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THE STATE-SPACE DESCRIPTION OF TWO-RATE NONLINEAR MULTIVARIABLE DISCRETE SYSTEMS

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Abstract. We approximated the behaviour of the systems, having the time-separation property, by two lower-order subsystems: a slow subsystem with large eigenvalues and a fast subsystem with small eigenvalues. The method proposed permits finding state equations for two-rate nonlinear discrete systems if an equivalent discrete system is presented in the canonical Jordan's form. For slow and fast discrete subsystems we obtained the decentralization error and upper limits of sampling periods. The theoretical analysis is illustrated by a fourth-order system model.

Key words: nonlinear, multivariable, continuous-time system, decentralization error, nonlinear equivalent discrete-time system, two-rate control, sampling period.

1. INTRODUCTION

Many physical and engineering problems are appropriately described by multirate dynamic models. Usually decentralized constraints arise in controlling large composite systems because a centralized controller, i.e., a single controller which observes all outputs of the system to control all inputs of the system, commonly requires excessive information gathering for practical application of such controller. Thus, the study of the control of multirate systems directly leads to the investigation of decentralized control systems.

A good combination of engineering judgment and analysis can be used to define in a reasonable, albeit ad hoc, way a special structure for the dynamic system. One should look for timescale separation (fast and slow dynamics), weak coupling, and similar phenomena, as they naturally lead to decentralizing of the system [1,2]. Linear discrete systems with fast and slow modes are discussed in

[3,4]. In [5], a systematic procedure for complete separation of slow and fast regulator designs is developed by extending the idea of two-stage eigenvalue assignment. In [6], a detailed study of nonconventional sampled-data control systems, based on a block multirate input–output model, is presented. The model is developed for multivariable multirate nonsynchronous sampled-data control systems and can be easily extended to multivariable systems. One of the main contributions of [4] is that it emphasizes the usefulness of the fast-slow separation in simplifying the design of stabilizing feedback controllers for linear discrete systems with inaccessible states. The objective of [7] is to introduce a decentralized control approach to multirate linear systems. In [8], the idea of multirate control is formulated and an algorithm for linear discrete systems is developed. In [9], a decentralized control technique is applied to the problems of the stabilization of periodic output feedback or pole assignment for a periodically time-varying discrete-time system.

While much progress has been made on analysis of two-rate systems both in continuous-time and discrete-time cases, all the studies focus only on the aspect of linear control of the two-rate problem. The method proposed in [10] allows for finding state and output equations and sampling periods for two-rate linear multivariable systems if an equivalent discrete system is presented in the canonical Jordan's form. This method is also a very simple design procedure.

In what follows we describe a new approach to two-rate nonlinear control, based on the idea of time-separation for fast and slow constituents of vectors. This approach leads to a new class of two-rate nonlinear discrete-time control systems which are based on elementary principles of modern control theory and are relatively easy to analyse and implement.

2. STATE EQUATIONS FOR THE NONLINEAR EQUIVALENT DISCRETE-TIME SYSTEM

Consider the nonlinear, continuous-time system

$$\dot{\mathbf{x}}(\tau) = A\mathbf{x}(\tau) + B\mathbf{u}(\tau) + \mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau)), \tag{2.1}$$

$$\mathbf{y}(\tau) = C\mathbf{x}(\tau), \tag{2.2}$$

where $\mathbf{x}(\tau) \in \mathrm{R}^n$, $\mathbf{u}(\tau) \in \mathrm{R}^m$, $\mathbf{y}(\tau) \in \mathrm{R}^p$, and $\mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau)) \in \mathrm{R}^n$ are the state, control input, output, and nonlinearity vectors, respectively.

Note that the study of nonlinear systems with concrete nonlinearities will not lead to good practical results [11]. This implies that the system (2.1)–(2.2) can be considered with the known nonlinearity vector $\mathbf{f}(\tau)$ [11]. The numerical values of the vector $\mathbf{f}(\tau)$ can be obtained from experimental results or using numerical methods [12].

Definition 2.1. *The nonlinear equivalent discrete system is the discrete model having a continuous-time nonlinear system with analog/digital and digital/analog converters.*

Using the well-known Laplace transform for (2.1), we obtain

$$\mathbf{x}(\tau)=\exp(A(\tau-\tau_0))\mathbf{x}(\tau_0)+\int_{\tau_0}^{\tau}\exp(A(\tau-t))B\mathbf{u}(t)dt+\int_{\tau_0}^{\tau}\exp(A(\tau-t))\mathbf{f}(t)dt. \tag{2.3}$$

Using Definition 2.1 and Eq. (2.3) for the system (2.1)–(2.2), we can easily show that the state equations for an equivalent discrete system are

$$\mathbf{x}[(t+1)\Delta]=F\mathbf{x}(t\Delta)+G\mathbf{u}(t\Delta)+T\mathbf{f}(t\Delta), \tag{2.4}$$

$$\mathbf{y}(t\Delta)=H\mathbf{x}(t\Delta), \quad t=0,1,2,\ldots, \tag{2.5}$$

where

$$F=\exp(A\Delta),\ G=\left[\int_0^{\Delta}\exp(Aq)dq\right]B,\ T=\int_0^{\Delta}\exp(Aq)dq,\ H=C.$$

3. DECENTRALIZATION OF DISCRETE-TIME NONLINEAR SYSTEMS

A. Transformations of the state equations

Suppose that we consider the linear transformation $\mathbf{q}(t)=Q\mathbf{x}(t)$, where Q is a nonsingular $n\times n$ matrix. It is easy to see that (2.4)–(2.5) are transformed into the equations

$$\mathbf{q}(t+1)=J_d\mathbf{q}(t)+\tilde{G}\mathbf{u}(t)+\tilde{T}\mathbf{f}(t), \tag{3.1}$$

$$\mathbf{y}(t)=\tilde{H}\mathbf{q}(t),\ t=0,1,2,\ldots, \tag{3.2}$$

where the matrix J_d indicates the canonical Jordan's form of F

$$J_d=QFQ^{-1},\quad \tilde{G}=QG,\quad \tilde{H}=HQ^{-1},\quad \tilde{T}=QT.$$

Equations (3.1)–(3.2) may be written in terms of submatrices as

$$\mathbf{z}_1(t+1)=F_1\mathbf{z}_1(t+1)=G_1\mathbf{u}(t)+T_1\mathbf{f}(t), \tag{3.3}$$

$$\mathbf{z}_2(t+1) = F_2\mathbf{z}_2(t) + G_2\mathbf{u}(t) + T_2\mathbf{f}(t), \tag{3.4}$$

$$\mathbf{y}(t) = H_1\mathbf{z}_1(t) + H_2\mathbf{z}_2(t), \ t = 0, 1, 2, ..., \tag{3.5}$$

where the eigenspectrum of the matrix F_2 consists of small eigenvalues and the eigenspectrum of the matrix F_1 consists of large eigenvalues within, including the unit circle

$$J_d = \begin{bmatrix} F_2 & 0 \\ 0 & F_1 \end{bmatrix}, \tilde{G} = \begin{bmatrix} G_2 \\ G_1 \end{bmatrix}, \tilde{H} = \begin{bmatrix} H_2 & H_1 \end{bmatrix}, \tilde{T} = \begin{bmatrix} T_2 \\ T_1 \end{bmatrix}, \mathbf{q}(t) = \begin{bmatrix} \mathbf{z}_2(t) \\ \mathbf{z}_1(t) \end{bmatrix}.$$

B. State equations for fast and slow subsystems

The next qualitative definition is given as

Definition 3.1. *The constituents of vectors are said to be slow constituents if these constituents are slowly varied constants between the sampling periods. Otherwise, the constituents of vectors are said to be fast constituents if they are strongly varied variables and important only during a short initial period.*

Assume that vectors have slow and fast constituents

$$\mathbf{y}(t) = \mathbf{y}_f(t) + \mathbf{y}_s(t), \tag{3.6}$$

$$\mathbf{u}(t) = \mathbf{u}_f(t) + \mathbf{u}_s(t), \tag{3.7}$$

$$\mathbf{f}(t) = \mathbf{f}_f(t) + \mathbf{f}_s(t), \tag{3.8}$$

$$\mathbf{z}_1(t) = \mathbf{z}_{1_f}(t) + \mathbf{z}_{1_s}(t), \tag{3.9}$$

$$\mathbf{z}_2(t) = \mathbf{z}_{2_f}(t) + \mathbf{z}_{2_s}(t). \tag{3.10}$$

If we now substitute (3.6)–(3.10) into (3.3)–(3.5), we find that, according to Definition 3.1,

$$\mathbf{z}_{1_s}(t+1) = F_1\mathbf{z}_{1_s}(t) + G_1\mathbf{u}_s(t) + T_1\mathbf{f}_s(t), \tag{3.11}$$

$$\mathbf{z}_{2_s}(t+1) = F_2\mathbf{z}_{2_s}(t) + G_2\mathbf{u}_s(t) + T_2\mathbf{f}_s(t), \tag{3.12}$$

$$\mathbf{y}_s(t) = H_1\mathbf{z}_{1_s}(t) + H_2\mathbf{z}_{2_s}(t). \tag{3.13}$$

According to Definition 3.1

$$\mathbf{z}_{2_s}(t) = \text{const.} \tag{3.14}$$

From (3.12) and (3.14), we get .

$$\mathbf{z}_{2_s}(t) = [I - F_2]^{-1} G_2 \mathbf{u}_s(t) + [I - F_2]^{-1} T_2 \mathbf{f}_s(t) \tag{3.15}$$

if $[I - F_2]^{-1}$ exists. We have from (3.13) and (3.15)

$$\mathbf{y}_s(t) = H_1 \mathbf{z}_{1_s}(t) + H_2 [I - F_2]^{-1} G_2 \mathbf{u}_s(t) + H_2 [I - F_2]^{-1} T_2 \mathbf{f}_s(t). \tag{3.16}$$

State equations for a slow subsystem are given by (3.11) and (3.16) and may be written as

$$\mathbf{z}_s(t+1) = F_s \mathbf{z}_s(t) + G_s \mathbf{u}_s(t) + T_{1_s} \mathbf{f}_s(t), \tag{3.17}$$

$$\mathbf{y}_s(t) = H_s \mathbf{z}_s(t) + E_s \mathbf{u}_s(t) + T_{2_s} \mathbf{f}_s(t), \; t = 0, 1, 2, \ldots, \tag{3.18}$$

where

$$\mathbf{z}_s(t) = \mathbf{z}_{1_s}(t), \; F_s = F_1, \; G_s = G_1, \; T_{1_s} = T_1, \; H_s = H_1,$$

$$E_s = H_2 [I - F_2]^{-1} G_2, \; T_{2_s} = H_2 [I - F_2]^{-1} T_2.$$

From (3.4), (3.7), (3.8), (3.10), and (3.12), we have

$$\mathbf{z}_{2_f}(t+1) = F_2 \mathbf{z}_{2_f}(t) + G_2 \mathbf{u}_f(t) + T_2 \mathbf{f}_f(t). \tag{3.19}$$

From (3.5), (3.6), (3.9), (3.10), and (3.13), we find that, according to Definition 3.1,

$$\mathbf{y}_f(t) = H_2 \mathbf{z}_{2_f}(t). \tag{3.20}$$

We note that state equations for a fast subsystem are given by (3.19) and (3.20) and may be written as

$$\mathbf{z}_f(t+1) = F_f \mathbf{z}_f(t) + G_f \mathbf{u}_f(t) + T_f \mathbf{f}_f(t), \tag{3.21}$$

$$\mathbf{y}_f(t) = H_f \mathbf{z}_f(t), \; t = 0, 1, 2, \ldots, \tag{3.22}$$

where

$$\mathbf{z}_f(t) = \mathbf{z}_{2_f}(t), \; F_f = F_2, \; G_f = G_2, \; T_f = T_2, \; H_f = H_2.$$

C. Decentralization error

Definition 3.2. *We call the norm* $\| \varepsilon \|$ *the decentralization error, that is,*

$$\| \varepsilon(t) \| = \| \mathbf{y}(t) - \mathbf{y}_f(t) - \mathbf{y}_s(t) \|. \tag{3.23}$$

Consider the system

$$\mathbf{x}(t+1) = F\mathbf{x}(t) + G\mathbf{u}(t) + T_1\mathbf{f}(t), \tag{3.24}$$

$$\mathbf{y}(t) = H\mathbf{x}(t) + E\mathbf{u}(t) + T_2\mathbf{f}(t), \quad t = 0,\ 1,\ 2,\ \ldots . \tag{3.25}$$

It is easy to see from (3.24)–(3.25) that

$$\mathbf{y}(t) = HF^t\mathbf{x}(0) + H\sum_{i=1}^{t} F^{i-1}G\mathbf{u}(t-i) + H\sum_{i=1}^{t} F^{i-1}T_1\mathbf{f}(t-i) + E\mathbf{u}(t) + T_2\mathbf{f}(t). \tag{3.26}$$

From (3.7), (3.8), (3.23), and (3.26), we find

$$\begin{aligned}\varepsilon(t) = {} & HF^t\mathbf{x}(0) + H\sum_{i=1}^{t} F^{i-1}G\mathbf{u}_f(t-i) + H\sum_{i=1}^{t} F^{i-1}G\mathbf{u}_s(t-i) + H\sum_{i=1}^{t} F^{i-1}T\mathbf{f}_f(t-i) \\ & + H\sum_{i=1}^{t} F^{i-1}T\mathbf{f}_s(t-i) - H_f F_f^t \mathbf{z}_f(0) - H_f\sum_{i=1}^{t} F_f^{i-1}G_f\mathbf{u}_f(t-i) \\ & - H_f\sum_{i=1}^{t} F_f^{i-1}T_f\mathbf{f}_f(t-i) - H_s F_s^t\mathbf{z}_s(0) - H_s\sum_{i=1}^{t} F_s^{i-1}G_s\mathbf{u}_s(t-i) \\ & - H_s\sum_{i=1}^{t} F_s^{i-1}T_{1_s}\mathbf{f}_s(t-i) - E_s\mathbf{u}_s(t) - T_{2_s}\mathbf{f}_s(t).\end{aligned} \tag{3.27}$$

After some algebra we have from (3.27)

$$\begin{aligned}\|\varepsilon(t)\| \le {} & \|H\|\|F\|^t\|\mathbf{x}(0)\| + \|H\|\|G\|\sum_{i=1}^{t}\|F\|^{i-1}\|\mathbf{u}_f(t-i)\| \\ & + \|H\|\|G\|\sum_{i=1}^{t}\|F\|^{i-1}\|\mathbf{u}_s(t-i)\| + \|H\|\|T\|\sum_{i=1}^{t}\|F\|^{i-1}\|\mathbf{f}_f(t-i)\| \\ & + \|H\|\|T\|\sum_{i=1}^{t}\|F\|^{i-1}\|\mathbf{f}_s(t-i)\| + \|H_f\|\|F_f\|^t\|\mathbf{z}_f(0)\| \\ & + \|H_f\|\|G_f\|\sum_{i=1}^{t}\|F_f\|^{i-1}\|\mathbf{u}_f(t-i)\| + \|H_f\|\|T_f\|\sum_{i=1}^{t}\|F_f\|^{i-1}\|\mathbf{f}_f(t-i)\| \\ & + \|H_s\|\|F_s\|^t\|\mathbf{z}_s(0)\| + \|H_s\|\|G_s\|\sum_{i=1}^{t}\|F_s\|^{i-1}\|\mathbf{u}_s(t-i)\| \\ & + \|H_s\|\|T_{1_s}\|\sum_{i=1}^{t}\|F_s\|^{i-1}\|\mathbf{f}_s(t-i)\| + \|E_s\|\|\mathbf{u}_s(t)\| + \|T_{2_s}\|\|\mathbf{f}_s(t)\|.\end{aligned} \tag{3.28}$$

4. SAMPLING PERIODS FOR THE TWO-RATE NONLINEAR DISCRETE-TIME SYSTEM

It is not easy to choose the sampling period Δ for the equivalent discrete system as this choice depends upon the process dynamics. If Δ is extremely small, then digital signals are nearly continuous, and continuous methods of analysis and design can be used. Many systems are originally conceived with fast sampling periods and the only way to accommodate the increased computer load is to slow down the sampling period. The selection of the best sampling period is a compromise. For example, we might use

$$\Delta_f \approx \Delta_{f_{\max}}, \tag{4.1}$$

$$\Delta_s \approx \Delta_{s_{\max}}. \tag{4.2}$$

From (3.24), (3.25), and using $t = 0$, $\mathbf{x}(0) = 0$, we find

$$\mathbf{y}[\Delta] = HG(\Delta)\mathbf{u}(0) + E(\Delta)\mathbf{u}[\Delta] + HT_1(\Delta)\mathbf{f}(0) + T_2(\Delta)\mathbf{f}[\Delta]. \tag{4.3}$$

If the maximum values of control input, output, and nonlinearities are given and Definition 2.1 is satisfied, then (4.3) may be written as

$$\mathbf{y}_{\max} = [HG(\Delta) + E(\Delta)]\mathbf{u}_{\max} + [HT_1(\Delta) + T_2(\Delta)]\mathbf{f}_{\max}. \tag{4.4}$$

From (4.1), (4.4), and (3.21)–(3.22), we have

$$\mathbf{y}_{f_{\max}} = \left[H_f G_f(\Delta_f)\right]\mathbf{u}_{f_{\max}} + \left[H_f T_f(\Delta_f)\right]\mathbf{f}_{f_{\max}}. \tag{4.5}$$

By (4.2), (4.4), and (3.17)–(3.18), we get

$$\mathbf{y}_{s_{\max}} = \left[H_s G_s(\Delta_s) + E_s(\Delta_s)\right]\mathbf{u}_{s_{\max}} + \left[H_s T_{1_s}(\Delta_s) + T_{2_s}(\Delta_s)\right]\mathbf{f}_{s_{\max}}. \tag{4.6}$$

Note that we can obtain Δ_f and Δ_s from matrix equations (4.5) and (4.6), respectively.

5. EXAMPLE

A numerical example is given to demonstrate the applicability of our results. The fourth-order system (2.1)–(2.2) is described by

$$A = \begin{bmatrix} -1.9374 & 1.9436 & -0.6813 & 2.4671 \\ 2.3577 & -1.7226 & 1.538 & -1.8748 \\ -0.782 & -2.0548 & -2.3693 & -3.0104 \\ -1.3489 & 0.6111 & -0.9831 & 0.5293 \end{bmatrix}, \quad B = \begin{bmatrix} -1.6969 & -1.0097 \\ 1.0891 & 0.3007 \\ 0.3717 & 1.3819 \\ 0.1199 & -0.6356 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.179 & 0.318 & -0.088 & -0.1 \\ 0.588 & 0.228 & 0.07 & -0.245 \end{bmatrix},$$

$$\mathbf{f}(\mathbf{x},\mathbf{u}) = \begin{bmatrix} 0.01 + 0.1(1-u_1^2)\cos x_1 \\ 0.03 - 0.05u_2^3 + 0.01\sin^2 x_2 \\ 0.02 + 0.06u_1^2 - 0.05\cos x_3 \\ 0.04 + u_2^2(1-\sin x_4) \end{bmatrix}.$$

This model has been discretized into the form (2.4)–(2.5) using $\Delta = 0.6$. From (4.5), we obtain

$$\Delta_f = 0.5,$$

where

$$\mathbf{u}_{f_{\max}}^T = [1.4 \;\; 2.7], \quad \mathbf{f}_{f_{\max}}^T = [1.1 \;\; 2.5 \;\; 4.2 \;\; 2.1], \quad \mathbf{y}_{f_{\max}}^T = [0.1 \; -0.5].$$

From (4.6), we obtain

$$\Delta_s = 7,$$

where

$$\mathbf{u}_{s_{\max}}^T = [3.4 \;\; 6.8], \quad \mathbf{f}_{s_{\max}}^T = [6.2 \;\; 5.1 \;\; 8.4 \;\; 6.5], \quad \mathbf{y}_{s_{\max}}^T = [5 \;\; 9.7].$$

Computation of the spectral norm of ε from (3.28) gives

$$\|\varepsilon(2)\| \le 0.7026,$$

where

$$\mathbf{x}^T(0) = [0.004 \;\; 0.001 \;\; 0.002 \;\; 0.006],$$

$$\mathbf{u}(t) = \begin{bmatrix} 0.01 - 0.03\cos t \\ 0.03 + 0.01\sin 2t \end{bmatrix}.$$

The fast-subsystem dynamics (3.21)–(3.22) is described by

$$F_f = \begin{bmatrix} 0.1653 & 0 \\ 0 & 0.2837 \end{bmatrix}, \; G_f = \begin{bmatrix} -0.2883 & 0.1522 \\ 1.0498 & 0.1702 \end{bmatrix},$$

$$T_f = \begin{bmatrix} 0.3203 & 0.0801 & 0.4004 & 0.1603 \\ -0.0002 & 0.7094 & 0.4254 & 0.9932 \end{bmatrix}, \; H_f = \begin{bmatrix} -0.1945 & 0.0529 \\ 0.0729 & -0.1371 \end{bmatrix}.$$

The slow subsystem (3.17)–(3.18) is specified by

$$F_s = \begin{bmatrix} 0.8555 & 0 \\ 0 & 0.9194 \end{bmatrix}, \quad G_s = \begin{bmatrix} -0.8324 & 1.0744 \\ 0.5292 & 0.2878 \end{bmatrix},$$

$$T_{1_s} = \begin{bmatrix} -0.4043 & -1.216 & 0.0004 & -1.6227 \\ -0.8055 & -0.6906 & -0.2301 & -0.0003 \end{bmatrix}, \quad H_s = \begin{bmatrix} 0.0521 & -0.315 \\ -0.0383 & -0.3402 \end{bmatrix},$$

$$E_s = \begin{bmatrix} 0.1447 & -0.0229 \\ -0.2261 & -0.0193 \end{bmatrix}, \quad T_{2_s} = \begin{bmatrix} -0.0747 & 0.0337 & -0.0619 & 0.036 \\ 0.028 & -0.1288 & -0.0465 & -0.1761 \end{bmatrix}.$$

6. CONCLUSIONS

The method proposed in this paper permits finding state equations for the fast nonlinear subsystem (3.21)–(3.22) and the slow nonlinear subsystem (3.17)–(3.18) if the equivalent discrete sy.tem (2.4)–(2.5) has a time-separation property. We obtained the decentralization error (3.28) and upper limits of sampling periods for the slow (4.6) and fast (4.5) discrete subsystems. Using the MATLAB program, we translated a detailed mathematical model into a computer model, because the problem solutions are expressed in MATLAB just as they are written mathematically, without traditional programming.

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OLEKURUUMI KAHEKIIRUSELISTE MITTELINEAARSETE MITMEMÕÕTMELISTE DISKREETSETE SÜSTEEMIDE KIRJELDAMINE

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Dünaamilise süsteemi olekumudel on Jordani kanoonilist kuju kasutades detsentraliseeritud aeglaseks ja kiireks alamsüsteemiks. On määratud vastavate alamsüsteemide diskreetimisperioodide ülemised piirid.