,$$

$$\underset{(n_i\times m)}{W_{ij}}=\left\|\begin{matrix}0\\ \cdot\\ \cdot\\ \cdot\\ 0\\ g_{n_1+\ldots+n_{i-1}+1}\\ \cdot\\ \cdot\\ \cdot\\ g_{n_1+\ldots+n_i-j}\end{matrix}\right\|,\quad G=\left\|\begin{matrix}g_1\\ \cdot\\ \cdot\\ \cdot\\ g_n\end{matrix}\right\|,\quad Z(z)=\left\|\begin{matrix}\mathrm{I}_m\\ z\mathrm{I}_m\\ \cdot\\ \cdot\\ \cdot\\ z^{n_M-1}\mathrm{I}_m\end{matrix}\right\|.$$

The substitution of (3) in (1) leads to the input-output description

$$\{(z\mathrm{I}-F)\,V(z)\}\,y(t)=\{(z\mathrm{I}-F)\,WZ(z)+B\}\,u(t). \qquad (4)$$

In representation (4), only the n_1th, (n_1+n_2)th, ..., nth equations are significant; the remaining ones are simple identities. The significant equations in (4) can be written in the form

$$A(z)\,y(t)=B(z)\,u(t),$$

where

$$A(z)=[a_{ij}(z)],\quad i,j=1,\ldots,p;\quad B(z)=[b_{ij}(z)],\quad \begin{matrix}i=1,\ldots,p,\\ j=1,\ldots,m.\end{matrix}$$

The polynomials of $A(z)$ can be immediately obtained by computating the left side of expression (4); it follows that

$$a_{ii}(z)=z^{n_i}-a^{ii}_{n_i}z^{n_i-1}-\ldots-a^{ii}_1,$$

$$a_{ij}(z)=a^{ij}_{n_j}z^{n_j-1}-\ldots-a^{ij}_1,\quad j<i,$$

$$a_{ij}(z)=0,\quad j>i.$$

The polynomials of $B(z)$ are obtained by computating the right side of expression (4); it follows that

$$b_{ij}(z)=b^{ij}_{v_i}z^{v_i-1}+\ldots+b^{ij}_1,$$

where the coefficients $b_k{}^{ij}$ are the elements of the matrix

$$B=MG=\left\|\begin{matrix}b^{11}_1 & \ldots & b^{1m}_1\\ \cdot & & \cdot\\ \cdot & & \cdot\\ \cdot & & \cdot\\ b^{11}_{v_1} & \ldots & b^{1m}_{v_1}\\ \cdot & \cdot\;\cdot\;\cdot & \cdot\\ b^{p1}_1 & \ldots & b^{pm}_1\\ \cdot & & \cdot\\ \cdot & & \cdot\\ \cdot & & \cdot\\ b^{p1}_{v_p} & \ldots & b^{pm}_{v_p}\end{matrix}\right\|,\quad v_i=\max\{(n_1-1),\ (n_2-1),\ \ldots,\ n_i\}$$

and the matrix M is given by

$$M=[M_{ij}], \quad i, j=1, \ldots, p,$$

$$\underset{(v_i \times n_i)}{M_{ii}=} \left\| \begin{matrix} -a_2^{ii} & -a_3^{ii} & \ldots & -a_{n_i}^{ii} & 1 \\ -a_3^{ii} & & \ldots & 1 & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ -a_{n_i}^{ii} & 1 & & & \\ 1 & & & & \\ 0 & & & & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & & & & \end{matrix} \right\|,$$

$$\underset{(v_i \times n_j)}{M_{ij}=} \left\| \begin{matrix} -a_2^{ij} & -a_3^{ij} & \ldots & -a_{n_j}^{ij} & 0 \\ -a_3^{ij} & & \ldots & 0 & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ -a_{n_j}^{ij} & 0 & & & \\ 0 & & & & \\ \cdot & & & & \\ \cdot & & & & \\ \cdot & & & & \\ 0 & & & & \end{matrix} \right\|, \quad j<i,$$

$$M_{ij}=0, \quad j>i.$$

The equivalence between the Bucy canonical state-space representation and the input-output representation has thus been established.

The structural indices $n_1, \ldots, n_p$ can be immediately deduced by inspection, indifferently from the knowledge of F or of $A(z)$; also from the parametric standpoint, F and $A(z)$ are equivalent. Matrix H can be directly written if $n_1 \ldots n_p$ are known.

To obtain the matrix $B(z)$ from the knowledge of the couple (F, G), it is necessary to construct the matrix M (by direct inspection of elements of F), the elements of $B=MG$ are then the coefficients of the polynomials $b_{ij}(z)$. Note that not always the elements of B are independent, it is only if for every i $n_i+1 \geqslant \max(n_1, \ldots, n_i)$.

To obtain the matrix G from the knowledge of $A(z)$, $B(z)$ it is first necessary to construct by direct inspection the matrices M and B, then find the nonsingular submatrix M_1 of order n from M; matrix G is then given by $G=M_1^{-1}B$. Note that it is always possible to find M_1 because of the structure of M, in fact, we can always choose the following rows of M:

$$1, 2, \ldots, n_1+n_2, v_1+v_2+1, \ldots, v_1+v_2+n_3, \ldots,$$

$$1+\sum_{i=1}^{p-1} v_i, \ldots, n_p+\sum_{i=1}^{p-1} v_i.$$

4. Example. Consider the canonical triple

$$F=\begin{Vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \overset{11}{a}_1 & \overset{11}{a}_2 & \overset{11}{a}_3 & 0 \\ \overset{21}{a}_1 & \overset{21}{a}_2 & \overset{21}{a}_3 & \overset{22}{a}_1 \end{Vmatrix}, \quad G=\begin{Vmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{Vmatrix}, \quad H=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In this case $n_1=3$, $n_2=1$ and it follows that

$$B=\begin{Vmatrix} \overset{11}{b}_1 \\ \overset{11}{b}_2 \\ \overset{11}{b}_3 \\ \overset{21}{b}_1 \\ \overset{21}{b}_2 \end{Vmatrix}=\begin{Vmatrix} -\overset{11}{a}_2 & -\overset{11}{a}_3 & 1 & 0 \\ -\overset{11}{a}_3 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ -\overset{21}{a}_2 & -\overset{21}{a}_3 & 0 & 1 \\ -\overset{21}{a}_3 & 0 & 0 & 0 \end{Vmatrix} \times \begin{Vmatrix} g_1 \\ g_2 \\ g_3 \\ g_4 \end{Vmatrix}.$$

Therefore the input-output representation is

$$\begin{bmatrix} z^3 \;\; -\overset{11}{a}_3 z^2 \;\; -\overset{11}{a}_2 z \;\; -\overset{11}{a}_1 & 0 \\ -\overset{21}{a}_3 z^2 \;\; -\overset{21}{a}_2 z \;\; -\overset{21}{a}_1 & z-\overset{22}{a}_1 \end{bmatrix} y(t)=$$

$$=\begin{bmatrix} g_1 z^2+(g_2-\overset{11}{a}_3 g_1) z+(g_3-\overset{11}{a}_3 g_2-\overset{11}{a}_2 g_1) \\ -\overset{21}{a}_3 g_1 z+(g_4-\overset{21}{a}_2 g_1-\overset{12}{a}_3 g_2) \end{bmatrix} u(t).$$

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