Proc. Estonian Acad. Sci. Phys. Math., 1999, **48**, 3/4, 230–238
https://doi.org/10.3176/phys.math.1999.3/4.06

JOINT EVOLUTION OF GENERALIZED AND CLASSICAL SPECTRA IN THE KINETIC THEORY

Tarmo SOOMERE

Estonian Marine Institute, Paldiski mnt. 1, 10137 Tallinn, Estonia

Received 4 March 1999

Abstract. Spectral evolution of wave systems described by a combination of generalized and classical functions in the framework of the kinetic theory is considered. Earlier results obtained for systems of Rossby waves are generalized to the case of arbitrary wave systems allowing three-wave interactions. Coexistence of singular and classical spectra typically causes suppressing of the spectral singularities. However, the presence of wave systems described by a singular spectrum accelerates the evolution of the smooth part towards thermodynamical equilibrium.

Key words: wave-wave interactions, weak nonlinearity, kinetic theory of waves, entropy.

1. INTRODUCTION

A most interesting feature of the temporal evolution of synoptic-scale motions in the Earth's atmosphere and oceans (geostrophic turbulence) is the tendency to be restructured to an anisotropic system of motions with the prevailing zonal component, while the only thermodynamically equilibrated state is spectrally isotropic (e.g. [1]). Detailed study of this peculiarity has revealed the existence of generalized anisotropic, thermodynamically equilibrated spectra of weak geostrophic turbulence which is equivalent to a complex system of weakly nonlinear Rossby waves [2]. It was proved in [2] that owing to the "balance" between the influence of the nonlinearity (trying to smooth the wave spectrum) and the β-effect (supporting flows with the prevailing zonal component), Rossby-wave systems evolve towards a special thermodynamically equilibrated state consisting of a superposition of a zonal flow and a spectrally isotropic wave system. Spectra of such motions can be described as a sum of a classical function and a delta-like additive. Although this was known already in the 1980s, to the knowledge of the author, no detailed studies are available to explain the consequences of this interesting phenomenon.

In the current study, the evolution of wave systems, spectrally represented by a combination of smooth and generalized functions in the case of arbitrary wave classes allowing three-wave resonance, is discussed. This is the basic mechanism of interaction for most of the cases in hydrodynamics.The kinetic framework serves here as an infrequent theoretical medium for handling joint evolution of singular and smooth objects, the evolution of both being governed by unique laws in an irreversible environment. The results of the study are valid in the case when motion can be split into any finite number of normal modes (e.g., Rossby waves in a vertically stratified medium; see [3,4]).

2. KINETIC EQUATION

Spectral evolution of weakly nonlinear wave systems is described, to the first approximation, by the kinetic equation. For barotropic Rossby waves it looks like [2]

$$\frac{\partial F}{\partial \tau} = 8\pi \int \frac{C_{\vec{\kappa}_1\vec{\kappa}_2}}{N} \mathbf{P}_{012}\left[C_{\vec{\kappa}_1\vec{\kappa}_2} F(\vec{\kappa}_1)F(\vec{\kappa}_2)\right]\delta(\omega_{012})\delta(\vec{\kappa}_{012})d\vec{\kappa}_{12}, \qquad (1)$$

where $F = F(\vec{\kappa},\tau)$ is the spectral density (spectrum) of the total energy, $\vec{\kappa} = (k,l)$ is the wave vector, $\tau = \varepsilon^{-2}t$ is slow time, $C_{\vec{\kappa}_1\vec{\kappa}_2}$ is the interaction coefficient, $N = (\kappa^2 + a^2)(\kappa_1^2 + a^2)(\kappa_2^2 + a^2)$, $\mathbf{P}_{012} f = f(\vec{\kappa},\vec{\kappa}_1,\vec{\kappa}_2) + f(\vec{\kappa}_1,\vec{\kappa}_2,\vec{\kappa}) + f(\vec{\kappa}_2,\vec{\kappa},\vec{\kappa}_1)$ is the operator of cyclic summation, $\omega_{012} = \omega(\vec{\kappa}) + \omega(\vec{\kappa}_1) + \omega(\vec{\kappa}_2)$, $\omega = \omega(\vec{\kappa})$ is the dispersion relation, $\vec{\kappa}_{012} = \vec{\kappa} + \vec{\kappa}_1 + \vec{\kappa}_2$, $d\vec{\kappa}_{12} = d\vec{\kappa}_1 d\vec{\kappa}_2$, and the integration is performed over the four-dimensional space $\mathbf{R}^2(\vec{\kappa}_1)\times\mathbf{R}^2(\vec{\kappa}_2)$. Kinetic equations for other wave systems allowing three-wave resonance differ from Eq. (1) in minor details, such as the form of interaction coefficients and the dispersion relation, the number of normal modes involved, etc.

The infinite integration area inconveniences treatment of stationary and equilibrated solutions to Eq. (1), since they all either have infinite energy or vanish identically. Also, the entropy of the wave system is typically infinite. Traditionally, the problem is avoided by considering a large but finite integration domain $\Omega(\vec{\kappa}_1)\times\Omega(\vec{\kappa}_2)$. Physically, this is equivalent to neglecting all the interactions involving at least one wave vector lying outside Ω.

The transfer to the symmetrical representation of the integrand of the kinetic equation is based on the Jacobi identities

$$\frac{C_{\vec{\kappa}_1\vec{\kappa}_2}}{\omega} = \frac{C_{\vec{\kappa}_2\vec{\kappa}}}{\omega(\vec{\kappa}_1)} = \frac{C_{\vec{\kappa}\vec{\kappa}_1}}{\omega(\vec{\kappa}_2)} = V_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2} \qquad (2)$$

which are satisfied at the resonance curves $\omega_{012}=0$, $\vec{\kappa}_{012}=\vec{0}$. Many derivations of the kinetic equation (e.g. [5,6]) ascertain that these identities proceed from the conservation laws. For Rossby-wave systems they could be proved in a straightforward way.

We can assume $F>0$, since in case $F=0$, Eq. (1) yields $\partial F/\partial\tau \geq 0$. Spectra which vanish in a certain area of the wave vector space will be considered below. With the use of Eqs. (2), Eq. (1) reduces to

$$\frac{\partial F}{\partial \tau}=8\pi\int \omega W\,\mathbf{P}_{012}\frac{\omega}{F}\delta^3 d\vec{\kappa}_{12}, \tag{3}$$

where $W=V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}N^{-1}FF(\vec{\kappa}_1)F(\vec{\kappa}_2)$ and $\delta^3=\delta(\omega_{012})\delta(\vec{\kappa}_{012})$. This representation makes it convenient to treat the spectral evolution in terms of the wave action $\hat{N}=F/\omega$ and allows us to use the symmetry of the arguments of the delta-functions.

Let us multiply Eq. (3) by an arbitrary function $S(\vec{\kappa})$, integrate it over $\vec{\kappa}$, and apply the operator $\mathbf{P}_{012}$ to both sides of it, first once, then twice. Combining the resulting three equations, we get

$$\frac{\partial}{\partial \tau}\int FS\,d\vec{\kappa}=\frac{8\pi}{3}\int W\,\mathbf{P}_{012}(\omega S)\,\mathbf{P}_{012}\frac{\omega}{F}\delta^3 d\vec{\kappa}_{012}, \tag{4}$$

where $d\vec{\kappa}_{012}=d\vec{\kappa}\,d\vec{\kappa}_{12}$ and both sides of Eq. (4) are totally symmetric with respect to the cyclic permutations of the wave vectors $\vec{\kappa}, \vec{\kappa}_1, \vec{\kappa}_2$.

3. ENTROPY, EQUILIBRIUM SOLUTIONS AND THEIR STABILITY

The conservation laws of energy and both components of wave impulse simply follow from Eq. (4): the choices $S=\text{const}$, $S=k/\omega$, and $S=l/\omega$ make $\mathbf{P}_{012}(\omega S)$ equal to an argument of the delta-functions δ^3. Another rich in content case is $S=F^{-1}$ that reduces Eq. (4) to

$$\int \frac{\partial F/\partial \tau}{F}d\vec{\kappa}=\frac{\partial}{\partial \tau}\int \ln F\,d\vec{\kappa}=\frac{8\pi}{3}\int W\left(\mathbf{P}_{012}\frac{\omega}{F}\right)^2\delta^3 d\vec{\kappa}_{012}\geq 0. \tag{5}$$

The latter inequality confirms the irreversibility of the spectral evolution and is equivalent to Boltzmann's H-theorem. The quantity $H=\int \ln F\,d\vec{\kappa}$ can be interpreted as the system entropy. It has a maximum value if the system reaches (thermodynamical) equilibrium and, provided $V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\neq 0$, corresponds to the case

$\mathbf{P}_{012}(\omega/F)=0$ at the resonance surfaces $\omega_{012}=0$, $\vec{\kappa}_{012}=\vec{0}$. The obvious solutions to this problem ${}^{(1)}F=\text{const}$, ${}^{(2)}F\sim\omega k^{-1}$, and ${}^{(3)}F\sim\omega l^{-1}$ were found and discussed already in the pioneering work [5] for the particular case of four-wave interactions of surface waves. Much later [7] it was proved that ${}^{(1)}F$ and ${}^{(2)}F$ are the only solutions to these equations among differentiable functions in the case of Rossby waves.

Physically, it is obvious that equilibrium states are stable with respect to small disturbances. Since the above definition of entropy differs from the classical one, this property need not be automatically true. The proof for barotropic Rossby waves [7] can be generalized to the case of arbitrary wave classes allowing three-wave resonance.

It is convenient to present Eq. (4) as follows:

$$\frac{\partial}{\partial\tau}\int FSd\vec{\kappa}=\frac{8\pi}{3}\int V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}N^{-1}\mathbf{P}_{012}(\omega S)\,\mathbf{P}_{012}(\omega F_1F_2)\,\delta^3 d\vec{\kappa}_{012}, \tag{6}$$

where $F_1=F(\vec{\kappa}_1)$, $F_2=F(\vec{\kappa}_2)$. Let us consider spectra $F=F_{eq}+G$, where $G \ll F_{eq}$ and F_{eq} is an equilibrated solution to the kinetic equation. Substituting F into Eq. (6), making use of the fact that $\mathbf{P}_{012}(\omega F_{1eq}F_{2eq})=0$, choosing $S=G/F^2_{eq}$, and dropping the terms $O(G^2)$, we have:

$$\frac{d}{d\tau}\int\frac{G^2}{F^2_{eq}}d\vec{\kappa}=\frac{8\pi}{3}\int\frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N}\mathbf{P}_{012}\frac{\omega G}{F^2_{eq}}\mathbf{P}_{012}\left[G(\omega_1F_{2eq}+\omega_2F_{1eq})\right]\delta^3 d\vec{\kappa}_{12}$$

$$=-\frac{8\pi}{3}\int\frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}F_{eq}F_{1eq}F_{2eq}}{N}\left(\mathbf{P}_{012}\frac{\omega G}{F^2_{eq}}\right)^2\delta^3 d\vec{\kappa}_{012}\le 0, \tag{7}$$

where $\omega_1=\omega(\vec{\kappa}_1)$, $\omega_2=\omega(\vec{\kappa}_2)$. The latter inequality means that the amplitude of any small disturbance of the equilibrium solutions does not increase in time and that the equilibrated solutions are stable with respect to small perturbations.

4. GENERALIZED SOLUTIONS

Rossby waves serve as an example of wave systems which yield reasonable and realizable spectra with a limited carrier. For example, the zonal flow is spectrally represented by $F_{zon}=f(\vec{\kappa},\tau)\delta(k)$. Here we consider $f(\vec{\kappa},\tau)\ge 0$ as an arbitrary continuous function.

Evolution of the combination spectra $\tilde{F} = f\tilde{\delta} + F$, where $F(\vec{\kappa},\tau) > 0$ is an arbitrary classical function, $\tilde{\delta} = \delta(\Phi)$ is an arbitrary generalized solution to the kinetic equation, and Φ is a subset of the wave vector plane, can be described by splitting the kinetic equation into two equations with respect to the singular and classical parts of the spectrum [8]:

$$\frac{\partial f}{\partial \tau} = 8\pi f \int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N} \omega(\omega_1 F_2 + \omega_2 F_1)\delta^3 d\vec{\kappa}_{12}, \tag{8}$$

$$\frac{\partial F}{\partial \tau} = 8\pi \int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\omega}{N} \left\{ \mathbf{P}_{012}(\omega F_1 F_2) + \mathbf{P}_{012}\left[f_1\tilde{\delta}_1(\omega F_2 + \omega_2 F) + f_2\tilde{\delta}_2(\omega F_1 + \omega_1 F)\right]\right\} \times \delta^3 d\vec{\kappa}_{12}, \tag{9}$$

where $f_i = f(\vec{\kappa}_i)$, $\tilde{\delta}_i = \tilde{\delta}(\vec{\kappa}_i)$, $i = 1, 2$.

The structure of Eq. (8) "prohibits" the development of wave subsystems with a delta-like spectrum during the evolution of an initially smooth spectrum: if $f(\vec{\kappa}^*) = 0$, then also $\partial f(\vec{\kappa}^*)/\partial\tau = 0$ and $f(\vec{\kappa}^*) \equiv 0$. Contrariwise, if $F(\vec{\kappa}^*) = 0$, then from Eq. (9) it follows that $\partial F(\vec{\kappa}^*)/\partial\tau \geq 0$, whereas energy flows into the wave with $\vec{\kappa}^*$ from both the classical and the generalized part of the spectrum. This property indicates that the interaction of the smooth and singular parts of the spectrum generally suppresses singularities.

Derivation of Eq. (6) and of the conservation laws obviously remains valid for the combination spectra. However, owing to the presence of delta-functions, the above definition of the system entropy is not appropriate and the choice $S = \tilde{F}^{-1}$ does not give a satisfactory result. Straightforward neglecting of the singular component is not particularly helpful, since the choice $S = F^{-1}$ in Eq. (6) does not lead to an analogue of Eq. (7).

However, it is possible to introduce entropy for the classical part of the spectrum. Application of the symmetrization procedure to Eq. (9), combined with the choice $S = F^{-1}$ yields:

$$\frac{\partial}{\partial \tau}\int \ln F d\vec{\kappa} = \frac{8\pi}{3}\int W\left(\mathbf{P}_{012}\frac{\omega}{F}\right)^2 \delta^3 d\vec{\kappa}_{012} + \frac{8\pi}{3}\int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N}\mathbf{P}_{012}\left[f\tilde{\delta}F_1F_2\left(\frac{\omega_1}{F_1} + \frac{\omega_2}{F_2}\right)^2\right]\delta^3 d\vec{\kappa}_{012} \geq 0. \tag{10}$$

Thus, the evolution of wave fields with the classical spectrum in the presence of arbitrary wave systems corresponding to generalized solutions of the kinetic equation is irreversible. The definition of entropy for such cases coincides with that given for the classical spectra.

The structure of Eq. (10) reveals an interesting feature of the evolution of wave systems with combined spectra. Namely, interactions with wave systems corresponding to a delta-like solution to the kinetic equation always accelerate the rest of the waves towards the thermodynamical equilibrium. This property was first mentioned in [2,9], but was related there only to the particular case of spectral symmetrization of Rossby-wave fields owing to interactions with zonal flow.

5. GENERALIZED EQUILIBRIUM SPECTRA

The first integral on the right-hand side of Eq. (10) vanishes if and only if $F = F_{eq}$. For this choice, Eq. (8) reduces to

$$\frac{\partial f}{\partial \tau} = -8\pi f \int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\,\omega^2 F_{1eq}\,F_{2eq}}{F_{eq}}\,\delta^3 d\vec{\kappa}_{12} \le 0. \qquad (11)$$

Inequality (11) once again confirms that the waves corresponding to the generalized solutions of the kinetic equation typically lose their energy to the rest of the wave field even if it is thermodynamically equilibrated. Thus, the evolution of such wave subsystems generally should end up with their merging with the rest of the wave field. Equilibrium of both spectral components will be achieved only in exceptional cases. The fact that only the classical part of the spectrum tends towards nontrivial equilibrium means that behaviour of a "limited" amount of waves is negligible in the statistical sense.

Equation (10) in the case $F = F_{eq}$ reduces to

$$\frac{\partial}{\partial \tau}\int \ln F d\vec{\kappa} = \frac{8\pi}{3}\int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N} F_{eq}F_{1eq}F_{2eq}\mathbf{P}_{012}\left(f\tilde{\delta}\,\frac{\omega^2}{F^3_{eq}} \right)\delta^3 d\vec{\kappa}_{012}. \qquad (12)$$

The right-hand side of Eq. (12) in the case of nonzero functions f vanishes if and only if $V_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\omega = 0$ at the manifold jointly defined by the delta-function $\tilde{\delta}$ and the resonance conditions $\omega_{012} = 0$, $\vec{\kappa}_{012} = \vec{0}$. For Rossby waves it happens if and only if $\tilde{\delta} \equiv \delta(k)$ [8].

The proof of the stability of generalized equilibrium solutions for arbitrary wave systems is shortly presented in the Appendix. Its structure and Eq. (A1) reveal another interesting feature of wave systems with possible generalized spectra. Namely, the actual presence of the generalized spectra additionally suppresses the evolution of the disturbances from the equilibrium.

6. CONCLUSIONS

We considered a case in which it was formally possible to analyse several thermodynamical properties of the "interaction" between the parts of the motions described by classical and singular functions in a unique framework. Although these parts can be distinguished mathematically rather than physically, their evolution scenarios are principally different. Wave subsystems with singular spectra are not generated during the evolution of wave fields with a smooth spectrum. Even interactions between the equilibrated classical spectrum and an arbitrary generalized spectrum generally lead to energy flow into the classical wave system.

The occurrence of spectral singularities accelerates the evolution of the classical spectrum towards thermodynamical equilibrium. Moreover, their presence additionally suppresses the amplitude of small disturbances of the classical equilibrium spectra. In other words, wave systems with generalized spectra serve as a catalyst for the evolution of the whole field towards equilibrium even if they do not change themselves.

Literally, one can say that singularities not only make our life more beautiful but they also always accelerate evolution. Another question is whether we like the direction of the evolution or evolution itself.

ACKNOWLEDGEMENT

This study was partially supported by the Estonian Science Foundation (grant 3504/98).

APPENDIX

STABILITY OF GENERALIZED EQUILIBRIUM SOLUTIONS

Let us again consider the perturbed spectra $F = f\hat{\delta} + F_{eq} + G,\ \ G \ll F_{eq}$, where $f\hat{\delta} + F_{eq}$ is a generalized equilibrium solution to the kinetic equation. Substituting F into Eq. (6), choosing $S = G\big/F_{eq}^2$, and dropping the terms $O(G^2)$, after a little algebra we have:

$$\frac{d}{d\tau}\int \frac{G^2}{F_{eq}^2}\, d\vec{\kappa} = -\frac{8\pi}{3}\int \frac{V_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}^2 F_{eq} F_{1eq} F_{2eq}}{N}\left(\mathbf{P}_{012}\frac{\omega G_p}{F_{eq}^2}\right)^2 \delta^3 d\vec{\kappa}_{012} + \Psi, \qquad \text{(A1)}$$

where

$$\Psi = \frac{8\pi}{3}\int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N}\mathbf{P}_{012}\left[f\hat{\delta}(\omega_1 G_2 + \omega_2 G_1)\right]\mathbf{P}_{012}\,\frac{\omega G}{F^2_{eq}}\,\delta^3 d\vec{\kappa}_{012}$$

and the first additive on the right-hand side of Eq. (A1) is familiar from Eq. (7). Here we have made use of the fact that $\mathbf{P}_{012}[\,\omega(f_1\hat{\delta}_1 + F_{1eq})(f_2\hat{\delta}_2 + F_{2eq})\,] = 0$, since functions $f\hat{\delta} + F_{eq}$ are equilibrium solutions to the kinetic equations. In further reduction of Eq. (A1) we shall repeatedly use the fact that terms containing factors $V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\hat{\delta}\omega$ do not contribute to Ψ, since, by the definition of generalized equilibrium spectra, $V_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\omega = 0$ at the manifold jointly defined by the delta-function $\tilde{\delta}$ and the resonance conditions. Dropping such terms gives:

$$\Psi = \frac{8\pi}{3}\int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N}\mathbf{P}_{012}\left[f\hat{\delta}(\omega_1 G_2 + \omega_2 G_1)\left(\frac{\omega_1 G_1}{F^2_{1eq}} + \frac{\omega_2 G_2}{F^2_{2eq}}\right)\right]\delta^3 d\vec{\kappa}_{012}. \tag{A2}$$

In the kernel of the right-hand side of Eq. (A2) the coefficient at $G_1 G_2$ is

$$\left(\frac{\omega_1}{F_{1eq}}\right)^2 + \left(\frac{\omega_2}{F_{2eq}}\right)^2 = \left(\frac{\omega_1}{F_{1eq}} + \frac{\omega_2}{F_{2eq}}\right)^2 - \frac{2\omega_1\omega_2}{F_{1eq}F_{2eq}} = \left(\frac{\omega}{F_{eq}}\right)^2 - \frac{2\omega_1\omega_2}{F_{1eq}F_{2eq}}.$$

Dropping the terms containing $V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\hat{\delta}\omega$, we reduce Eq. (A2) to

$$\Psi = \frac{8\pi}{3}\int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N}\mathbf{P}_{012}\left[f\hat{\delta}\omega_1\omega_2\left(\frac{G_1}{F_{1eq}} - \frac{G_2}{F_{2eq}}\right)^2\right]\delta^3 d\vec{\kappa}_{012}.$$

Substituting $\omega(\vec{\kappa}_2) = -\omega(\vec{\kappa}_1) - \omega$ into the latter equation and again dropping the terms containing $V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}\hat{\delta}\omega$, we finally have:

$$\Psi = -\frac{8\pi}{3}\int \frac{V^2_{\vec{\kappa}\vec{\kappa}_1\vec{\kappa}_2}}{N}\mathbf{P}_{012}\left[f\hat{\delta}\omega_1^2\left(\frac{G_1}{F_{1eq}} - \frac{G_2}{F_{2eq}}\right)^2\right]\delta^3 d\vec{\kappa}_{012} \le 0. \tag{A3}$$

Comparison of Eqs. (7), (A1), and (A3) shows that the presence of singular spectra additionally suppresses amplitudes of small disturbances of the equilibrium states.

REFERENCES

1. Rhines, P. Waves and turbulence on a β-plane. *J. Fluid Mech.*, 1975, **69**, 417–443.
2. Reznik, G. M. and Soomere, T. On generalized spectra of weakly nonlinear Rossby waves. *Oceanology*, 1983, **23**, 692–694.
3. Kozlov, O. V., Reznik, G. M. and Soomere, T. Weak turbulence on the β-plane in a two-layer ocean. *Izv. Acad. Sci. USSR Atmos. Oceanic Phys.*, 1987, **23**, 869–874.
4. Soomere, T. Kinetic equation for Rossby waves in multi-layer ocean, 1999 (in press).
5. Hasselmann, K. On the nonlinear energy transfer in a gravity-wave spectrum. Part 1. General theory. *J. Fluid Mech.*, 1962, **12**, 481–500.
6. Balk, A. M., Zakharov, V. E. and Nazarenko, S. M. On non-local turbulence of the drift waves. *Soviet Phys., JETP*, 1990, **71**, 249–260.
7. Reznik, G. M. On the properties of the equilibrium spectra of weakly nonlinear Rossby waves. *Oceanology*, 1984, **24**, 869–873.
8. Soomere, T. Generalized stationary solutions of the kinetic equation of barotropic Rossby waves. *Oceanology*, 1987, **27**, 407–409.
9. Reznik, G. M. and Soomere, T. Numerical investigation of the evolution of the energy spectrum of weakly nonlinear Rossby waves. *Oceanology*, 1984, **24**, 287–294.

KLASSIKALISTE JA SINGULAARSETE SPEKTRITE ÜHINE EVOLUTSIOON KINEETILISES TEOORIAS

Tarmo SOOMERE

Kineetilise teooria raames on analüüsitud lainesüsteeme, mille energiaspekter on kirjeldatav klassikalise ning üldistatud funktsioonide superpositsioonina. Varasemad Rossby lainete jaoks saadud tulemused on üldistatud suvaliste triaad-interaktsiooni võimaldavate lainesüsteemide juhule. On tõestatud, et singulaarse ja klassikalise spektri vastasmõju viib üldjuhul singulaarse osa kadumisele, singulaarsetele spektritele vastavad lainesüsteemid kiirendavad aga siledate funktsioonidega kirjeldatava osa evolutsiooni termodünaamilise tasakaalu-seisundi suunas.