Proc. Estonian Acad. Sci. Phys. Math., 1997, **46**, 3, 209–220
https://doi.org/10.3176/phys.math.1997.3.05

GROWTH AND DECAY OF WAVES IN ONE-DIMENSIONAL SELF-ADAPTIVE MICROSTRUCTURES

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Received 2 October 1996, accepted 3 April 1997

Abstract. Wave propagation in one-dimensional bodies with a scalar microstructure is discussed. Under suitable constitutive assumptions on the microstructure, i.e., dry friction dissipation, and on the body, i.e., purely elastic or viscoelastic, a strong absorbing effect on the propagation of disturbances is shown. The presence of a threshold for the amplitude of the incoming wave, in case of shocks, allows for undamped propagation of initially large disturbances, namely a wave with constant amplitude, while initially small amplitude shocks are dissipated. As the critical value depends on the state of deformation ahead of the wave, a self-adaptive behaviour arises.

Key words: microstructured bodies, shock waves, nonlinear dissipation.

1. INTRODUCTION

A passive self-adaptive structure is able to modify its response to external disturbances both in dependence from varying external controls and from the characteristics of the external disturbance itself. On the other hand, many composite or complex materials (e.g., liquid crystals, microfractured materials, polymers, and so on) may be described in terms of microstructures.

This work discusses the possibility that a simple model of a microstructure, with internal dissipation of viscous or dry friction type, may give an account of a passive self-adaptive behaviour. According to the usual definitions the microstructure is assumed to be of the scalar type (see, for instance, Capriz [1], Gurtin and Podio-Guidugli [2], Kunin [3], and Mindlin [4]). The procedure applied here for constructing the model is based on the use of the second law of thermodynamics to obtain constitutive restrictions (cf. Ericksen [5] and Leslie [6] for liquid crystals).

We examine in this work a one-dimensional model of a medium with a scalar microstructure with a strongly nonlinear Lipschitz dissipation function which may describe, for instance, a microfractured material. Our model could also be applied to explain the long-range absence of decay observed in seismology (cf. the "dilaton" introduced by Engelbrecht [7,8]. The basic ingredient of the model is a dry-friction type of dissipation in the microstructure.

In this case a self-adaptive behaviour arises, in that while the propagation of a small amplitude shock is governed by the usual jump relations for an elastic dissipative material, a large amplitude shock, with the amplitude above a well-defined threshold value which depends on the state of deformation ahead of the wave only, is "damped" to the threshold value, and then propagates without dissipation. The threshold may be modified by hysteresis cycles under which the material may undergo external loadings.

Thus, a material with this kind of dry-friction microscopic dissipation may be used for devices which absorb large disturbances, reducing them to stationary waves which propagate without damping, while small disturbances are unaffected. This behaviour may be classified as self-adaptive, in that the state of deformation modifies the response threshold and the wave amplitude. Such a behaviour has already been described by Leugering [9] for wave propagation in networks of beams and strings with dry-friction joints.

In case of a vector microstructure, we have previously investigated a strongly nonlinear (smooth) viscous friction dissipation function (see Cermelli and Pastrone [10,11]). Here also a threshold effect for the propagation of disturbances may be described, but it results mainly from the anisotropy caused by the orientation of the microstructure ahead of the wave front.

2. ONE-DIMENSIONAL MODEL

Consider a one-dimensional continuum $\mathcal{B}$, with x a generic point in a reference configuration of $\mathcal{B}$. A *scalar microstructure* is a real valued field $d = d(x)$ on $\mathcal{B}$, and a deformation of $\mathcal{B}$ is an invertible mapping $y = y(x)$ of $\mathcal{B}$ into $\mathbb{R}$, with the *displacement gradient* $u = u(x) = \partial_x y - 1$. A *motion* is a time-dependent family of deformation-microstructure pairs $(y(t), d(t))$; the corresponding velocity pair is denoted by $v = y_t$ and $w = d_t$. We will refer to the map

$$t \mapsto (u(t), v(t), d(t), w(t)) \tag{1}$$

compatible with the requirement

$$u_t = v_x, \qquad d_t = w,$$

as an admissible *kinematic process* for $\mathcal{B}$.

A *strong singularity* for the motion is a time-parametrized curve $\xi = \xi(t)$ in $\mathcal{B}$, such that, for each t, the fields u and d_t (together with their derivatives) are discontinuous at $\xi(t)$. Analogously, a *weak singularity* is a time-parametrized curve such that now the fields u_t and d_t are discontinuous together with their derivatives. The velocity $\dot{\xi}$ of the singularity will be denoted by V.

The basic dynamic fields of this theory are the *free energy* W, the *Piola stress* σ, and the *internal force* κ. With the introduction of the *body external force* b_{ext} and the *microstructural external force* b_{mic}, for any portion $\mathcal{P} \subset \mathcal{B}$, the balance equations have the form

$$\int_{\partial\mathcal{P}} \sigma + \int_{\mathcal{P}} b_{\text{ext}} = \frac{d}{dt}\int_{\mathcal{P}} v, \qquad (2)$$
$$\int_{\mathcal{P}} \kappa + \int_{\mathcal{P}} b_{\text{mic}} = 0.$$

Localization of the integral balances yields the field equations

$$\begin{aligned} u_t &= v_x, \\ v_t &= \sigma_x + b_{\text{ext}}, \\ 0 &= \kappa + b_{\text{mic}}, \end{aligned} \qquad (3)$$

to be satisfied away from possible singularities, and the jump condition

$$V[\![v]\!] + [\![\sigma]\!] = 0 \qquad (4)$$

at the singularity. Note that at a weak singularity this condition becomes

$$[\![\sigma]\!] = 0. \qquad (5)$$

The dynamic fields above are related to the kinematic fields by a suitable set of constitutive equations: to account for dissipation we assume here that

$$W = \tilde{W}(u,d), \qquad \sigma = \tilde{\sigma}(u, v, v_x, d, w), \qquad \kappa = \tilde{\kappa}(u, v, v_x, d, w),$$

with $\tilde{W}$ and $\tilde{\sigma}$ C^2 and C^1 functions, respectively, and $\tilde{\kappa}$ continuous with respect to (u, d). We shall omit from now on the tilde over the constitutive functions.

A *constitutive process* is a kinematic process together with the corresponding dynamic fields computed using the constitutive equations. The dissipation inequality in this context states that, for any constitutive process and portion $\mathcal{P}$ of the body $\mathcal{B}$,

$$\frac{d}{dt}\left\{\int_{\mathcal{P}} W + \frac{1}{2}\int_{\mathcal{P}} v^2\right\} - \int_{\partial\mathcal{P}} \sigma v - \int_{\mathcal{P}} b_{\text{ext}} v - \int_{\mathcal{P}} b_{\text{mic}} w = -\int_{\mathcal{P}} \Delta \le 0, \qquad (6)$$

with Δ the *total dissipation density*.

Use of the balance equations and the arbitrariness of the constitutive process yields the following constitutive restrictions

$$\sigma(u, d, v_x, w) = \sigma_{\text{eq}}(u, d) + \hat{\sigma}(u, d, v_x, w),$$
$$\kappa(u, d, v_x, w) = \kappa_{\text{eq}}(u, d) - \hat{\kappa}(u, d, v_x, w),$$

with

$$\sigma_{\text{eq}} = \frac{\partial W}{\partial u}, \qquad \kappa_{\text{eq}} = -\frac{\partial W}{\partial d},$$

and the jump relation across a strong singularity

$$-([\![W]\!] - \langle\sigma\rangle[\![u]\!])V \leq 0, \tag{7}$$

with $\langle\sigma\rangle$ the average of σ across the singularity.

Thus, the total dissipation Δ is determined by the nonequilibrium constitutive functions through

$$\Delta = \hat{\sigma}(u, d, v_x, w)v_x + \hat{\kappa}(u, d, v_x, w)w \geq 0.$$

The structure of the nonequilibrium stress and force can be made more precise under the assumptions that: (i) the dissipative internal force $\hat{\kappa}$ is absolutely continuous with respect to v_x, of (regular) bounded variation with respect to w, and continuous away from $w = 0$; (ii) the stress $\hat{\sigma}$ is differentiable with respect to its arguments.

These hypotheses yield in general a Lipschitz dissipation density and are compatible with the consideration that, if the microstructure is not taken into account, the dissipative mechanism should be smooth, of the viscous friction type.

Under the above hypotheses the following representations hold:

$$\hat{\sigma}(u, d, v_x, w) = A(u, d, v_x, w)v_x + B(u, d, v_x, w)w,$$
$$\hat{\kappa}(u, d, v_x, w) = C(u, d, v_x, w)v_x + E(u, d, v_x, w),$$

with $E(u, d, v_x, w)$ a bounded variation (BV) function of w.

Now, a standard theorem asserts that any (regular) BV function may be decomposed as the sum of an absolutely continuous and a jump component, so that (omitting the additional arguments)

$$E(w) = D(w)w + \Phi(w),$$

where we may take $\Phi(w)$ given by

$$\begin{aligned} &\Phi \in [-\Phi_c, \Phi_c], \quad &&\text{if } w = 0, \\ &\Phi = \Phi_c \text{sgn}(w), \quad &&\text{if } w \neq 0. \end{aligned} \tag{8}$$

In what follows we shall assume that the coefficients of the nonequilibrium fields do not depend on the kinematic fields, so that

$$\begin{aligned} \hat{\sigma} &= Av_x + Bw, \\ \hat{\kappa} &= Cv_x + Dw + \Phi(w), \end{aligned} \tag{9}$$

with A, B, C, D the entries of a positive definite matrix. Thus, for Lipschitz dissipation functions, only two types of behaviour are possible, namely a dry friction and a viscous dissipative mechanism.

The field equations have the explicit form

$$\begin{aligned} u_t &= v_x, \\ v_t &= (\partial_u W(u,d))_x + Av_{xx} + Bw_x, \\ 0 &= \partial_d W(u,d) + Cv_x + Dw + \Phi(w). \end{aligned} \tag{10}$$

In our applications we shall consider an energy function quadratic in the displacement gradient

$$W = W(u,d) = \frac{1}{2}c^2(u - \lambda d)^2 + \varphi(d), \tag{11}$$

with c, λ given constants and φ a given function. For fixed $d = d_0$, this is a typical one-well convex energy function, with the equilibrium state $u_0 = \lambda d_0$. The field equations become then

$$\begin{aligned} u_t &= v_x, \\ v_t &= c^2 u_x - \lambda c^2 d_x + Av_{xx} + Bw_x, \\ 0 &= -\lambda c^2 u + \lambda^2 c^2 d + \varphi'(d) + Cv_x + Dw + \Phi(w). \end{aligned} \tag{12}$$

Notice that this form of the energy induces a hysteretic behaviour in the stress-against-displacement gradient plane. Consider the quasi-static equations

$$\begin{aligned} \sigma &= \text{const}, \\ \kappa_{\text{eq}} &= \Phi \end{aligned}$$

and let for simplicity $\varphi = 0$ and $\lambda > 0$. The following diagram shows such a behaviour: quasi-static lines $d = \text{const}$ are shown for $d = 0$ and $d = d_0$, along which the stress $\sigma = c^2(u - \lambda d)$ and the internal force $\kappa_{\text{eq}} = -\lambda c^2(u - \lambda d)$ increase (or decrease) monotonically, until the internal force reaches the threshold value given by $\pm\Phi_c$. Along the horizontal lines the internal force and stress remain constant, while the microstructural density and the displacement gradient vary.

We shall assume from now on that

$$\lambda > 0,$$

as this choice is compatible with a naive model of plasticity suggested by the diagram below:

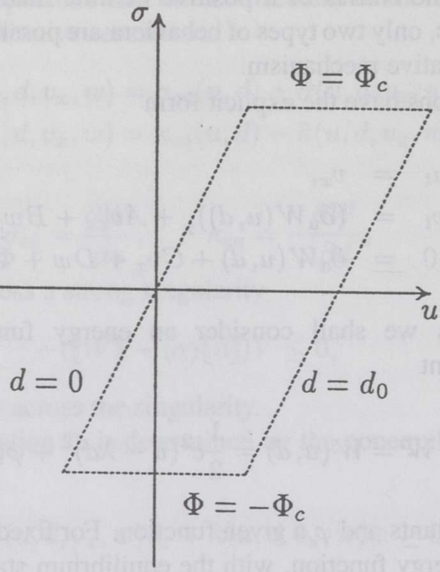

Fig. 1. The $\sigma - u$ diagram.

3. WEAK SINGULARITIES

It is well known that a linear dissipative material does not allow for acceleration waves to propagate. This result has an analogue in the case we are considering here. Consider a weak singularity across which u_x, u_t, and d_t are discontinuous, while u, v, and d are continuous.

The basic jump conditions at this weak singularity are the continuity of the traction across the singular surface,

$$[\![\sigma]\!] = A[\![v_x]\!] + B[\![w]\!] = 0, \tag{13}$$

and the equation obtained from the microforce balance,

$$D[\![w]\!] + C[\![v_x]\!] = -[\![\Phi(w)]\!]. \tag{14}$$

Notice that, if $w = 0$, then Φ is indeterminate and its value is given by

$$\Phi = \Phi(u, d, v_x) = -\partial_d W(u, d) - C v_x = \kappa_{\text{eq}}(u, d) - C v_x.$$

Assume now that the state ahead of the singularity is at rest, i.e., $v^+ = w^+ = 0$, so that

$$-\Phi_c < \kappa_{\text{eq}}(u, d) < \Phi_c.$$

The jump relations above become

$$\begin{aligned} Av_x^- + Bw^- &= 0, \\ Cv_x^- + Dw^- &= \kappa_{\text{eq}}(u,d) - \Phi_c \operatorname{sgn}(w^-). \end{aligned} \tag{15}$$

A weak singularity may propagate in this type of material only if its amplitude reaches the threshold value (recall that the dissipation matrix is positive definite by hypothesis)

$$v_x^- = -\frac{B}{AD-BC}\left(\kappa_{\text{eq}}(u,d) - \Phi_c \operatorname{sgn}(w^-)\right),$$

with $\kappa_{\text{eq}}(u,d)$ only determined by the state ahead of the wave. On the other hand, as

$$w^- = \frac{A}{AD-BC}\left(\kappa_{\text{eq}}(u,d) - \Phi_c \operatorname{sgn}(w^-)\right),$$

then it is easy to see that, for instance, $w^- > 0$ if and only if $\kappa_{\text{eq}}(u,d) - \Phi_c > 0$. This is impossible due to the continuity of the displacement gradient and microstructural density, and the hypothesis that the state ahead of the wave is at rest, which implies $-\Phi_c < \kappa_{\text{eq}}(u,d) < \Phi_c$.

4. STRONG SINGULARITIES

Consider now the propagation of strong singularities (shock waves), so that the displacement gradient u and the velocity v are discontinuous across the singularity, but the microstructural density d is continuous. We assume that the state ahead of the wave is at equilibrium below the threshold value, so that, as before,

$$-\Phi_c < \kappa_{\text{eq}}(u^+,d) < \Phi_c. \tag{16}$$

The jump conditions now consist of the momentum balance (4)

$$V[\![v]\!] + [\![\sigma_{\text{eq}}]\!] + A[\![u_t]\!] + B[\![w]\!] = 0, \tag{17}$$

with V the velocity of the singularity, the dissipation inequality (7)

$$-\left([\![W]\!] - \langle\sigma_{\text{eq}}\rangle[\![u]\!] + \frac{1}{2}(A[\![u_t]\!] + B[\![w]\!])[\![u]\!]\right) V \leq 0 \tag{18}$$

(note that the average of the equilibrium stress across the interface yields the last terms, containing the jumps, in the above inequality as a consequence of the fact that the state ahead of the wave is at rest); the microforce balance $(11)_3$ evaluated behind the interface

$$C[\![u_t]\!] + D[\![w]\!] + \kappa_{\text{eq}}(u^-,d) + \Phi_c \operatorname{sgn}[\![w]\!] = 0. \tag{19}$$

Notice that the dissipative term Φ, if $w^- \neq 0$ and $V > 0$, is given by

$$\Phi = \Phi(w^-) = \Phi_c \text{sgn}\,(w^-) = -\Phi_c \text{sgn}\, [\![w]\!] = \Phi_c \text{sgn}\,\gamma.$$

We list here also an additional condition obtained by taking the jump of the momentum balance $(11)_2$ at the singularity

$$\begin{aligned} V^2\beta - 2V\frac{d\alpha}{dt} - \alpha\frac{dV}{dt} &= (\sigma_{\text{eq}})_u(u^-, d)\beta + (\sigma_{\text{eq}})_d(u^-, d)\gamma \\ &\quad + A\left(\frac{d\beta}{dt} - V\epsilon\right) + B\left(\frac{d\gamma}{dt} - V\delta\right), \end{aligned} \tag{20}$$

where

$$[\![u]\!] = \alpha, \quad [\![u_t]\!] = -V\overline{\beta}, \quad [\![u_x]\!] = \beta, \quad [\![d_x]\!] = \gamma, \quad [\![u_{xx}]\!] = \epsilon, \quad [\![d_{xx}]\!] = \delta$$

and we have used the jump compatibility condition

$$[\![u_t]\!] = -V\beta + \frac{d\alpha}{dt}$$

as well as others of the same nature (cf., e.g., Wright [12]).

Consider now the special case of an energy function quadratic in strain of the form (11): the jump conditions (17), (18), and (19) become

$$\begin{aligned} AV\overline{\beta} + BV\gamma &= (c^2 - V^2)\alpha, \\ (A\overline{\beta} + B\gamma)\alpha V^2 &\leq 0, \\ CV\overline{\beta} + DV\gamma &= -\lambda c^2\alpha + \kappa_{\text{eq}}(u^+, d) - \Phi. \end{aligned} \tag{21}$$

The first equation, together with the second inequality, implies

$$(c^2 - V^2)\alpha^2 V \leq 0,$$

and assuming without loss in generality that $V > 0$, this implies that either (i) $\alpha = 0$, in which case no shock may propagate, or (ii) $V^2 > c^2$, so that the shock is supersonic, or finally (iii)

$$V^2 = c^2,$$

which requires that $A\overline{\beta} + B\gamma = 0$, so that the complete list of jump conditions has the form

$$\begin{aligned} A\overline{\beta} + B\gamma = 0, \\ Cc\overline{\beta} + Dc\gamma = -\lambda c^2\alpha + \kappa_{\text{eq}}(u^+, d) - \Phi. \end{aligned} \tag{22}$$

In what follows we shall assume that (iii) holds, so that we exclude a priori the presence of supersonic shocks in our material.

5. DISCUSSION

From now on, we assume that the energy function is quadratic in the displacement gradient, the state ahead of the wave is at equilibrium, and the propagation velocity V is positive. We shall discuss in this section some easy consequences of the equations described above. The main point is that, in general, the presence of a dry-friction term in the microstructure has a strong absorbing effect on the propagation of disturbances, in that the shock amplitude is necessarily bounded from above. On the other hand, the presence of the threshold-activation mechanism allows for undamped propagation of initially large disturbances, so that above-threshold shocks persist without decay.

5.1. Absence of viscous dissipation

In this (oversimplified) case we assume that

$$A = B = C = D = 0,$$

so that the only source of dissipation in this model is the dry-friction term $\Phi = \Phi(w)$ depending on the microvelocity. Notice that, as the state ahead of the wave is at equilibrium, (16) holds. Across a strong singularity the following jump relations hold:

$$\begin{aligned} V^2 = c^2 \quad &\Rightarrow \quad V = +c, \\ \lambda c^2 \alpha = \kappa_{\text{eq}}&(u^+, d) - \Phi, \end{aligned} \tag{23}$$

so that we have two cases:

(i) If

$$-\Phi_c < \kappa_{\text{eq}}(u^-, d) = -\lambda c^2 \alpha + \kappa_{\text{eq}}(u^+, d) < \Phi_c, \tag{24}$$

then the microstructure is not activated: the internal force κ_{eq} does not reach the threshold value, and as $d_t^- = -\gamma = 0$, the amplitude α of the shock is constant but otherwise arbitrary (determined by the initial conditions only).

(ii) If

$$\kappa_{\text{eq}}(u^+, d) - \lambda c^2 \alpha \leq -\Phi_c \quad \text{or} \quad \Phi_c \geq \kappa_{\text{eq}}(u^+, d) - \lambda c^2 \alpha, \tag{25}$$

then the microstructure is activated, but the shock amplitude is *completely determined* by the state ahead of the wave through the relation

$$\lambda c^2 \alpha = \kappa_{\text{eq}}(u^+, d) - \Phi_c \text{sgn}(\gamma) \tag{26}$$

and (25) must hold with equality sign.

In this case the microstructure has a strong absorbing effect on the amplitude of a shock singularity propagating into a rest state. This property might be used as a device to absorb vibrations.

Notice that the threshold value itself for the shock α, obtained by (24), is determined by the state ahead of the wave, and may be modified by deforming the material or varying the microstructural density.

5.2. Presence of viscous dissipation in the microstructure only

Here $A = B = C = 0$, but $D > 0$. The second equation of system (23) becomes now

$$Dc\gamma = -\lambda c^2\alpha + \kappa_{\text{eq}}(u^+, d) - \Phi, \tag{27}$$

and, as before, two cases are possible.

(i) If the jump amplitude is small, so that (24) holds, then the same conclusions as in case (i) above hold.

(ii) Conversely, if the microstructure is activated and (25) holds, then the initial jump α may assume arbitrary values above the threshold, but it decays eventually to the value given by (26). In fact, by (20)

$$\frac{d\alpha}{dt} = \frac{\lambda c}{2}\gamma$$

and (27), we have

$$\frac{d\alpha}{dt} = \frac{\lambda}{2D}\left(-\lambda c^2\alpha + \kappa_{\text{eq}}(u^+, d) - \Phi_c \text{sgn}(\gamma)\right),$$

so that

$$\lambda c^2\alpha \to \kappa_{\text{eq}}(u^+, d) - \Phi_c \text{sgn}(V\gamma)$$

as $t \to +\infty$.

5.3. Viscous dissipation in both the micro- and the macrostructure

We are now considering the general case where the dissipation matrix, with coefficients A, B, C, and D, is nonsingular. The jump relations at the singularity are given by (22) (we omit, as before, the necessary condition $V = c$). We may distinguish between two cases.

(i) If

$$-\Phi_c < \kappa_{\text{eq}}(u^-, d) - Cc\overline{\beta} = \kappa_{\text{eq}}(u^+, d) - Cc\overline{\beta} - \lambda c^2\alpha < \Phi_c, \tag{28}$$

then the microstructure is not activated, and thus the only nontrivial jump condition is $\overline{\beta} = 0$. This implies in turn that α may be arbitrary (only determined by the initial conditions) provided

$$-\Phi_c < \kappa_{\text{eq}}(u^+, d) - \lambda c^2\alpha < \Phi_c. \tag{29}$$

(ii) Conversely, if the jump is above the threshold value, then the system (22) may be solved in the form

$$\overline{\beta} = -\frac{B/c}{AD - BC}\left(-\lambda c^2 \alpha + \kappa_{\mathrm{eq}}(u^+, d) - \Phi_c \operatorname{sgn}(\gamma)\right),$$

$$\gamma = \frac{A/c}{AD - BC}\left(-\lambda c^2 \alpha + \kappa_{\mathrm{eq}}(u^+, d) - \Phi_c \operatorname{sgn}(\gamma)\right).$$

Here the shock jump is necessarily bounded from below in the absolute value. To see this, consider for instance the case $\gamma > 0$: by the positive definiteness of the dissipation matrix,

$$\lambda c^2 \alpha < \kappa_{\mathrm{eq}}(u^+, d) - \Phi_c < 0.$$

If this value is reached during the evolution of the system, it is easy to notice that, by uniqueness, the shock amplitude may not increase above this value.

ACKNOWLEDGEMENTS

The authors would like to thank M. L. Tonon for useful discussions. This work was completed under support of the M.U.R.S.T. and the C.N.R. of Italy.

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LAINETE VÕIMENDUMINE JA SUMBUMINE ÜHEMÕÕTMELISTES ADAPTEERUVATES MIKROSTRUKTUURIDES

Paolo CERMELLI ja Franco PASTRONE

On uuritud lainelevi ühemõõtmelistes skalaarse mikrostruktuuriga kehades. Mikrostruktuur allub kuiva hõõrde tüüpi dissipatsioonile, makrostruktuur on kas elastne või viskoelastne. On näidatud tugeva dissipatsiooni olemasolu. Mõnda tüüpi olekuvõrrandite puhul on lainelevi seotud läve olemasoluga: alates amplituudi teatud väärtusest lööklained levil võimenduvad, kuid lööklained, mille amplituud on väiksem kui lävi, sumbuvad. Lävi sõltub deformatsioonist lööklaine ees.