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MORPHOLOGICAL INSTABILITY OF GRAIN BOUNDARIES IN SOLIDS

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Abstract. Morphological instability of grain boundaries in solids is studied with the help of the second energy variation approach. The second variations of the elastic energy prove to be certain integro-differential quadratic forms, the structure and properties of which depend essentially on the physical nature of the interfaces. The grains are assumed to be chemically different substances, each with the ability of recrystallization. It is demonstrated that the ability of rearrangement (recrystallization) dramatically diminishes the morphological stability of interfaces with that ability (which might experience the Biot surface morphological instability at deformation of the order of 1). An explicit criterion of morphological stability of two prestressed nonlinear isotropic elastic half-spaces is established. In the case of small prestresses the criterion is formulated in terms of Lamé elastic modules.

Key words: morphological instabilities, thermodynamics of heterogeneous systems, microstructures in solids.

1. INTRODUCTION

Morphological instabilities of different microstructures in solids are actively studied in many disciplines like geomechanics, metallurgy, materials science, etc. (see [1] and references therein). In this paper the morphological instability of grain boundaries is discussed in the framework of the exact nonlinear theory of elasticity on the basis of variational approach going back to Gibbs [2]. The study relies on the criterion of sign-definiteness of the second energy variation of the heterogeneous elastic system (the variation is understood in the spirit of the approach presented in [3]). The grains are treated as one-component, chemically different solids without mass exchange across the separation boundary between them. We assume, however, that in the immediate vicinity of the interface the material particles of each body are able to change their relative positions by means of, say, the interface diffusion

within the (microscopic) intergrain space. The study, though, is purely static (thermodynamic) and does not rely explicitly on any specific kinetic mechanism of mass rearrangement. The role of mass rearrangement is widely discussed in materials science, and the reader can find many useful references in [1].

In the present paper (i) for arbitrarily stressed nonlinear isotropic elastic solid grains the secular equation – (5.3) – is derived, determining the spectrum of singular values of the second energy variation (hence, the stability criterion), and (ii) the explicit formula – (5.6) – of the critical (neutral) wavelength of the surface corrugations is established: the surface disturbances of the wavelength exceeding the critical one appears to be morphologically unstable with respect to mass rearrangement (this formula is established in the framework of $2D$ approach, and the grains are treated as slightly stressed isotropic elastic half-planes).

2. EQUILIBRIUM EQUATIONS (THE FIRST ENERGY VARIATION)

We use here the Euler description of continuous medium and the notation, variational technique, and some results of [3]. The reference frame is referred to the affine Euler coordinates z^i with the metric tensor z^{ij} used for "juggling" the Latin indices.

The grains, treated as simple elastic solids, are supposed to be immobile on the external boundary S of the spatial domain Ω (Fig. 1). Therefore, the virtual velocities $f^i(z^k)$ of the material particles vanish on S. We denote by Ξ the surface of the contact of the grains. At the grain boundary Ξ the material particles of both substances can migrate freely, thus making the recrystallization process possible. We assume that no macroscopic voids exist between the grains. Also, we assume that the system is maintained at constant absolute temperature by certain external thermal agents. The total (free) energy of the heterogeneous system includes two ingredients: a) the bulk (elastic) energy of the grains and b) the surface energy of the grain boundary Ξ which is proportional (with the surface tension coefficient σ) to the area of Ξ in the actual configuration. Thus, we arrive at the following formula of E:

$$E = \widehat{\int_{\Omega}} d\Omega \rho e\left(\nabla_i u_j\right) + \int_{\Xi} d\Xi \sigma, \tag{2.1}$$

where $\rho_{\pm}(z^k)$ are the actual density distributions of the bodies; e is the elastic energy density per unit mass; $u_j(z^k)$ are the Euler components of the displacements; ∇_i and ∇_α are the symbols of covariant differentiation in space and at the actual intergrain surface Ξ (the Latin and Greek indices take on the values $1, 2, 3$ and $1, 2$, respectively); $\Omega_{\pm}$ are the spatial domains occupied by the bodies (which can change due to the rearrangement); $\widehat{\int}$ is the symbol of summation of the integrals over the domains $\Omega_{\pm}$ (Fig. 1).

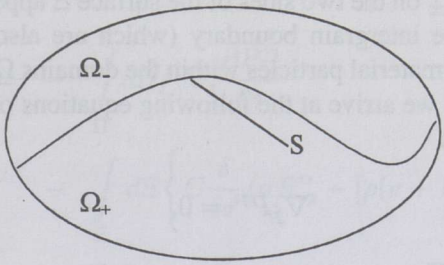

Fig. 1. The geometry of the grain microstructure.

The equilibrium and stability of such a system can be studied on the basis of minimization of the total free energy E under the constraints that the total masses $M_\pm$ of the grains remain fixed:

$$M_\pm = \int\limits_{\Omega_\pm} d\Omega \rho_\pm. \tag{2.2}$$

Thus, we arrive at the problem of minimization of the functional Φ

$$\begin{aligned} \Phi &= E + L^+ M_+ + L^- M_- \\ &= \widehat{\int\limits_{\Omega}} d\Omega \rho e + \int\limits_{\Xi} d\Xi \sigma + L^+ \int\limits_{\Omega_+} d\Omega \rho_+ + L^- \int\limits_{\Omega_-} d\Omega \rho_-, \end{aligned} \tag{2.3}$$

where $L^\pm$ are the Lagrange multipliers associated with the isoperimetric constraints (2.2). Following the scheme of [3], we arrive at the formula of the first variation:

$$\begin{aligned} \delta\Phi = &- \widehat{\int\limits_{\Omega}} d\Omega f^i \nabla_j P^{ji} \\ &- \int\limits_{\Xi} d\Xi \left\{ C \left(\sigma B^\alpha_{.\alpha} - [\rho(e+L)]^+_- \right) + [\rho(\chi^{ji} + L\delta^{ij}) f_i]^+_- N_j \right\}. \end{aligned} \tag{2.4}$$

Here P^{ji} is the Cauchy stress tensor, C is the virtual velocity of the intergrain boundary Ξ, $B^\alpha_{.\alpha}$ is the doubled mean curvature of Ξ (the trace of the second quadrics $B^{\alpha\beta}$ of Ξ), N_j are the components of a unit normal of Ξ, χ^{ji} is the Bowen chemical potential tensor. We recall the following definitions of the tensors P^{ji} and χ^{ji}:

$$P^{ji} = \rho \frac{\partial e}{\partial \nabla_j u_k} (\delta^i_k - \nabla_k u^i), \quad \chi^{ji} = e z^{ij} - \frac{1}{\rho} P^{ji}. \tag{2.5}$$

In the case of two recrystallizing solids of different chemical composition, the virtual velocity C of the boundary Ξ and the limit values of the virtual velocities of

the material particles $f_{i\pm}$ on the two sides of the surface Ξ appear to be independent functions defined at the intergrain boundary (which are also independent of the virtual velocities of the material particles within the domains $\Omega_{\pm}$). Separating these independent variations, we arrive at the following equations of equilibrium:

a) within the grains

$$\nabla_j P^{ji} = 0, \tag{2.6}$$

b) at the boundary Ξ

$$[\rho\,(e+L)]^{+}_{-} = \sigma B^{\alpha}_{.\alpha}, \tag{2.7}$$

$$\rho(\chi^{ji} + L\delta^{ij})_{\pm} N_j = 0. \tag{2.8}$$

The boundary conditions (2.7), (2.8) show, in particular, that, at equilibrium, the boundary Ξ appears to be free of traction. In the case of an intergrain boundary of plane shape, the remaining equilibrium conditions are the following:

$$\rho_+(e_+ + L_+) = \rho_-(e_- + L_-), \tag{2.9}$$

$$e_{\pm} + L_{\pm} - \frac{P^n_{\pm}}{\rho_{\pm}} = 0, \tag{2.10}$$

where $P^n_{\pm}$ are the normal components of the equilibrium stresses.

Equation (2.10) implies the following formulas of the Lagrange multipliers:

$$L_{\pm} = -\left(e_{\pm} - \frac{P^n_{\pm}}{\rho_{\pm}}\right). \tag{2.11}$$

Inserting (2.11) into (2.9), we arrive at the following standard equation of mechanical equilibrium:

$$P^n_+ = P^n_-. \tag{2.12}$$

3. THE SECOND ENERGY VARIATION

In what follows, we investigate the stability of uniformly stressed grains separated by a flat intergrain boundary, using the method of the second energy variation. Differentiating (2.2) one more time and using the equilibrium equations (2.6)–(2.8), we arrive at the following formula:

$$\delta^2\Phi = -\widehat{\int_\Omega} d\Omega f^i \nabla_j \frac{\partial P^{ji}}{\partial \tau}$$
$$- \int_\Xi d\Xi \left\{ C \frac{\delta}{\delta \tau} \left(\sigma B^{\alpha}_{.\alpha} - [\rho(e+L)]^+_- \right) \right.$$
$$\left. + \left[\frac{\delta}{\delta \tau} \left(\rho \left(e + \Lambda \right) z^{ji} - P^{ij} \right) N_j \right]^+_- \right\}. \tag{3.1}$$

For further reduction, we need the following formulas which are valid in the vicinity of piecewise uniform configuration with a flat intergrain boundary (see [3] for details):

$$\frac{\partial P^{ji}}{\partial \tau} = c^{ijkl} \nabla_l f_k,$$
$$\frac{\delta}{\delta \tau} \left\{ (\rho(e+\Lambda) z^{ji} - P^{ij}) N_j \right\} = -c^{ijkl} N_j \nabla_l f_k \tag{3.2}$$
$$-\nabla_\alpha \left\{ C z^{\alpha}_{j.} (\rho(e+L) z^{ji} - P^{ji}) \right\} - N_i \nabla_j f^k \left\{ \rho(e+L) z^{ji} - P^{ji} \right\},$$

where c^{ijkl} is the tensor of "instant" elastic modulae.

Combining (3.1), (3.2), we can reduce the second energy variation to the following form:

$$\delta^2\Phi = -\widehat{\int_\Omega} d\Omega c^{ijkl} \nabla_j f^i \nabla_l f_k$$
$$- \int_\Xi d\Xi \left\{ \sigma \nabla^\alpha \nabla_\alpha C + 2C [D^{ij} \nabla_i f_j]^+_- - [f^k D^{ij} \nabla_i f_j]^+_- N_k \right\}, \tag{3.3}$$

where the tensor D^{ij} is defined as

$$D^{ij} = P^{ij} - \rho(e+L) z^{ij}. \tag{3.4}$$

For the equilibrium configuration in question to be stable, the second variation of energy $\delta^2\Phi$ should be non-negative for all virtual velocities obeying the following constraints (which are the "linearized" version of the isoperimetric constraints (2.2)):

$$\mathcal{M}_\pm = \int_\Xi d\Xi \rho_\pm (C - N^i f_{i\pm}) = 0. \tag{3.5}$$

4. THE SPECTRAL PROBLEM

One of the standard approaches to analyze sign-definiteness of the integro-differential quadratic form is to study its extreme values on the set of kinematically admissible virtual velocities f^i and C of the material particles and of the intergrain boundary, respectively, satisfying the normalization condition

$$G = \widehat{\int\limits_{\Omega}} d\Omega \rho f^i f_i = 1. \tag{4.1}$$

The last question is reduced to the investigation of the unconstrained minimum of the functional $\Pi = \delta^2\Phi - \pi G - \varpi_{\pm}\mathcal{M}_{\pm}$, where π is the Lagrange multiplier associated with the constraint (4.1) and $\varpi_{\pm}$ are the Lagrange multipliers associated with the constraint (3.5), respectively.

Varying the functional Π in the set of the kinematically admissible fields, we get

$$\begin{aligned}
\delta\Pi &= 2\widehat{\int\limits_{\Omega_{\pm}}} d\Omega \left\{C^{*ijkl}\nabla_j(\delta f_i)\nabla_l f_k - \pi\rho f^i \delta f_i\right\} \\
&\quad + 2\int\limits_{\Xi} d\Xi \left\{\begin{array}{c} \sigma\nabla^\alpha C\nabla_\alpha(\delta C) + \delta C\left[D^{ij}\nabla_i f_j\right]_-^+ \\ +C\left[D^{ij}\nabla_i\delta f_j\right]_-^+ - \left[\varpi\rho\left(\delta C - N^i\delta f_i\right)\right]_-^+ \\ -\frac{1}{2}\left[\delta f^k D^{ij}\nabla_i f_j + f^k D^{ij}\nabla_i(\delta f_j)\right]_-^+ N_k \end{array}\right\} \\
&= -2\widehat{\int\limits_{\Omega_{\pm}}} d\Omega\delta f_i\left\{\nabla_j\left(C^{*ijkl}\nabla_l f_k\right) + \pi\rho f^i\right\} \\
&\quad + 2\int\limits_{\Xi} d\Xi \left\{\begin{array}{c} \left[C^{*ijkl}(\delta f_i)\nabla_l f_k\right]_-^+ N_j \\ +\delta C\left(-\sigma\nabla_\alpha\nabla^\alpha C + \left[D^{ij}\nabla_i f_j - \varpi\rho\right]_-^+\right) - \nabla_\alpha C\left[D^{ij}z_{i\cdot}^{\alpha}\delta f_j\right]_-^+ \\ -\frac{1}{2}\left[\delta f^k D^{ij}\nabla_i f_j + \delta f_j D^{ij}z_{i\cdot}^{\alpha}\nabla_\alpha f^k\right]_-^+ N_k + \left[\varpi\rho N^i\delta f_i\right]_-^+ \end{array}\right\}.
\end{aligned} \tag{4.2}$$

Separating independent variations in (4.2), we arrive at the following linear boundary value problem:

a) within the solids

$$\nabla_j(C^{*ijkl}\nabla_l f_k) + \pi\rho f^i = 0; \tag{4.3}$$

b) at the intergrain boundary

$$-\sigma\nabla_\alpha\nabla^\alpha C + \left[D^{ij}\nabla_i f_j - \varpi\rho\right]_-^+ = 0, \tag{4.4}$$

$$C_{\pm}^{*mjkl}\nabla_l f_{k\pm}N_j - \nabla_\alpha C D_{\pm}^{mj} z_{j\cdot}^{\cdot\alpha} - \frac{1}{2}(z^{km}D_{\pm}^{ij}\nabla_i f_{j\pm} + D_{\pm}^{im}z_{i\cdot}^{\cdot\alpha}\nabla_i f_{\pm}^{k})N_k + \varpi_{\pm}\rho_{\pm}N^m = 0. \tag{4.5}$$

The non-negativeness of the singular values π is the necessary condition of the non-negative definiteness of the second energy variation, and, thus, of the stability. This standard assertion can be proved following [3]. It reveals the connection between the second energy variation sign-definiteness and the singular numbers (the eigenvalues) of the uniform linear boundary value problem (4.3)–(4.5).

5. THE SPECTRAL PROBLEM AND THE STABILITY CRITERION FOR THE NONLINEAR ISOTROPIC SOLIDS

In what follows, we limit ourselves to the study of isotropic solids in the framework of the $2D$ approach (all fields depend on the variables $x^1 = x$ and $x^2 = z$ only and f^3 vanishes identically). Using the notation Λ_a, P_a of the principal elongations and stresses, we arrive at the following expressions of the nonzero components of the tensor of instant elasticities C^{*ijkl}:

$$\begin{aligned} C^{*1111} &= \Lambda_1\frac{\partial P^1}{\partial\Lambda_1} = c_{11}, \quad C^{*2222} = \Lambda_2\frac{\partial P^2}{\partial\Lambda_2} = c_{22}, \\ C^{*2211} &= \Lambda_1\frac{\partial P^2}{\partial\Lambda_1} = c_{21}, \quad C^{*1122} = \Lambda_2\frac{\partial P^1}{\partial\Lambda_2} = c_{12}, \\ C^{*1221} &= \Lambda_1^2\frac{P^1 - P^2}{\Lambda_1^2 - \Lambda_2^2} = b_{21}, \quad C^{*1212} = \Lambda_2^2\frac{P^1 - P^2}{\Lambda_1^2 - \Lambda_2^2} = b_{12}... \end{aligned} \tag{5.1}$$

With the help of the notation (5.1) the two nontrivial partial differential equations in the set (4.1) can be rewritten to read

$$\begin{aligned} &\frac{\partial}{\partial x}\left(c_{11}\frac{\partial f^1}{\partial x} + c_{12}\frac{\partial f^2}{\partial z}\right) + \frac{\partial}{\partial z}\left(b_{12}\frac{\partial f^2}{\partial x} + b_{21}\frac{\partial f^1}{\partial z}\right) + \pi\rho f^1 = 0, \\ &\frac{\partial}{\partial x}\left(b_{12}\frac{\partial f^2}{\partial x} + b_{21}\frac{\partial f^1}{\partial z}\right) + \frac{\partial}{\partial z}\left(c_{21}\frac{\partial f^1}{\partial x} + c_{22}\frac{\partial f^2}{\partial z}\right) + \pi\rho f^2 = 0. \end{aligned} \tag{5.2}$$

Using the same notation, the boundary condition (4.4) and the two nontrivial boundary conditions (4.5) take the following forms:

$$\sigma\frac{\partial^2 C}{\partial x^2} + \left[R\frac{\partial f^1}{\partial x^1} - \varpi\rho\right]_-^+ = 0, \tag{5.3}$$

$$
\begin{aligned}
b_{21\pm}\frac{\partial f^1_\pm}{\partial z} + b_{12\pm}\frac{\partial f^2_\pm}{\partial x} - R_\pm\frac{\partial C}{\partial x} + \frac{1}{2}R_\pm\frac{\partial f^2_\pm}{\partial x} = 0,\\
c_{21\pm}\frac{\partial f^1_\pm}{\partial z} + c_{22\pm}\frac{\partial f^2_\pm}{\partial z} - \frac{1}{2}R_\pm\frac{\partial f^1_\pm}{\partial x} + \varpi_\pm\rho_\pm = 0,
\end{aligned}
\tag{5.4}
$$

where $R = P^n_1 - P^n_2$.

Let us consider the solutions of (5.2)–(5.4) of the following form:

$$
f^1(x,z) = F_1(z)e^{ikx}, \quad f^2(x,z) = F_2(z)e^{ikx}, \quad C(x) = Ge^{ikx}. \tag{5.5}
$$

Inserting (5.5) into (5.2) and making a rather simple routine computation, we arrive at the following solutions (which exponentially decay in the upper and lower half-planes, respectively):

$$
\begin{aligned}
f^1_+(x,z) &= (K^-_1 e^{-k|r_1|z} + K^-_2 e^{-k|r_2|z})e^{ikx},\\
f^2_+(z) &= i(\Theta_1 K^-_1 e^{-k|r_1|z} + \Theta_2 K^-_2 e^{-k|r_2|z})e^{ikx},\\
f^1_-(x,z) &= (K^+_1 e^{k|r_1|z} + K^+_2 e^{k|r_2|z})e^{ikx},\\
f^2_-(z) &= i(\Theta_1 K^+_1 e^{k|r_1|z} + \Theta_2 K^+_2 e^{k|r_2|z})e^{ikx},
\end{aligned}
\tag{5.6}
$$

where $|r_{1,2}|$ are two different positive roots of the biquadratic equation

$$
r^4 - r^2\frac{\alpha_1^2 + \alpha_2^2 - (c_{12}+b_{12})(c_{21}+b_{21})}{c_{22}b_{21}} + \frac{\alpha_1^2\alpha_2^2}{c_{22}b_{21}}(\pi' - c_{11})(\pi' - b_{12}) = 0, \tag{5.7}
$$

where $\pi' = \pi\,\rho k^{-2}$, and $\Theta_{1,2}$ are defined as

$$
\Theta_{1,2} = -\frac{\pi' - c_{11} + |r_{1;2}|b_{21}}{(c_{12}+b_{12})|r_{1;2}|}. \tag{5.8}
$$

Inserting (5.5), (5.6) into the boundary conditions (5.3), (5.4), we arrive at the following linear algebraic system of the unknown coefficients $K^\pm_{1;2}$:

$$
\begin{aligned}
&\sigma k^2 G + ik\left\{R_+(K^-_1 + K^-_2) - R_-(K^+_1 + K^+_2)\right\} = 0;\\
&b_{21+}k(|r_{1+}|K^-_1 + |r_{2+}|K^-_2) + b_{12+}k(\Theta_{1+}K^-_1 + \Theta_{2+}K^-_2)\\
&\qquad + ikGR_+ + \frac{1}{2}R_+k(\Theta_{1+}K^-_1 + \Theta_{2+}K^-_2) = 0,\\
&b_{21-}k(|r_{1-}|K^+_1 + |r_{2-}|K^+_2) + b_{12-}k(\Theta_{1-}K^+_1 + \Theta_{2-}K^+_2)\\
&\qquad - ikGR_+ + \frac{1}{2}R_-k(\Theta_{1-}K^+_1 + \Theta_{2+}K^+_2) = 0;\\
&c_{21+}(K^-_1 + K^-_2) - c_{22+}(\Theta_{1+}|r_{1+}|K^-_1 + \Theta_{2+}|r_{2+}|K^-_2) - \frac{1}{2}R_+(K^-_1 + K^-_2) = 0,\\
&c_{21-}(K^+_1 + K^+_2) - c_{22-}(\Theta_{1-}|r_{1-}|K^+_1 + \Theta_{2-}|r_{2-}|K^+_2) - \frac{1}{2}R_-(K^+_1 + K^+_2) = 0.
\end{aligned}
\tag{5.9}
$$

The linear uniform system (5.9) has nonzero solutions if and only if its determinant vanishes. This gives us the following equation of the spectral values π:

$$\left\{ \begin{array}{c} b_{21+}|r_{2+}| + b_{12+}\Theta_{2+} + \frac{1}{2}R_+\Theta_{2+} - \frac{R_+^2}{\sigma k} \\ +L_+\left(b_{21+}|r_{1+}| + b_{12+}\Theta_{1+} + \frac{1}{2}R_+\Theta_{1+} - \frac{R_+^2}{\sigma k}\right) \end{array} \right\}$$

$$\times \left\{ \begin{array}{c} b_{21-}|r_{2-}| + b_{12-}\Theta_{2-} + \frac{1}{2}R_-\Theta_{2-} - \frac{R_-^2}{\sigma k} \\ +L_-\left(b_{21-}|r_{1-}| + b_{12-}\Theta_{1-} + \frac{1}{2}R_-\Theta_{1-} - \frac{R_-^2}{\sigma k}\right) \end{array} \right\}$$

$$= \left(\frac{R_+R_-}{\sigma k}\right)^2 (1+L_+)(1+L_-), \tag{5.10}$$

where $L_\pm$ are defined as follows:

$$L_\pm = -\frac{c_{21\pm} - c_{22\pm}\Theta_{2\pm}|r_{2\pm}| - \frac{1}{2}R_\pm}{c_{21\pm} - c_{22\pm}\Theta_{1\pm}|r_{1\pm}| - \frac{1}{2}R_\pm}. \tag{5.11}$$

For the stability of the intergrain boundary all solutions π of Eq. (5.10) must be non-negative. The equilibrium state at which one of the spectral values changes its sign is the neutral state. To find the neutral configuration (i.e., the critical wavenumber k_{ne} for the system under study), we find the limit of (5.9) at π approaching zero.

In the vicinity of undistorted configurations of the isotropic solids the following approximate formulas are valid:

$$\begin{gathered} b_{12\pm} = b_{21\pm} = c_\pm^{1212} = c_\pm^{1221} = \mu_\pm, \\ c_{21\pm} = c_\pm^{2211} = \lambda_\pm, \quad c_{22\pm} = c_\pm^{2222} = \lambda_\pm + 2\mu_\pm, \end{gathered} \tag{5.12}$$

where $\lambda_\pm$ and $\mu_\pm$ are the Lamé modules of the grains. Inserting (5.12) into the equation of neutral equilibrium, we arrive at the final result

$$(1-\nu_+)\frac{R_+^2}{\mu_+} + (1-\nu_-)\frac{R_-^2}{\mu_-} = \sigma k_{ne}, \tag{5.13}$$

where $\nu_\pm$ are the Poisson ratios of the solids: $\nu_\pm = \lambda_\pm/2\,(\lambda_\pm + \mu_\pm)$.

The formula (5.13) is quite close to the formula of the critical wavelength relating to the morphological instability of the boundary "solid-melt" (see [3] and references therein). It shows, in particular, that the boundary separating recrystallizing grains is less stable than the boundary "solid-melt" (in the sense that the wider domain of the unstable corrugations corresponds to the first of two heterogeneous systems).

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The topic of this paper lies at intersection of the theories of nonlinear continuum mechanics, nonlinear wave propagation, and surface instabilities. For more than two decades, I was very fortunate to learn, discuss, and develop different aspects of these theories with Jüri Engelbrecht. Our numerous encounters have always been the source of inspiration and enthusiasm.

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TAHKE KEHA MIKROSTRUKTUURI TERADE PINNA MORFOLOOGILINE EBASTABIILSUS

Michael GRINFELD

Elastse tahke keha mikrostruktuuri terade pinna morfoloogilise ebastabiilsuse uurimisel on vaadeldud meelevaldset staatilist pingeseisundit. Probleem on taandatud mittelineaarse isotroopse poolruumi ülesandeks ja lahendatud energia avaldise teist järku variatsiooni abil. On näidatud, et kui teradel on omadus rekristalliseeruda, siis väheneb tunduvalt nende pinna morfoloogiline stabiilsus. On tuletatud avaldis kriitilise lainepikkuse määramiseks. Kui pinnahäirituste lainepikkus ületab kriitilise piiri, muutub pind ebastabiilseks.