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THE WONDERFUL HOOKE'S LAW: HISTORY, THEORY, AND STORY

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Abstract. A brief history of Hooke's law and its generalizations are presented. The deep essence and perfection of fundamental laws of nature are pointed out within the framework of modified mathematical models.

Key words: Hooke's law.

1. PREFACE

The life of the most fundamental laws of physics is more interesting and more exciting than any other story of mankind.

Take, for instance, the celebrated Fourier's law of heat conduction. It has been used for centuries as one of the most efficient laws of physics. Even the contradiction it contains was first mentioned by Maxwell in 1867, and hundreds and hundreds of attempts have been made to modify it. To describe its indelible imprint on physics, nothing is better than the name of all attempts: modified Fourier's law. Or a prophecy: upon resurrection he will be able to recognize his law.

As another example, let us consider Hooke's law for an elastic body. It is even older than the previous example, more basic, and much more attempts have been made to modify it. Even this law does not contain any contradictions. But it has required a lot of completion in both physical and mathematical sense. Concerning the latter one, we can use an analogy taken from music. Hooke was the composer and his followers, first of all mathematical physicists, made the score and the masterpiece was ready.* On the basis of the mathematical

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* By the way, the same applies to the field of electrodynamics. Faraday, Ampere, and Volta created the laws and Maxwell gave them the mathematical formulation.

apparatus, which has been built into the original Hooke's law, I suspect he would never be able to recognize his law again.

The study of Hooke's law is a profound experience. The deeper one delves in it, the more he or she appreciates the great master whose unparalleled work has left a prominent landmark.

Let us describe shortly the history, which inevitably contains also some theory, of the law that is huge from a bird's eye view and, as all great thoughts it, too, has satellites, which are collected in the successive chapters as different stories.

2. HISTORY AND THEORY

In the history of the theory of elasticity started by Galileo Galilei (1564–1642), the greatest landmark is undoubtedly the discovery of Hooke's law in 1660.

The notion of elasticity was first announced by Robert Hooke (1635–1703) in 1676 in the form of an anagram [1],

$$\text{ceiiinosssttuv.} \tag{1}$$

He explained it only two years later, in 1678, as

$$\text{ut tensio sic vis.} \tag{2a}$$

The approximate English translation of this Latin sentence is:

The power of any springy body is in the same proportion with the extension. (2b)

This and nothing else was the start of the celebrated Hooke's law.

Let us pause here for a moment to mention that an identical law was enunciated independently by Edmé Mariotte (1620–84) two years later, i.e. in 1680. So, as it often happens in science, the discovery was once again duplicated, but the history refers only to Hooke. By the way, the history of science is lucky, because in case of the reverse order nobody would know who Hooke is while Mariotte is well known from another field of physics.

The mathematical theory, developed and extended to other materials since that time, is associated with the names of practically all great mathematical physicists of the last three centuries and forms one of the most important parts of classical physics. The line of inquiry has never been broken, and in recent times we witness the most vigorous developments.

Without mentioning all items of the history of Hooke's law, let us give here the principal milestones.

The most primitive and widely used version of this law says that stress is proportional to strain:

$$\sigma = E\varepsilon \tag{3}$$

and the factor of proportionality is the modulus of elasticity or Young's modulus, announced by Thomas Young (1773–1829) in 1807. It means that almost one and half a century was needed to complete the law well known today.

The second greatest landmark is the isotropic–isothermal Hooke's law [2]:

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}, \; i, j = 1, 2, 3. \tag{4}$$

(We use the notation of indices and the summation convention introduced by Einstein.) This law, with $c = \lambda = \mu$ because of an inadequate molecular model, was proposed to the Paris Academy by Louis M. H. Navier (1785–1836) in 1821. In 1823 the two constant versions $(c_1 = \lambda;\; c_2 = 2\mu)$ were presented to the Academy by Augustin L. Cauchy (1789–1857), who formulated the continuum theory of linearized elasticity in virtually the same form as it stands today. This law, of course, largely generalizes the observations of Hooke and Young [3]. Siméon-Denis Poisson (1781–1840) developed a molecular theory of elasticity and arrived at the same equation as Navier. Both Navier and Poisson based their analysis on the Newtonian conception of the constitution of bodies and assumed certain laws of intermolecular forces. Cauchy's general reasoning, however, made no use of the hypothesis of material particles.

For an anisotropic body in the isothermal case the following form is valid:

$$\sigma_{ij} = c_{ijrs} \varepsilon_{rs}, \; i, j, r, s = 1, 2, 3. \tag{5}$$

Here the elastic coefficient matrix c_{ijrs} contains 81 constants, but they are not all independent. If we take into account the symmetry of the stress and strain tensors, there remain only 36 independent constants. Let us enumerate the elements and instead of Eq. (5) write Eq. (6):

$$\sigma_k = c_{k\ell} \varepsilon_\ell, \; k, \ell = 1, \ldots, 6, \tag{6}$$

where $c_{k\ell}$ are the 36 independent elastic coefficients. If the concept of strain energy introduced by George Green (1793–1841) is valid, then $c_{k\ell}$ is symmetrical and the number of independent elements is reduced to 21.

The case of orthotropic symmetry according to Love [4] was discussed by Barré de Saint-Venant (1797–1886). In this case the number of the independent elements of $c_{k\ell}$ decreases to 9. If the body is isotropic, then the number of independent constants decreases to 2 in Eq. (4) with the so-called Lamé constants λ and μ, introduced by Gabriel Lamé (1795–1870).

If the phenomenon is not isothermal, then we have to use the linear thermoelastic constitutive equation

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} - \beta \Delta T \delta_{ij}, \tag{7}$$

which is attributed to J. M. C. Duhamel (1797–1872) and Franz Neumann (1798–1895) by Sokolnikoff [5] and called as Duhamel–Neumann equation.

If the material we are dealing with is thermohygroelastic, then the proper equation instead of Eq. (7) (see, e.g., [6,7]) is the following:

$$\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} - (\beta\Delta T + \beta_m\Delta m)\,\delta_{ij}. \tag{8}$$

The strong impact of Hooke's law (through the elasticity and solid mechanics) on civilization is felt through its application in engineering. Beside the scientists already mentioned above, let us name some of the most famous users of this law: Jacques Bernoulli (1654–1705) – beam theory (1705); Leonhard Euler (1707–1783) – elastic stability (1778); Charles A. Coulomb (1736–1806) – failure criterion (1784); Joseph L. Lagrange (1736–1813) – bending and vibration of plates (1773); and many others only in the first period (1638–1820) of the law's life.

We are not able even to mention all the scientists connected with Hooke's law in the further periods of its life. Just some examples should be given to emphasize its importance nowadays. Almost one quarter of the celebrated book on continuum mechanics by Malvern [2] deals with the constitutive laws, from our point of view with Hooke's law. A large portion of investigations in the field of solid mechanics is devoted to the constitutive laws. To get a deeper insight, open any journal on mechanics and related fields and look at the table of contents.

As all unparalleled ideas, Hooke's law also has satellites. Now let us finish this brief historic overview and little piece of theory and discuss some of the thoughts or, as we call them, stories aroused by the law. The family tree of generalized Hooke's law is shown in Fig. 1.

3. (SHORT STORY) CHARM

Look first at Eq. (1) and then at expression (8) of Hooke's law. Compare them and decide whether it is a wonder or not. Notice the little difference in their essence in spite of substantial alteration in the form. By the way, versions (4)–(6), but not (7), are all called generalized Hooke's law and I do not know why (7) is an exception.

From another viewpoint, is it not fascinating that somebody creates one of the biggest law of mechanics of the age or, as it turned out, of the whole following period including also the present day, and meanwhile he has spirits, power, and patience to play. Well, is it not charming?

Undoubtedly the greatest landmark in mechanics, or in the whole physics, Sir Isaac Newton's (1642–1727) *Principia* is a real scientific work without any art in the sense mentioned above. Nothing tells us more about the eternal relationship between art and science than the somewhat mean remark of a

NOT ISOTHERMAL

THERMOHYGROELASTIC

ANISOTROPIC: $\sigma_{ij} = c_{ljkl}\varepsilon_{kl} - \beta_{ij}\Delta T - \beta_{ij}^{m}\Delta m$

ISOTROPIC: $\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \beta\Delta T - \beta_m \Delta m)\delta_{ij}$

DUHAMEL 1838, NEUMANN 1841
$\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda\varepsilon_{kk} - \beta\Delta T)\delta_{ij}$

Elastic potential

Symmetry of stress σ_{ij} and strain ε_{ij}

ISOTHERMAL

Number of independent coefficients

81	36	21	9	2
$\sigma_{ij} = c_{ijkl}\varepsilon_{kl}$	$\sigma_k = c_{kl}\varepsilon_l$	GREEN 1839 $\sigma_k = c_{kl}\varepsilon_l$	SAINT-VENANT 1865 $\sigma_k = c_{kl}\varepsilon_l$	LAMÉ 1852 $\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}$
ANISOTROPIC		ORTHOTROPIC		ISOTROPIC

CAUCHY 1823
$\sigma_{ij} = C_1\varepsilon_{ij} + C_2\varepsilon_{kk}\delta_{ij}$

NAVIER 1821
$\sigma_{ij} = C(2\varepsilon_{ij} + \varepsilon_{kk}\delta_{ij})$

POISSON 1828
$\sigma_{ij} = C(2\varepsilon_{ij} + \varepsilon_{kk}\delta_{ij})$

COULOMB 1784
Failure criterion

YOUNG 1807
$\sigma = E\varepsilon$

J. BERNOULLI 1705
Beam theory

EULER 1778
Elastic stability

LAGRANGE 1773
Bending and vibration of plates

HOOKE 1678
Ut tensio sic vis

HOOKE 1676
Ceiiinosssttuv

MARIOTTE 1680

GALILEI

Fig. 1. Family tree of generalized Hooke's law.

scientist: analysing the history of some modern equipment, he points out that although the equipment traces back about one century, Leonardo da Vinci (1452–1519) should be mentioned, as his drawings and sketches certainly make some reference to that equipment as well. Mean as this comment may be, knowing Leonardo, it is not altogether unjustified.

I am a little bit worried when I compare Hooke's destiny with that of a contemporary scientist. The real scientist is not a proletarian, but a nobleman of science, or, by the contemporary notion, a real bourgeois. He does not have to struggle for everyday science. He has time to enjoy also the beauty of science, to create art at the same time. But many publications per year in oral and written form take up all his efforts: there is no time and energy to follow Hooke's way. There may be some exceptions, e.g., Lánczos [8], Engelbrecht [9], or Simonyi [10], and others.

4. (LONG STORY) MATHEMATICS

As one can figure out from the above, one point of view in the history of Hooke's law is nothing else but a story of mathematics. Since mathematics is also, in a general sense, one of the most profound parts of our civilization, let us deal with it in relation to Hooke's law.

A few years ago the highly regarded and world-famous scientist, Janos Selye made a rather curious statement. He argued that a physician does not need mathematics, as the logic of mathematics is harmful to medical thinking. This I have ever since regarded as nonsense and have some ill feelings towards Janos Selye. Is it possible that he has retained his aversion towards mathematics from childhood all the way until becoming a world-renown scientist? Who knows?

Mathematics is a tricky issue. There is no other science which would divide so markedly into two groups those getting in touch with it. Those who need mathematics, having got to do with it, cannot imagine their work without it. Other people, however, consider it completely superfluous, and object to it. This division is for sure not coincidental, the explanation for it lies in the very nature of mathematics. It is an extremely abstract, complex science, which for some people is better to avoid. On the other hand, it is hardly possible to avoid, as it defines the most basic laws of the universe, thus one bumps into it in all areas. And this has been so for thousands of years! As Richard P. Feynman put it very nicely, nature talks to us in the language of mathematics.

In [11] Joja says that the whole of modern science, starting with Leonardo da Vinci and Copernicus was born under the auspices of Pythagoras, Plato, and especially Archimedes. Pythagorism advocated that everything can be derived from the number and the proportion: this was the first panmathematism. The Pythagorean Philolaus said that the nature of the number, similarly to harmony, does not bear anything false, as that is not its own. Truth is inherent to the number... Archimedes otherwise considered himself a platonist and a Pythagorean. Galilei talks about Archimedes with fascination: the superhuman Archimedes whose name can never be uttered without deep emotions.

I feel that mathematics and languages are to some extent related. Those who have lived their whole life without ever speaking any other language than their mother tongue, will never have this sense of having missed something. Those, however, who start learning foreign languages will sense a growing urge to learn more and more languages.

Let us now ponder a little bit about the relationship between mathematics and the technical and natural sciences. The more basic relations can easily be described with words, the more complicated ones, however, call for mathematical apparatus.

The constitutive law of linearly elastic, homogeneous, isotropic bodies, for example, was first defined by Hooke in a very simple form (see expressions (1)–(3)). According to him, stress and strain are in proportion, the factor of proportionality is characteristic of the material. If we think of the simple tension test, this is correct. Introducing the notions of stress and strain, Hooke's law can be written in the form of Eq. (3).

The first problem presents itself if we take a closer look at the experiment, as strain occurs not only longitudinally but transversely as well. Thus the transversal size is reduced. It may be stated that the alteration of the transversal size is in proportion with the longitudinal one, and the factor of proportionality is characteristic of the material in this case as well. Based on the above, we have

$$\varepsilon_t = -\nu\varepsilon \,, \tag{9}$$

where ν is Poisson's factor that can be established from the measurement.

Let us now move on. Let us this time imagine a twisted bar instead of a stretched one. In this case torque appears instead of tension and, naturally, the phenomenon is twisting instead of stretching. Introducing the specific quantities similarly to the previous case, we get the relation

$$\tau = G\gamma, \tag{10}$$

where G is another material characteristic which can also be defined through experiments.

Let us make yet another step forward. Instead of a rod let us now picture a three-dimensional body, and let us assume that we load this body with forces and moments working along all three axes. In this case the relationship between the stresses and strains cannot be described with words, only with a system of equations with six unknowns. Up to this point the mathematical skills acquired at high school will be sufficient. Knowing matrix calculation, however, these equations can well be condensed:

$$\underline{\underline{\varepsilon}} = \frac{1+\nu}{E}\left(\underline{\underline{\sigma}} - \frac{\nu}{1+\nu}\sigma_I \underline{\underline{I}}\right), \; G = \frac{E}{2(1+\nu)}. \tag{11}$$

This system comprises everything but requires advanced mathematical skills. If you compare Eqs. (4) and (11), then you recognize soon total analogy. While in Eq. (4) we used Lamé's coefficients, then in Eqs. (11) those of Young and Poisson are introduced.

In case of anisotropy, Eq. (5) is valid and the problem is hardly understandable without higher mathematics. The series of examples could be continued or extended. Describing certain phenomena in the language of mathematics may show that the phenomena are related. This leads us to analogies which provide special experimental and computing technical opportunities, e.g., the analogy of mechanical and electric vibration systems, or the analogy of Fourier's and Fick's laws. Or, through the examination of the mathematical apparatus (model) describing the phenomenon, we may actually reach the criticism of the original physical model (e.g., modified law of heat conduction).

5. (COMPLICATED STORY) POLITICS AND ECONOMICS

As I have already mentioned, the real scientist is not a proletarian who has to struggle for everyday science. To become and to exist as a real scholar, certain financial circumstances and, based on this, great intellectual independence, too, are required. It means that in feudalism most of the scientists were noblemen, in early capitalism bourgeois from the upper middle classes, and nowadays most of them are researchers. It is not by chance that in the first period of the history of Hooke's law one can not find any female scientist. Before the emancipation it was hardly possible for a woman to become a scientist. Of course, there are always several exceptions, e.g. Aesop, the great narrator, was a slave, but his field was the arts and during slavery the situation was fundamentally different.

I think that one of the reasons, if not the first and foremost one, for the failure of communism was the destruction of the middle class and the lack of respect for the intelligentsia, first of all the researchers. The communists did not only have a political reason for this, but also economical: keeping thousands and thousands of researchers without any results. But you never know who improves later to become "a Hooke" and, the probability being very small, there is no other choice but to keep countless researchers and "trust in God". In case of an industrial enterprise the best investment is said to be the R&D (research and development). You can reach the highest recovery in this field. I do not know exact figures, but I guess the same applies to "social enterprises".

No doubt, playing in childhood is part of the process of becoming an adult with the ability to work and to create. Later, for a grown-up, somehow art takes up this role. But for this, time is needed. If one goes, not having time to play, to deal with art as, for example, Hooke and several others did, I doubt that anyone would have a chance to reach a real scientific result. That is the reason for my bad feeling concerning a scientist with 10–20 publications per year, as I mentioned before. By the way, very good positive examples are the best writers of science-fiction, who have been also first-class researchers in their own field.

6. (ENDLESS STORY) COMPUTERS

I do not give a story here but only a hint to start this endless way, namely: try to fit the story of computers and computational methods to the history of Hooke's law.

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My enthusiasm for Hooke's law was born in the Institute of Cybernetics, Tallinn; it grew at the Virginia Tech, Blacksburg, and was realized at Oulu University, Oulu, during my stay as a visiting researcher.

Part of the text is taken word for word from the references. I hope the authors understand me and the readers appreciate it.

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IMEPÄRANE HOOKE'I SEADUS: AJALUGU, TEOORIA JA LUGU

Andras SZEKERES

Fundamentaalsete loodusseaduste avastamise ajalugu võib olla huvitavamgi kui mõne teise inimtegevuse valdkonna oma. Essees on esitatud elastsusteooria alguse – Hooke'i seaduse lugu ja mõtteid, mida see lugu on uurijas esile kutsunud.