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CONSTITUTIVE RATE-DEPENDENT THEORY OF THERMOVISCOELASTICITY

1. Introduction

The most used theory of heat conduction dates back to the beginning of the nineteenth century when Fourier proposed his well-known law

$$Q_A = k_{AB} G_B \tag{1.1}$$

where $G_B = T_{,B}$ is the temperature gradient. Here we refer to a fixed set of rectangular Cartesian axes in the initial configuration. The usual index notation operates throughout, and Latin upper case indices range over the values 1, 2, 3. A comma followed by the index B denotes differentiation with respect to X_B where X_B are the components of a typical position vector in the initial configuration. The components of the heat flux vector are Q_A and the conductivity matrix is given by its components k_{AB}.

This law leads to the paradox of infinite thermal wave-speeds in the dynamic theory of heat conducting continuous media. The paradox has, in the past ten years, been solved in several ways. The first approach dates back to Maxwell [1], who deduced a rate-type law of heat conduction

$$\tau_0 \dot{Q}_A + Q_A = k_{AB} G_B \tag{1.2}$$

where τ_0 is a relaxation time, and a superposed dot indicates time differentiation. The rate-type law was employed later for gases by Vernotte [2], Cattaneo [3], and Grad [4]. Chester [5] has postulated the modified heat conduction law in a discussion of second sound in solids. Further work in the theory of the rate-type heat conduction has been presented in subsequent years by, amongst others, Lord and Shulman [6], Norwood and Warren [7], Achenbach [8], Luikov [9], Kiltschinskaya [10], Kaliski [11] and Beevers [12]. Surveys of the problem have been presented by Francis [13] and Lindholm and Francis [14].

A more sophisticated approach has been outlined by Gurtin and Pipkin [15], McCarthy [16] and Nunziato [17], who have considered memory effects in continuum mechanics. In effect, the rate-type law of heat conduction assumes that memory effects are important in the heat flux only (see Achenbach [8]). The general history-dependent theories are much more difficult to employ.

The third approach is that of Green and Lindsay [18]. In that theory, the authors introduce a new functional Φ which depends upon T and its material time rate $\dot{T}$. The unknown functional Φ appears in an extended version of the Clausius-Duhem inequality and requires a constitutive assumption of its own. Green and Lindsay [18] show that their equations predict a finite thermal wave-speed.

In this paper we present a nonlinear theory which takes into consideration both the temperature rate and the deformation rate. This approach follows from the principle of equipresence: "a variable present as an independent variable in one constitutive equation should be so present in all" (p. 704 [19]). Further we show with the help of the entropy inequality the possibility of neglecting some terms. We will adopt a procedure similar to that of Fox [20], who has deduced a consistent theory by proposing constitutive assumptions for $\dot{q}_i$, the material time rate of the heat flux vector. He is primarily concerned with fluids, and so the Eulerian description is used in [20]. In following we use the Lagrangian description since we are concerned with solids and derive the nonlinear dynamic rate-dependent theory which, according to our principles, is the theory of thermoviscoelasticity.

In Section 2 we briefly state the governing equations before presenting in Section 3 the constitutive theory. In Section 4 an example of a mathematical model of nonlinear theory of thermoviscoelasticity is given.

2. Basic theory

Continuum mechanics is based on balance laws, an entropy inequality and constitutive equations. We use the Lagrangian description so that we have

$$x_i = x_i(X_A, t) \tag{2.1}$$

where X_A are the initial components of the particle which of time t is at x_i. The velocity v_i and deformation F_{iA} are then

$$v_i = \dot{x}_i, \quad F_{iA} = x_{i,A}. \tag{2.2}$$

The balance of mass in Lagrangian description is given by

$$\varrho = \varrho_0 \det(F_{iA}) \tag{2.3}$$

where ϱ, ϱ_0 are the current and initial densities respectively.

Further, the conservation law for momentum states that

$$L_{Ai,A} + \varrho_0 f_i = \varrho_0 a_i \tag{2.4}$$

where L_{Ai} are the components of the unsymmetric Piola-Kirchhoff stress tensor, a_i are the components of acceleration vector, and f_i are the components of the body force per unit mass. The balance of energy gives

$$\varrho_0 r + Q_{A,A} - \varrho_0 \dot{W} + L_{Ai} v_{i,A} = 0 \tag{2.5}$$

where r is the heat supply per unit mass, W is the internal energy per unit mass and Q_A are the components of the pseudoheat flux vector in the Lagrangian description.

If we define the symmetric Kirchhoff stress tensor with components K_{AB} and the Green strain tensor with components E_{AB} by

$$K_{AB} = F_{iA}^{-1} L_{Bi}, \quad E_{AB} = \frac{1}{2}(F_{iA}F_{iB} - \delta_{AB}) \tag{2.6}$$

then the energy equation (2.5) becomes

$$\varrho_0 r + Q_{A,A} - \varrho_0 \dot{W} + K_{AB}\dot{E}_{AB} = 0 \tag{2.7}$$

where we use the property $K_{AB} = K_{BA}$.

Finally, from the second law of thermodynamics we get the Clausius-Duhem inequality in point-wise form

$$\varrho_0 \dot{S} - \frac{\varrho_0 r}{T} - \left(\frac{Q_A}{T}\right)_{,A} \geqslant 0 \tag{2.8}$$

where S is the entropy per unit mass. Using the energy equation to eliminate r and assuming $T>0$, inequality (2.8) becomes

$$\varrho_0 T\dot{S} - \varrho_0 \dot{W} + K_{AB}\dot{E}_{AB} + \frac{Q_A G_A}{T} \geqslant 0. \tag{2.9}$$

We introduce the free energy F by

$$F = W - ST \tag{2.10}$$

and the inequality (2.9) yields

$$-\varrho_0(\dot{F} + S\dot{T}) + K_{AB}\dot{E}_{AB} + \frac{Q_A G_A}{T} \geqslant 0. \tag{2.11}$$

3. The constitutive theory

3.1. The fundamental assumptions and general relations. The theory given by the equations (2.4), (2.7) and the inequality (2.11) is underspecified and needs some constitutive assumptions to be added. These assumptions could be listed in following way: (i) the constitutive equations in the form of the stress-strain relations and the heat conduction law must be proposed; (ii) the assumption of the existing of an initial state: the following relations find place at $t=0$:

$$U_I = 0,\quad \dot{U}_I = 0,\quad T = T_0,\quad \dot{T} = 0,\quad K_{AB} = 0, \\ S = 0,\quad F = 0,\quad Q_A = 0. \tag{3.1}$$

Here U_I are the components of the displacement vector in the Lagrangian description; (iii) the assumption about the stress: the elastic part of the pseudostress tensor K_{AB} depends only on the function of the free specific energy F at the determined time t, and K_{AB} can be determined from the special case $S = \text{const}$, $Q_A = \text{const}$.

There are two possibilities to fill the first assumption [21]. First, the structure of the internal energy can be approximated as the function of strain and temperature (Green's method). The constitutive equations are then derivable from the energy function. This method is usually of use for elastic and thermoelastic solids. Another alternative is based on the assumption that the stress is a function of strain (Cauchy method). This method finds application for dissipative systems, while Green's method fails. Further we use both methods following [22].

We note that according to the approach of Green and Lindsay [18], in the inequality (2.11) T has been replaced by the unknown functional Φ which later is given by a constitutive equation of its own, the heat conduction law being then a consequence of entropy inequality.

The choice problem here is of great importance. There are a lot of possibilities to construct the heat conduction law and the stress-strain relations which are subject to entropy inequality and material frame indifference. The theories of elasticity, hypoelasticity, viscoelasticity, etc., for solids, fluids, and gases are found in many monographs, including, for example, those of Eringen [22] and Sedov [23]. The theory of thermoelasticity is derived by Nowacki [24], Kovalenko [25] and Luikov [26]. Some choice problems are analysed by Nigul and Engelbrecht [27, 28].

We invoke the principle of equipresence here and state that

$$W,\ F,\ S,\ K_{AB},\ \dot{Q}_A \tag{3.2}$$

must depend, in general, upon

$$F_{iA},\ \dot{F}_{iA},\ T,\ G_A,\ \dot{T},\ Q_A. \tag{3.3}$$

The list (3.3), invoking material indifference, must be replaced by

$$E_{AB},\ \dot{E}_{AB},\ G_A,\ T,\ \dot{T},\ Q_A. \tag{3.4}$$

Now, according to [22], we assume that the stress K_{AB} is determined as a sum of dissipative part ${}_DK_{AB}=\tau_{AB}$ and nondissipative part ${}_EK_{AB}=\sigma_{AB}$:

$$K_{AB}=\sigma_{AB}+\tau_{AB}. \tag{3.5}$$

The general assumption (ii) about the initial state needs then precising — the dissipative stress is zero when deformation speed reaches zero. Following [22], the dissipation function ψ is determined by

$$\psi=\tau_{AB}\dot{E}_{AB}. \tag{3.6}$$

Substituting (3.4) in (2.11) and keeping in the mind the general assumptions, we have

$$-\varrho_0\left(\frac{\partial F}{\partial E_{AB}}\dot{E}_{AB}+\frac{\partial F}{\partial T}\dot{T}+\frac{\partial F}{\partial \dot{T}}\ddot{T}+\frac{\partial F}{\partial G_A}\dot{G}_A+\frac{\partial F}{\partial Q_A}\dot{Q}_A+S\dot{T}\right)+$$
$$+\sigma_{AB}\dot{E}_{AB}+\psi+\frac{Q_AG_A}{T}\geqslant 0. \tag{3.7}$$

Now, following Fox [29], we propose a rate-type law of heat conduction

$$\dot{Q}_A=\Gamma_{ABC}\dot{E}_{BC}+\gamma_{AB}\dot{G}_B+\alpha_A\dot{T}+\beta_A\ddot{T}+h_A(T,\dot{T},E_{BC},G_B,Q_B). \tag{3.8}$$

Fox has shown the equipresence of such a law in Eulerian description. The equation (3.8) and the inequality (3.7) now provide

$$-\varrho_0\left(\frac{\partial F}{\partial T}+S-\frac{\partial F}{\partial Q_A}\alpha_A\right)\dot{T}-\varrho_0\left(\frac{\partial F}{\partial \dot{T}}+\frac{\partial F}{\partial Q_A}\beta_A\right)\ddot{T}+$$
$$+\dot{E}_{AB}\left(\sigma_{AB}-\varrho_0\frac{\partial F}{\partial E_{AB}}-\frac{1}{2}\varrho_0\frac{\partial F}{\partial Q_C}\Gamma_{CAB}-\frac{1}{2}\varrho_0\frac{\partial F}{\partial Q_C}\Gamma_{CBA}\right)-$$
$$-\varrho_0\left(\frac{\partial F}{\partial G_A}+\frac{\partial F}{\partial Q_B}\gamma_{AB}\right)\dot{G}_A-\varrho_0\frac{\partial F}{\partial Q_A}h_A+\psi+\frac{Q_AG_A}{T}\geqslant 0. \tag{3.9}$$

Since we vary $\dot{E}_{AB}$ independently, we must have

$$\sigma_{AB}=\varrho_0\frac{\partial F}{\partial E_{AB}}+\frac{1}{2}\varrho_0\frac{\partial F}{\partial Q_C}(\Gamma_{CAB}+\Gamma_{CBA}). \tag{3.10}$$

This stress equation immediately shows the complexity involved in our constitutive assumptions.

If we follow Fox and assume that F does not depend on Q_A, then

$$\frac{\partial F}{\partial \dot{T}}=\frac{\partial F}{\partial G_A}=0. \tag{3.11}$$

Green and Lindsay [18] do not use the direct way in the form of relations (3.11) but give the functional relations containing the coefficients and field quantities so as to get the restrictions on the coefficients. Under their assumption, the free energy depends upon the temperature derivatives. Our restriction (3.11) does not allow such dependence.

The residual inequality is

$$-\varrho_0\left(\frac{\partial F}{\partial T}+S\right)\dot{T}+\psi+\frac{Q_AG_A}{T}\geqslant 0. \tag{3.12}$$

Further we specialize by taking $\Gamma_{ABC}=0$, $\gamma_{AB}=0$, $\alpha_A=0$, $\beta_A=0$. Thus

$$\begin{aligned}\dot{Q}_A&=h_A(T,\dot{T},E_{AB},G_A,Q_A)=\\&=B_A\dot{T}+\alpha k_{AB}G_B-\alpha Q_A+a_{ABC}E_{BC}\end{aligned}\tag{3.13}$$

derived directly from the corresponding relation in Eulerian description.

Indeed, let us consider, for example, the simplest form of the heat conduction law in Eulerian description following Fox [20]:

$$\dot{q}_i-\omega_{ij}q_j-\gamma d_{ij}q_j=\alpha(k_{ij}G_j+b_i\dot{T}-q_i+a_{ikm}e_{km})\tag{3.14}$$

where d_{ij}, ω_{ij} are rate tensors [22] and e_{km} is the Cauchy deformation tensor. Making use of the relations between the field quantities in Eulerian and Lagrangian description, we get

$$\dot{Q}_A=B_A\dot{T}+\alpha k_{AB}G_B-\alpha Q_A+a_{ABC}E_{BC}+R_{AB}Q_B\tag{3.15}$$

where

$$B_A=JX_{A,i}b_i;\quad a_{ABC}=Ja_{ikm}X_{A,i}X_{B,k}X_{C,m},$$
$$k_{AB}=JX_{A,i}k_{ij}X_{B,j};\quad R_{AB}=(\omega_{ij}+\gamma d_{ij})X_{A,i}F_{jB}$$

and J is the Jacobian. If heat conduction is supposed to be linear, then the term with R_{AB} in (3.15) is neglected and (3.13) holds.

With (3.13) the inequality (3.12) yields

$$\begin{aligned}&\left[-\varrho_0\left(\frac{\partial F}{\partial T}+S\right)+\frac{B_AG_A}{\alpha T}\right]\dot{T}+\psi+\frac{k_{AB}}{T}G_AG_B+\\&+\frac{a_{ABC}}{\alpha T}E_{BC}G_A-\frac{\dot{Q}_AG_A}{\alpha T}\geqslant 0.\end{aligned}\tag{3.16}$$

The inequality (3.16) must hold for all temperature and displacement fields; therefore we obtain $a_{ABC}=0$ and

$$S=-\frac{\partial F}{\partial F}+\frac{B_AG_A}{\alpha\varrho_0T},\tag{3.17}$$

$$\psi+\left(k_{AB}G_B-\frac{1}{\alpha}\dot{Q}_A\right)\frac{G_A}{T}\geqslant 0.\tag{3.18}$$

We note that with $\alpha=\tau_0^{-1}$, $B_A=\alpha b_A$ equation (3.13) now reads

$$\tau_0\dot{Q}_A=-Q_A+k_{AB}G_B+b_A\dot{T}\tag{3.19}$$

and

$$S=-\frac{\partial F}{\partial T}+\frac{b_AG_A}{\varrho_0T}.\tag{3.20}$$

For a material with a centre of symmetry $b_A=0$.

Under the assumption made above, the energy equation yields

$$\begin{aligned}&\varrho_0T\left[\frac{\partial^2F}{\partial T^2}\dot{T}+\frac{\partial^2F}{\partial T\,\partial E_{AB}}\dot{E}_{AB}\right]+b_A\dot{G}_A-\frac{b_AG_A}{T}\dot{T}+\\&+\varrho_0r+\psi+Q_{A,A}=0.\end{aligned}\tag{3.21}$$

3.2. The stress-strain relation. In addition to fundamental assumptions given in Sect 3.1, supplementary assumptions must be made. For widely used materials these fundamental assumptions are right when the following supplementary assumptions are used:

$$|U_{A,B}| \ll 1, \qquad (3.22)$$

$$|T - T_0|\, T_0^{-1} \ll 1. \qquad (3.23)$$

The elastic part σ_{AB} of the whole stress K_{AB} is derived from the specific free energy by (3.10). According to the invariance conditions and to the assumptions (3.22), (3.23), the function of specific energy is expanded into the series with regard to the initial state. For isotropic material we get

$$F = F(I_1, I_2, I_3; T) \qquad (3.24)$$

where I_i are the Green deformation tensor invariants determined by

$$I_1 = E_{AA}, \quad I_2 = E_{AB}E_{AB}, \quad I_3 = E_{AB}E_{BC}E_{CA}. \qquad (3.25)$$

The dissipative part τ_{AB} of the whole stress K_{AB} must be determined independently under invariance conditions. Here we restrict our attention with linear dissipation. Then, following Eringen [22], we get for simple materials

$$\tau_{AB} = \tau_{AB}(E_{KL}, \dot{E}_{KL}). \qquad (3.26)$$

Cauchy's law (2.4) in terms of K_{AB}

$$(K_{AB}F_{iB})_{,A} + \varrho_0 f_i - \varrho_0 a_i = 0 \qquad (3.27)$$

and equations (3.14), (3.21) with relations (3.5), (3.6), (3.10), (3.24), (3.26) and restriction (3.18) form a closed system of equations.

4. Nonlinear theory of thermoviscoelasticity

In this section, an example of a mathematical model of the theory of thermoviscoelasticity is presented. The derived model is correct to the second order in E_{AB} and to the first order in temperature, its derivatives and deformation-rate terms. We assume $r = 0$.

The function of the specific free energy must be expanded now correct to the third order in E_{AB} and to the second order in T. Using the finite deformation theory [27, 28], we get

$$\varrho_0 F = \frac{1}{2}\lambda I_1^2 + \mu I_2 + \nu_1 I_1^3 + \nu_2 I_1 I_2 + \nu_3 I_3 - \varkappa_1 I_1 (T - T_0) - \varkappa_2 (T - T_0)^2 \qquad (4.1)$$

where

$$\varkappa_1 = (3\lambda + 2\mu)\,\alpha_T, \qquad \varkappa_2 = \frac{1}{2}\varrho_0 c_E T_0^{-1}$$

and λ, μ are the Lamé parameters, ν_1, ν_2, ν_3 are the third order elastic moduli, c_E is the specific heat, α_T is the bulk coefficient of thermal conductivity.

Now the elastic part of the stress tensor K_{AB} is determined by (3.10) and (4.1):

$$\begin{aligned}\sigma_{AB} = {} & \lambda U_{M,M}\delta_{AB} + \mu(U_{A,B} + U_{B,A}) + 3\nu_1 U_{M,M}\delta_{AB} + \\ & + \frac{1}{2}(\lambda + \nu_2)\, U_{N,M}^2 \delta_{AB} + \nu_2 \Big(U_{A,B}U_{M,M} + U_{B,A}U_{M,M} + \\ & + \frac{1}{2} U_{M,N}U_{N,M}\delta_{AB} \Big) + \Big(\frac{1}{2}\nu_3 + \mu \Big) U_{M,B}U_{M,A} + \\ & + \frac{3}{4}\nu_3 \Big(U_{M,B}U_{A,M} + \frac{4}{3} U_{B,M}U_{A,M} + U_{B,M}U_{M,A} \Big) - \\ & - \varkappa_1 (T - T_0)\,\delta_{AB} + O(U_{M,N}^3). \end{aligned} \qquad (4.2)$$

The dissipative part is determined following [22]

$$\tau_{AB}=\eta_1\dot{U}_{M,M}\delta_{AB}+\eta_2(\dot{U}_{A,B}+\dot{U}_{B,A}) \tag{4.3}$$

where η_1, η_2 are the coefficients of viscosity.

The whole set of equations, correct to the second order of displacement gradients and the first order of temperature and rate terms then reduces to

$$\varrho_0\ddot{U}_A-C_{AKMN}U_{K,MN}-D_{AKMN}\dot{U}_{K,MN}=B_{AM}T_{,M}, \tag{4.4}$$

$$g\dot{T}+H_{MN}\dot{U}_{M,N}+F_M\dot{T}_{,M}=Q_{K,K}, \tag{4.5}$$

$$\tau_0\dot{Q}_A+Q_A=k_{AB}T_{,B}+b_A\dot{T} \tag{4.6}$$

where

$$\begin{aligned}C_{AKMN}=&(\lambda+\mu)\delta_{AM}\delta_{KN}+\mu\delta_{AK}\delta_{MN}+(6\nu_1+\nu_2)U_{I,I}\delta_{AN}\delta_{KM}+\\&+(\lambda+\nu_2)(U_{A,N}\delta_{KM}+U_{K,M}\delta_{AN}+U_{I,I}\delta_{AK}\delta_{MN})+\\&+\nu_2(U_{N,A}\delta_{KM}+U_{M,K}\delta_{AN})+\\&+\mu[(U_{A,K}+U_{K,A})\delta_{MN}+\\&+(U_{N,M}+U_{M,N})\delta_{AK}+U_{A,M}\delta_{KN}+U_{K,N}\delta_{AM}]+\\&+\nu_3\left[\left(\frac{3}{4}U_{A,K}+\frac{1}{2}U_{K,A}\right)\delta_{MN}+\left(U_{N,M}+\frac{3}{4}U_{M,N}\right)\delta_{AK}+\right.\\&\left.+\left(U_{A,M}+\frac{3}{4}U_{M,A}\right)\delta_{KN}+\left(\frac{1}{2}U_{K,N}+\frac{3}{4}U_{N,K}\right)\delta_{AM}\right]\end{aligned}$$

$$\begin{aligned}&D_{AKMN}=\eta_1\delta_{KM}\delta_{AN}+\eta_2(\delta_{AK}\delta_{MN}+\delta_{KN}\delta_{AM}),\\&B_{AM}=\varkappa_1(\delta_{AM}+U_{A,M})\\&g=\varkappa_2T,\\&H_{MN}=\varkappa_1(\delta_{MN}+U_{M,N}),\\&F_M=-b_M(\varrho_0T)^{-1}.\end{aligned} \tag{4.7}$$

The system of equations (4.4), (4.5) and (4.6) is the quasilinear system of 7 equations with unknowns

$$U_A,\ T,\ Q_A \quad (A=1,\ 2,\ 3). \tag{4.8}$$

From the assumptions (3.22) and (3.23) it follows that the part of the system (4.4)—(4.6) consisting of terms up to the second order only belongs to the class of hyperbolic equations.

As a simple extension let us consider the one-dimensional problem of linear uncoupled theory. The temperature distribution is then governed by the equations

$$\varrho_0c_E\dot{T}-b_1(\varrho_0T)^{-1}\dot{T}_{,1}=Q_{1,1}, \tag{4.9}$$

$$\tau_0\dot{Q}_1+Q_1=k_{11}T_{,1}+b_1\dot{T}. \tag{4.10}$$

This system can be developed eliminating Q_1 and arriving at the following equation:

$$\tau_0\varrho_0c_E\ddot{T}+\varrho_0c_E\dot{T}-k_{11}T_{,1}-b_1[1+(\varrho_0T)^{-1}]\dot{T}_{,1}=\tau_0b_1(\varrho_0T)^{-1}\ddot{T}_{,1}. \tag{4.11}$$

Correct to the second order terms, this equation is of a hyperbolic type and therefore leads to the finite speed of thermal waves.

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DEFORMATSIOONI JA SOOJUSE MUUTUMISE KIIRUSEST SÕLTUV TERMOVISKO-ELASTSUSTEOORIA

On tuletatud mittelineaarse termovisko-elastsusteooria põhiseosed, mis arvestavad deformatsiooni muutumise kiirust ja soojuse levimise lõplikku kiirust. Selleks on kasutatud Lagrange'i muutujaid, mis sobivad tahkes deformeeruvas kehas toimuvate protsesside kirjeldamiseks, kusjuures on lähtutud oletusest, et kõik sama järku sõltumatud muutujad peavad olema olekuvõrrandeis esindatud. Näitena on toodud üleminekuprotsessi kirjeldav matemaatiline mudel, milles on arvestatud lõplikke deformatsioone. On esitatud üleminek mittelineaarselt temperatuurivõrrandilt Euleri muutujates vastavale võrrandile Lagrange'i muutujates.

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ТЕОРИЯ ТЕРМОВЯЗКОУПРУГОСТИ, ЗАВИСЯЩАЯ ОТ СКОРОСТИ ДЕФОРМАЦИИ И ТЕПЛА

Развивается нелинейная теория термовязкоупругости с учетом скорости изменения деформации и конечной скорости распространения тепла. Общие зависимости представлены в лагранжевом описании для твердых деформируемых тел. Исходным является предположение, что все независимые переменные одного порядка учтены в конституционных уравнениях. В качестве примера приведена математическая модель для описания переходных волновых процессов, в которой деформация учтена с точностью до членов второго порядка. Показан переход от нелинейного уравнения теплопроводности в эйлеровом описании к соответствующему уравнению в лагранжевом описании.