Proc. Estonian Acad. Sci. Phys. Math., 1997, **46**, 3, 197–208
https://doi.org/10.3176/phys.math.1997.3.04

THE DISCRETE CONTROL OF TWO-RATE NONLINEAR MULTIVARIABLE CONTINUOUS-TIME SYSTEMS

Igor ASTROV and Ennu RÜSTERN

Department of Computer Control, Tallinn Technical University, Ehitajate tee 5, EE-0026 Tallinn, Estonia; e-mail: iastr@edu.ttu.ee

Received 18 January 1996; revised 14 March 1996, 23 January 1997, and 3 June 1997; accepted 5 June 1997

Abstract. We describe a method for the decomposition of multivariable nonlinear continuous-time systems with the time-separation property on two-rate continuous-time aggregative models in state space. The method can be applied to solve problems connected with the design of the digital control systems in the industry. We obtained the decomposition error and upper limits of sampling periods for slow and fast discrete-time subsystems. A numerical example is considered in order to emphasize the effectivity of our method.

Key words: multivariable, nonlinear, continuous-time system, decomposition, equivalent discrete system, two-rate discrete-time system, state equations, sampling period.

1. INTRODUCTION

Many physical and engineering problems are appropriately described by multirate dynamic models. The computational efforts required for control analysis of such models are quite excessive. It is therefore considered desirable to develop reduced-order models that approximate the dynamic behaviour of multirate systems. A lot of work is currently being done along this direction, including multi-timescale approaches [1–3]. The results of [4,5] show that the singular perturbation theory for difference equations involves the reduction, a separation of timescales, and boundary layer phenomena. The paper [6] surveys the literature on decentralized and hierarchical control, and methods for the study of large-scale systems. The purpose of [7] is to demonstrate two methods for the analysis of singularly perturbed difference equations in the context of linear-control problems. These methods are the technique of matched asymptotic expansions and the multitime method. In [8], an efficient algorithm is developed for designing decentralized dynamic compensators for large-scale systems. The Gauss–Seidel method and the Newton–Raphson method are presented to solve a set of nonlinear equations with the compensator parameters as their unknowns. In [9], several analytical results are introduced to characterize the set of decentralized controllers which achieve desired eigenvalue assignment. A method is proposed to simultaneously assign eigenvalues and eigenvectors of a linear system using decentralized control.

Much progress has been made on analysis of two-rate systems both in continuous-time as well as in discrete-time cases. All these studies focus only on the linear control aspect of the two-rate problem.

In [10], an efficient method for computing state and output equations and sampling periods for two-rate linear multivariable systems based on the canonical Jordan's form of the original system is introduced. In the present work we expand the main ideas of [10]. We describe a new method for designing a class of two-rate nonlinear discrete-time control systems. This class of systems has the characteristic of the sampling rate for the fast subsystem being faster than the sampling rate for the slow subsystem.

2. NONLINEARITIES IN THE CONTROL SYSTEM

Consider the nonlinear, continuous-time system

$$\dot{\mathbf{x}}(\tau) = A\mathbf{x}(\tau) + B\mathbf{u}(\tau) + \mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau)) \,, \tag{2.1}$$

$$\mathbf{y}(\tau) = C\mathbf{x}(\tau) \,, \tag{2.2}$$

where $\mathbf{x}(\tau) \in R^n$, $\mathbf{u}(\tau) \in R^m$, $\mathbf{y}(\tau) \in R^p$, and $\mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau)) \in R^n$ are the state, control input, output, and known nonlinearity vectors [11], respectively. The numerical values of the nonlinearity vector can be obtained from experimental results or using numerical methods [12].

3. DECOMPOSITION OF CONTINUOUS-TIME NONLINEAR SYSTEMS

A. Transformations of the state equations

Let us consider the linear transformation $\mathbf{q}(\tau) = T\mathbf{x}(\tau)$, where T is a nonsingular $n \times n$ matrix. It is easy to see that (2.1)–(2.2) are transformed into the equations

$$\dot{\mathbf{q}}(\tau) = J_C \mathbf{q}(\tau) + \tilde{B}\mathbf{u}(\tau) + T\,\mathbf{f}(\tau), \tag{3.1}$$

$$\mathbf{y}(\tau) = \tilde{C}\mathbf{q}(\tau), \tag{3.2}$$

where the matrix J_C indicates the canonical Jordan's form of A:

$$J_C = TAT^{-1}, \quad \tilde{B} = TB, \quad \tilde{C} = CT^{-1}.$$

Equations (3.1)–(3.2) may be written in terms of submatrices as

$$\dot{\mathbf{z}}_1(\tau) = A_1\mathbf{z}_1(\tau) + B_1\mathbf{u}(\tau) + T_1\mathbf{f}(\tau), \tag{3.3}$$

$$\dot{\mathbf{z}}_2(\tau) = A_2\mathbf{z}_2(\tau) + B_2\mathbf{u}(\tau) + T_2\mathbf{f}(\tau), \tag{3.4}$$

$$\mathbf{y}(\tau) = C_1\mathbf{z}_1(\tau) + C_2\mathbf{z}_2(\tau), \tag{3.5}$$

where

small eigenvalues of a matrix $A_1 : |\lambda(A_1)| < 1$,

large eigenvalues of a matrix $A_2 : |\lambda(A_2)| > 1$,

$$J_C = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}, \tilde{B} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \tilde{C} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}, T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \mathbf{q}(t) = \begin{bmatrix} \mathbf{z}_1(\tau) \\ \mathbf{z}_2(\tau) \end{bmatrix}.$$

B. State equations for fast and slow subsystems

The next qualitative definition is given as

Definition 3.1. *A function f is said to be a fast function if the derivative of f is large. Otherwise, a function f is said to be a slow function if the derivative of f is small.*

Assume that vectors have slow and fast constituents

$$\mathbf{u}(\tau) = \mathbf{u}_f(\tau) + \mathbf{u}_s(\tau), \tag{3.6}$$

$$\mathbf{y}(\tau) = \mathbf{y}_f(\tau) + \mathbf{y}_s(\tau), \tag{3.7}$$

$$\mathbf{f}(\tau) = \mathbf{f}_f(\tau) + \mathbf{f}_s(\tau), \tag{3.8}$$

$$\mathbf{z}_1(\tau) = \mathbf{z}_{1_f}(\tau) + \mathbf{z}_{1_s}(\tau), \tag{3.9}$$

$$\mathbf{z}_2(\tau) = \mathbf{z}_{2_f}(\tau) + \mathbf{z}_{2_s}(\tau). \tag{3.10}$$

If we now substitute (3.6)–(3.10) into (3.3)–(3.5), we find that, according to Definition 3.1,

$$\dot{\mathbf{z}}_{1_s}(\tau) = A_1 \mathbf{z}_{1_s}(\tau) + B_1 \mathbf{u}_s(\tau) + T_1 \mathbf{f}_s(\tau), \tag{3.11}$$

$$\dot{\mathbf{z}}_{2_s}(\tau) = A_2 \mathbf{z}_{2_s}(\tau) + B_2 \mathbf{u}_s(\tau) + T_2 \mathbf{f}_s(\tau), \tag{3.12}$$

$$\mathbf{y}_s(\tau) = C_1 \mathbf{z}_{1_s}(\tau) + C_2 \mathbf{z}_{2_s}(\tau). \tag{3.13}$$

We have from (3.4), (3.6), (3.8), (3.10), and (3.12)

$$\dot{\mathbf{z}}_{2_f}(\tau) = A_2 \mathbf{z}_{2_f}(\tau) + B_2 \mathbf{u}_f(\tau) + T_2 \mathbf{f}_f(\tau). \tag{3.14}$$

According to Definition 3.1

$$\mathbf{z}_1(\tau) = \mathbf{z}_{1_s}(\tau). \tag{3.15}$$

From (3.5), (3.7), (3.10), (3.13), and (3.15), we find that

$$\mathbf{y}_f(\tau) = C_2 \mathbf{z}_{2_f}(\tau). \tag{3.16}$$

We note that state equations for a fast subsystem are given by (3.14) and (3.16) and may be written as

$$\dot{\mathbf{z}}_f(\tau) = A_f \mathbf{z}_f(\tau) + B_f \mathbf{u}_f(\tau) + T_f \mathbf{f}_f(\tau), \tag{3.17}$$

$$\mathbf{y}_f(\tau) = C_f \mathbf{z}_f(\tau), \tag{3.18}$$

where

$$\mathbf{z}_f(\tau) = \mathbf{z}_{2_f}(\tau),\ A_f = A_2,\ B_f = B_2,\ T_f = T_2,\ C_f = C_2.$$

According to Definition 3.1

$$\dot{\mathbf{z}}_{2_f}(t) = 0. \tag{3.19}$$

From (3.12), (3.13), and (3.19), we obtain

$$\mathbf{y}_s(\tau) = C_1 \mathbf{z}_{1_s}(\tau) + C_2 A_2^{-1} B_2 \mathbf{u}_s(\tau) - C_2 A_2^{-1} T_2 \mathbf{f}_s(\tau) \tag{3.20}$$

if A_2^{-1} exists.

State equations for a slow subsystem are given by (3.11) and (3.20) and may be written as

$$\dot{\mathbf{z}}_s(\tau) = A_s \mathbf{z}_s(\tau) + B_s \mathbf{u}_s(\tau) + T_{s_1} \mathbf{f}_s(\tau), \tag{3.21}$$

$$\mathbf{y}_s(\tau) = C_s \mathbf{z}_s(\tau) + D_s \mathbf{u}_s(\tau) + T_{s_2} \mathbf{f}_s(\tau), \tag{3.22}$$

where

$$\mathbf{z}_s(\tau) = \mathbf{z}_{1_s}(\tau),\ A_s = A_1,\ B_s = B_1,\ T_{s_1} = T_1,\ C_s = C_1,$$

$$D_s = -C_2 A_2^{-1} B_2, \quad T_{s_2} = -C_2 A_2^{-1} T_2.$$

C. Decomposition error

Definition 3.2. *We call the norm* $\| \varepsilon \|$ *the decomposition error, that is,*

$$\| \varepsilon(\tau) \| = \left\| \mathbf{y}(\tau) - \mathbf{y}_f(\tau) - \mathbf{y}_s(\tau) \right\|. \tag{3.23}$$

Consider the system

$$\dot{\mathbf{x}}(\tau) = A\mathbf{x}(\tau) + B\mathbf{u}(\tau) + T_1\mathbf{f}(\tau), \tag{3.24}$$

$$\mathbf{y}(\tau) = C\mathbf{x}(\tau) + D\mathbf{u}(\tau) + T_2\mathbf{f}(\tau). \tag{3.25}$$

Using the well-known Laplace transform for the system (3.24)–(3.25), we can easily show that

$$\mathbf{y}(\tau) = C\exp(A\tau)\mathbf{x}(0) + \int_0^\tau C\exp(A(\tau - t))B\mathbf{u}(t)dt$$

$$+\int_0^\tau C\exp(A(\tau - t))T_1\mathbf{f}(t)dt + D\mathbf{u}(\tau) + T_2\mathbf{f}(\tau). \tag{3.26}$$

In convolution representation, (3.26) can be written as

$$\mathbf{y}(\tau) = \mathbf{y}_0(\tau) + (G_{\mathbf{yu}} * \mathbf{u})(\tau) + (G_{\mathbf{yf}} * \mathbf{f})(\tau) + D\mathbf{u}(\tau) + T_2\mathbf{f}(\tau), \tag{3.27}$$

where

$$\mathbf{y}_0(\tau) = C\exp(A\tau)\mathbf{x}(0), \tag{3.28}$$

$$G_{\mathbf{yu}}(\tau) = C\exp(A\tau)B, \tag{3.29}$$

$$G_{\mathbf{yf}}(\tau) = C\exp(A\tau)T_1. \tag{3.30}$$

From (2.2), (3.6), (3.8), (3.18), (3.22), (3.23), and (3.27)–(3.30), we find that

$$\varepsilon = \mathbf{y}_0 - \mathbf{y}_{0_f} - \mathbf{y}_{0_s} + (G_{\mathbf{yu}} - G_{\mathbf{y}_f\mathbf{u}_f})*\mathbf{u}_f + (G_{\mathbf{yu}} - G_{\mathbf{y}_s\mathbf{u}_s})*\mathbf{u}_s$$

$$+(G_{\mathbf{yf}} - G_{\mathbf{y}_f\mathbf{f}_f})*\mathbf{f}_f + (G_{\mathbf{yf}} - G_{\mathbf{y}_s\mathbf{f}_s})*\mathbf{f}_s - D_s\mathbf{u}_s - T_{s_2}\mathbf{f}_s, \qquad (3.31)$$

where

$$\mathbf{y}_0(\tau) = C\exp(A\tau)\mathbf{x}(0),$$

$$\mathbf{y}_{0_f}(\tau) = C_f\exp(A_f\tau)\mathbf{z}_f(0),$$

$$\mathbf{y}_{0_s}(\tau) = C_s\exp(A_s\tau)\mathbf{z}_s(0),$$

$$G_{\mathbf{yu}}(\tau) = C\exp(A\tau)B,$$

$$G_{\mathbf{y}_f\mathbf{u}_f}(\tau) = C_f\exp(A_f\tau)B_f,$$

$$G_{\mathbf{y}_s\mathbf{u}_s}(\tau) = C_s\exp(A_s\tau)B_s,$$

$$G_{\mathbf{yf}}(\tau) = C\exp(A\tau),$$

$$G_{\mathbf{y}_f\mathbf{f}_f}(\tau) = C_f\exp(A_f\tau)T_f,$$

$$G_{\mathbf{y}_s\mathbf{f}_s}(\tau) = C_s\exp(A_s\tau)T_{s_1}.$$

From (3.31), we find, after some algebra, that

$$\|\varepsilon(\tau)\| \le \|\mathbf{y}_0(\tau)\| + \|\mathbf{y}_{0_f}(\tau)\| + \|\mathbf{y}_{0_s}(\tau)\| + \int_0^\tau \left\|G_{\mathbf{yu}}(\tau - t) - G_{\mathbf{y}_f\mathbf{u}_f}(\tau - t)\right\| \|\mathbf{u}_f(t)\| dt$$

$$+\int_0^\tau \left\|G_{\mathbf{yu}}(\tau - t) - G_{\mathbf{y}_s\mathbf{u}_s}(\tau - t)\right\| \|\mathbf{u}_s(t)\| dt + \int_0^\tau \left\|G_{\mathbf{yf}}(\tau - t) - G_{\mathbf{y}_f\mathbf{f}_f}(\tau - t)\right\| \|\mathbf{f}_f(t)\| dt$$

$$+\int_0^\tau \left\|G_{\mathbf{yf}}(\tau - t) - G_{\mathbf{y}_s\mathbf{f}_s}(\tau - t)\right\| \|\mathbf{f}_s(t)\| dt + \|D_s\| \|\mathbf{u}_s(\tau)\| + \|T_{s_2}\| \|\mathbf{f}_s(\tau)\|. \qquad (3.32)$$

4. STATE EQUATIONS FOR THE TWO-RATE NONLINEAR DISCRETE-TIME SYSTEM

Definition 4.1. *The nonlinear equivalent discrete system is the discrete model having a continuous-time nonlinear system with analog/digital and digital/analog converters.*

Consider the system (3.24)–(3.25). Using the well-known Laplace transform for (3.24), we obtain that

$$\mathbf{x}(\tau) = \exp(A(\tau - \tau_0))\mathbf{x}(\tau_0) + \int_{\tau_0}^{\tau} \exp(A(\tau - t))B\mathbf{u}(t)dt$$

$$+ \int_{\tau_0}^{\tau} \exp(A(\tau - t))T_1\mathbf{f}(t)dt. \tag{4.1}$$

Using Definition 4.1 and Eq. (4.1) for the system (3.24)–(3.25), we can easily show that the state equations for the equivalent discrete system are

$$\mathbf{x}\left[(t+1)\Delta\right] = F\mathbf{x}(t\Delta) + G\mathbf{u}(t\Delta) + T_{d_1}\mathbf{f}(t\Delta), \tag{4.2}$$

$$\mathbf{y}(t\Delta) = H\mathbf{x}(t\Delta) + E\mathbf{u}(t\Delta) + T_{d_2}\mathbf{f}(t\Delta), \quad t = 0, 1, 2, \ldots, \tag{4.3}$$

where

$$F = \exp(A\Delta), \quad G = \left[\int_0^{\Delta} \exp(Aq)dq\right]B, \quad T_{d_1} = \left[\int_0^{\Delta} \exp(Aq)dq\right]T_1,$$

$$H = C, \quad E = D, \quad T_{d_2} = T_2.$$

We note that the state equations for the equivalent fast subsystem, which was obtained from (3.17)–(3.18), are given in the form (4.2)–(4.3) and may be written as

$$\mathbf{z}_f\left[(t+1)\Delta_f\right] = F_f\mathbf{z}_f(t\Delta_f) + G_f\mathbf{u}_f(t\Delta_f) + T_{d_f}\mathbf{f}_f(t\Delta_f), \tag{4.4}$$

$$\mathbf{y}_f(t\Delta_f) = H_f\mathbf{z}_f(t\Delta_f), \ t = 0, 1, 2, \ldots, \tag{4.5}$$

where

$$F_f = \exp(A_f \Delta_f), \quad G_f = \left[\int_0^{\Delta_f} \exp(A_f q) dq \right] B_f,$$

$$T_{d_f} = \left[\int_0^{\Delta_f} \exp(A_f q) dq \right] T_f, \quad H_f = C_f.$$

The state equations for the equivalent slow subsystem, which was obtained from (3.21)–(3.22), are given in the form (4.2)–(4.3) and may be written as

$$\mathbf{z}_s\left[(t+1)\Delta_s\right] = F_s \mathbf{z}_s(t\Delta_s) + G_s \mathbf{u}_s(t\Delta_s) + T_{d_{s_1}} \mathbf{f}_s(t\Delta_s), \tag{4.6}$$

$$\mathbf{y}_s(t\Delta_s) = H_s \mathbf{z}_s(t\Delta_s) + E_s \mathbf{u}_s(t\Delta_s) + T_{d_{s_2}} \mathbf{f}_s(t\Delta_s), \quad t = 0, 1, 2, \ldots, \tag{4.7}$$

where

$$F_s = \exp(A_s \Delta_s), \quad G_s = \left[\int_0^{\Delta_s} \exp(A_s q) dq \right] B_s, \quad T_{d_{s_1}} = \left[\int_0^{\Delta_s} \exp(A_s q) dq \right] T_{s_1},$$

$$H_s = C_s, \quad E_s = D_s, \quad T_{d_{s_2}} = T_{s_2}.$$

4. SAMPLING PERIODS FOR THE TWO-RATE NONLINEAR DISCRETE-TIME SYSTEM

The choice of the sampling period Δ for the equivalent discrete system depends upon the system dynamics. A small Δ used to be associated with serious problems owing to quantization and roundoff. On the other hand, if a system has slow dynamics, then it might dictate a larger Δ. The selection of the best sampling periods is a compromise. For example, we might use

$$\Delta_f \approx \Delta_{f_{\max}}, \tag{5.1}$$

$$\Delta_s \approx \Delta_{s_{\max}}. \tag{5.2}$$

From (4.2), (4.3), we find that

$$\mathbf{y}[(t+1)\Delta] = HF(\Delta)\mathbf{x}(t\Delta) + HG(\Delta)\mathbf{u}(t\Delta) + HT_{d_1}(\Delta)\mathbf{f}(t\Delta)$$
$$+E\mathbf{u}[(t+1)\Delta] + T_{d_2}\mathbf{f}[(t+1)\Delta]. \tag{5.3}$$

Using $t = 0$, $\mathbf{x}(0) = 0$, and (5.3), we find that

$$\mathbf{y}[\Delta] = HG(\Delta)\mathbf{u}(0) + E\mathbf{u}[\Delta] + HT_{d_1}(\Delta)\mathbf{f}(0) + T_{d_2}\mathbf{f}[\Delta]. \tag{5.4}$$

If the maximum values of control input, output, and nonlinearities are given and Definition 4.1 is satisfied, then (5.4) may be written as

$$\mathbf{y}_{\max} = [HG(\Delta) + E]\mathbf{u}_{\max} + \left[HT_{d_1}(\Delta) + T_{d_2}\right]\mathbf{f}_{\max}. \tag{5.5}$$

From (5.1), (5.5), and (4.4)–(4.5), we find

$$\mathbf{y}_{f_{\max}} = \left[H_f G_f(\Delta_f)\right]\mathbf{u}_{f_{\max}} + \left[H_f T_{d_f}(\Delta_f)\right]\mathbf{f}_{f_{\max}}. \tag{5.6}$$

From (5.2), (5.5), and (4.6)–(4.7), we have

$$\mathbf{y}_{s_{\max}} = \left[H_s G_s(\Delta_s) + E_s\right]\mathbf{u}_{s_{\max}} + \left[H_s T_{d_{s_1}}(\Delta_s) + T_{d_{s_2}}\right]\mathbf{f}_{s_{\max}}. \tag{5.7}$$

Note that we can obtain Δ_f and Δ_s from matrix equations (5.6) and (5.7), respectively.

6. EXAMPLE

Consider the fourth-order system (2.1)–(2.2) given by

$$A = \begin{bmatrix} -0.1259 & -1.0116 & 0.5584 & -2.2093 \\ -2.3202 & -2.5385 & -1.0378 & -0.4024 \\ -2.2824 & -0.2158 & -1.9373 & 3.2799 \\ 2.6355 & 1.5343 & 1.3487 & -1.6382 \end{bmatrix}, \quad B = \begin{bmatrix} -1.4252 & -1.5087 \\ 0.9051 & 1.4777 \\ 2.7818 & 0.9982 \\ -1.3959 & -1.1119 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.937 & 0.252 & 0.56 & 0.255 \\ 0.048 & 0.396 & 0.024 & 0.296 \end{bmatrix},$$

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} 0.01 + 0.04u_1^2 + 0.06\sin x_1 \\ 0.03 + 0.02u_2^3 \cos x_2 \\ 0.02 + 0.05u_1^3 \sin x_3^2 \\ 0.05 + (1 - u_2^2)\sin x_4 \end{bmatrix}$$

to demonstrate the implementation of our decomposition method. Computation of the largest singular value of ε from (3.32) gives

$$\|\varepsilon(3)\| \le 0.1273,$$

where

$$\mathbf{x}^T(0) = \begin{bmatrix} 0.001 & 0.003 & 0.002 & 0.005 \end{bmatrix},$$

$$\mathbf{u}(\tau) = \begin{bmatrix} 0.05 + 0.01\sin 2\tau \\ 0.04 + 0.03\sin 3\tau \end{bmatrix}.$$

From (5.6), we obtain

$$\Delta_f = 0.35,$$

where

$$\mathbf{u}_{f_{\max}}^T = \begin{bmatrix} 1.3 & 2.6 \end{bmatrix}, \quad \mathbf{f}_{f_{\max}}^T = \begin{bmatrix} 1.2 & 2.4 & 4.3 & 2.5 \end{bmatrix}, \quad \mathbf{y}_{f_{\max}}^T = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}.$$

From (5.7), we have

$$\Delta_s = 2.9,$$

where

$$\mathbf{u}_{s_{\max}}^T = \begin{bmatrix} 2.8 & 1.5 \end{bmatrix}, \quad \mathbf{f}_{s_{\max}}^T = \begin{bmatrix} 6.3 & 5.2 & 8.1 & 7.2 \end{bmatrix}, \quad \mathbf{y}_{s_{\max}}^T = \begin{bmatrix} 14 & 2.45 \end{bmatrix}.$$

The fast-subsystem dynamics (4.4)–(4.5) is described by

$$F_f = \begin{bmatrix} 0.3455 & 0.1751 \\ -0.1751 & 0.3455 \end{bmatrix}, \quad G_f = \begin{bmatrix} -0.1305 & -0.0448 \\ 0.113 & 0.0387 \end{bmatrix},$$

$$T_{d_f} = \begin{bmatrix} 0.0748 & 0.1363 & 0.0132 & 0.1318 \\ 0.145 & 0.1098 & 0.0659 & -0.0264 \end{bmatrix}, \quad H_f = \begin{bmatrix} 0.19 & 0.57 \\ 0.28 & 0.16 \end{bmatrix}.$$

The slow subsystem (4.6)–(4.7) is specified by

$$F_s = \begin{bmatrix} 0.419 & 0 \\ 0 & 0.2214 \end{bmatrix}, \quad G_s = \begin{bmatrix} 0.2324 & -1.3558 \\ 0.4642 & 0.3893 \end{bmatrix},$$

$$T_{d_{s1}} = \begin{bmatrix} 0.9684 & 0 & 0.9684 & 0.7747 \\ 0 & 0.7487 & 0.4492 & 1.0482 \end{bmatrix}, \quad H_s = \begin{bmatrix} 1 & -0.37 \\ -0.24 & 0.32 \end{bmatrix},$$

$$E_s = \begin{bmatrix} 0.0931 & 0.0319 \\ -0.0063 & -0.0022 \end{bmatrix}, \quad T_{d_{2s}} = \begin{bmatrix} 0.1324 & 0.1045 & 0.0591 & -0.0163 \\ 0.0739 & 0.0829 & 0.0265 & 0.0357 \end{bmatrix}.$$

7. CONCLUSIONS

The decomposition method proposed by us permits finding discrete-time state equations for the fast nonlinear subsystem (4.4)–(4.5) and the slow nonlinear subsystem (4.6)–(4.7) if the original continuous-time system (2.1)–(2.2) has a time-separation property. We obtained the decomposition error (3.32) and high limits of sampling periods for the slow (5.7) and fast (5.6) discrete subsystems. The translation of a mathematical model of a fourth-order system into a computer model is implemented using the MATLAB program, because solutions to the problem are expressed in the MATLAB almost as exactly as mathematically.

REFERENCES

1. Mahmoud, M. S. Design of observer-based controllers for a class of discrete systems. *Automatica*, 1982, **18**, 3, 323–328.
2. Sezer, M. E. and Siljak, D. D. Decentralized multirate control. *IEEE Trans. Automat. Control*, 1990, **35**, 1, 60–65.
3. Saif, M. and Guan, Y. Decentralized state estimation in large-scale interconnected dynamical systems. *Automatica*, 1992, **28**, 1, 215–219.
4. Kokotovic, P. V. and Yackel, R. A. Singular perturbation of linear regulator: Basic theorems. *IEEE Trans. Automat. Control*, 1972, **17**, 1, 29–37.
5. Kokotovic, P. V., O'Malley, R. E., and Sannuti, P. Singular perturbations and order reduction in control theory – an overview. *Automatica*, 1976, **12**, 2, 123–132.
6. Sandell, N. R., Varaiya, P., Athans, M., and Safonov, M. G. Survey of decentralized control methods for large scale systems. *IEEE Trans. Automat. Control*, 1978, **23**, 2, 108–128.
7. Blankenship, G. Singularly perturbed difference equations in optimal control problems. *IEEE Trans. Automat. Control*, 1981, **26**, 4, 911–917.
8. Chen, C.-L. and Hsu, Y.-Y. Applying two numerical methods to the decentralized modal control problem. *IEEEE Trans. Automat. Control*, 1991, **36**, 5, 627–629.
9. Lu, J., Chiang, H. D., and Thorp, J. S. Eigenstructure assignment by decentralized feedback control. *IEEE Trans. Automat. Control*, 1993, **38**, 4, 587–594.
10. Astrov, I. I. Opisanie dvukhskorostnogo diskretnogo ob''ekta upravleniya. *Proc. Acad. Sci. USSR. Technical Cybernetics*, 1990, 5, 224 (in Russian).
11. Voronov, A. A. *Ustoichivost', upravlyaemost', nablyudaemost'*. Nauka, Moscow, 1979 (in Russian).
12. Forsythe, G. E., Malcolm, M. A., and Moler, C. B. *Computer Methods for Mathematical Computations*. Prentice Hall, New York, 1977.

KAHEKIIRUSELISTE MITTELINEAARSETE MITMEMÕÕTMELISTE PIDEVA AJA SÜSTEEMIDE SÜSTEEMNE JUHTIMINE

Igor ASTROV ja Ennu RÜSTERN

On esitatud meetod, mis võimaldab diskreetaja olekuruumis dekomponeerida mitmemõõtmelisi mittelineaarseid pideva aja süsteeme kiireks ja aeglaseks alamsüsteemiks. Selle meetodi abil saab projekteerida nüüdisaegse tööstuse automaatjuhtimissüsteeme. On leitud dekomponeerimisvea hinnang ning kiire ja aeglase alamsüsteemi diskreetimissammude ülemised rajad. Meetodi kasutatavust on demonstreeritud neljandat järku süsteemi varal.