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DYNAMIC DISTURBANCE DECOUPLING FOR DISCRETE-TIME NONLINEAR SYSTEMS: A SOLUTION IN TERMS OF SYSTEM INVARIANTS*

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Abstract. The paper considers the dynamic disturbance decoupling problem for a discrete-time nonlinear system locally around its equilibrium point. The complete solution to the problem, that is necessary and sufficient conditions as well as the procedure for constructing the compensator, is given both for the cases of measurable and unmeasurable input disturbances under certain regularity assumptions of the equilibrium point. Under the mentioned regularity assumptions of the equilibrium point the solvability conditions for the dynamic disturbance decoupling problem via regular or nonregular dynamic state feedback are shown to be the same. Instrumental in the problem solution are the two versions of the inversion algorithm for a discrete-time nonlinear system with disturbances which define two finite sequences of uniquely defined integers, the so-called invertibility indices, either with respect to both inputs or with respect to the controls only. Firstly, the solvability conditions are expressed in terms of the invertibility indices. Secondly, the proof of the existence and construction of the dynamic state feedback compensator relies on this algorithm.

Key words: discrete-time systems, nonlinear control systems, disturbance decoupling, inversion algorithm, invertibility indices.

1. INTRODUCTION

In this paper we shall consider the disturbance decoupling problem with the dynamic state feedback for discrete-time nonlinear systems. The problem deals with the situation in which, by means of the dynamic state feedback compensator, we want to achieve decoupling between the input disturbances entering the system and the outputs leaving the system. Clearly, the dynamic disturbance decoupling problem (DDDP) is essentially a nonlinear problem: for linear systems the DDDP is solvable if and only if the disturbance decoupling problem is solvable by static state feedback [1]. In [2] the DDDP has been solved locally around an equilibrium point of the discrete-time nonlinear system under two

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additional assumptions. The first assumption was that the original system is square, that is the number of the controls equals the number of the outputs. The second assumption was that the system under zero disturbances is invertible. The solution (both for measurable and unmeasurable disturbances) presented in [2] roughly says that the problem is locally solvable if and only if it is solvable by means of a certain compensator that is obtained from the inversion algorithm applied to the system under zero disturbances. In [3] the earlier results have been extended to nonsquare and noninvertible systems. The main result has been formulated in terms of the certain functions appearing in the inversion algorithm and, besides necessary and sufficient conditions, it also provides a constructive procedure for solving the DDDP in the case of unmeasurable disturbances (DDDPud).

The purpose of this paper is to give a complete solution (that is a solution without any further assumption on the discrete-time nonlinear system) of the DDDP also in the case of measurable disturbances (DDDPmd). Another purpose of this paper is to translate necessary and sufficient conditions for a local solvability of the DDDPud and the DDDPmd (DDDP in the case of measurable disturbances) stated in terms of certain functions appearing in the inversion algorithm into necessary and sufficient conditions stated in terms of invertibility indices. Using the vector space technique introduced in [4], it is not difficult to show that the latter conditions are actually system – intrinsic and algorithm-independent conditions stated in terms of a certain structure at infinity either of the original system (for the DDDPmd) or some auxiliary system (for the DDDPud).

2. PROBLEM STATEMENT

Consider a discrete-time nonlinear plant P described by equations of the form

$$x(t+1) = f(x(t), u(t), w(t)), \; x(0) = x_0,$$

$$y(t) = h(x(t)), \tag{2.1}$$

where the states $x(t)$ belong to an open subset X of R^n, the controls $u(t)$ belong to an open subset U of R^m, the disturbances $w(t)$ (either measurable or unmeasurable) belong to an open subset W of R^r, and the outputs $y(t)$ belong to an open subset Y of R^p. The mappings f and h are supposed to be real analytic.

Throughout the paper we shall adopt a local viewpoint. However, contrary to the continuous-time case, in the discrete-time case local study is impossible around an arbitrary initial state, since even in one step the state can move far away from the initial state regardless of the small control and disturbance values. For this reason we shall work in a neighbourhood of an equilibrium point of the system (2.1), that is around

$(x^0, u^0, w^0) \in X \times U \times W$ such that $f(x^0, u^0, w^0) = x^0$. Working around the point (x^0, u^0, w^0) and on a finite-time interval $0 \le t \le t_F$, then, using the control sequence $u(0), u(1), ..., u(t_F)$ with each $u(t)$ sufficiently close to u^0, and provided that in the disturbance sequence $w(0), w(1), ..., w(t_F)$ each $w(t)$ is sufficiently close to w^0, we can assure that the states $x(t)$ are sufficiently close to x^0 and the outputs $y(t)$ are sufficiently close to $y^0 = h(x^0)$, both for $0 \le t \le t_F$.

We shall consider separately the cases of unmeasurable and measurable disturbances. In the case of unmeasurable disturbances the compensator C (dynamic state feedback) used to control the plant P is a discrete-time nonlinear system described by the equations of the form

$$z(t+1) = \psi(z(t), x(t), v(t)),\ z(0) = z_0,$$

$$u(t) = \varphi(z(t), x(t), v(t)), \tag{2.2}$$

with the state $z(t) \in Z$, an open subset of R^{μ}, with a new m-dimensional control $v(t) \in V$, an open subset of R^m, and real analytic ψ and φ.

We call the compensator C described by equation (2.2) r e g u l a r if the dynamical system

$$x(t+1) = f(x(t), \varphi(z(t), x(t), v(t)), w(t)),$$

$$z(t+1) = \psi(z(t), x(t), v(t)), \tag{2.3}$$

$$u(t) = \varphi(z(t), x(t), v(t))$$

with inputs $v(t)$ and outputs $u(t)$ is invertible (see [4, 5] for details about the notion of invertibility), or equivalently that (2.3) defines a one-to-one (x, z, w)-dependent correspondence between the input variable v and output variable u. The analogous definition of regular dynamic state feedback for continuous-time nonlinear systems has been used in [6].

The closed-loop system (2.1), (2.2), that is the system

$$x(t+1) = f(x(t), \varphi(z(t), x(t), v(t)), w(t)),$$

$$z(t+1) = \psi(z(t), x(t), v(t)), \tag{2.4}$$

$$y(t) = h(x(t)),$$

initialized at (x_0, z_0) is denoted by $P \circ C$.

D e f i n i t i o n 2.1. **Local dynamic disturbance decoupling problem in the case of the unmeasurable disturbances (DDDPud).** *Given the system* (2.1) *around an equilibrium point* (x^0, u^0, w^0) *find, if possible, a compensator* C *defined by equations of the form* (2.2) *together with an initial state* z_0 *and neighbourhoods* $V_1 = Z^0 \times X^0 \times V^0$ *of* (z^0, x^0, v^0) in $Z \times X \times V$ *and* V_2 *of* u^0 *in* U, *being domain and codomain of* C, *as well as a neighbourhood* W^0 *of* w^0 *so that the outputs of the closed-loop system*

$$y^{P \circ C}(t; x_0, z_0, w(0), ..., w(t-1), v(0), ..., v(t-1)),\ t \le t_F$$

do not depend on disturbances $w(t)$ *for every* $x_0 \in X^0$, *all* $v(t) \in V^0$, *all* $w(t) \in W^0$, *and for some finite* t_F.

In the case of measurable disturbances the compensator C_w (dynamic state feedback) used to control the plant is a discrete-time nonlinear system described by

$$z(t+1) = \psi(z(t), x(t), v(t), w(t)), \quad z(0) = z_0, \tag{2.5}$$

$$u(t) = \varphi(z(t), x(t), v(t), w(t))$$

with the state $z(t) \in Z \subset R^{\mu}$, with a new m-dimensional control $v(t) \in V \subset R^m$ and real analytic ψ and φ. The r e g u l a r i t y of (2.5) means that the dynamical system

$$x(t+1) = f(x(t), \varphi(z(t), x(t), v(t), w(t)), w(t)),$$

$$z(t+1) = \psi(z(t), x(t), v(t), w(t)), \tag{2.6}$$

$$u(t) = \varphi(z(t), x(t), v(t), w(t))$$

with the controls $v(t)$ and the outputs $u(t)$ is invertible, or equivalently that it defines a one-to-one (x, z, w)-dependent correspondence between the control variable v and the output variable u.

The closed-loop system (2.1), (2.5), that is the system

$$x(t+1) = f(x(t), \varphi(z(t), x(t), v(t), w(t)), w(t)),$$

$$z(t+1) = \psi(z(t), x(t), v(t), w(t)), \tag{2.7}$$

$$y(t) = h(x(t))$$

initialized at (x_0, z_0) is denoted by $P \circ C_w$.

D e f i n i t i o n 2.2. **Local dynamic disturbance decoupling problem in the case of the measurable disturbances (DDDPmd).** *Given the system* (2.1) *around an equilibrium point* (x^0, u^0, w^0) *find, if possible, a compensator* C_w *defined by equations of the form* (2.5) *together with an initial state* z_0 *and neighbourhoods* $V_1 = Z^0 \times X^0 \times V^0 \times W^0$ *of* (z^0, x^0, v^0, w^0) in $X \times Z \times V \times W$ *and* V_2 *of* u^0 *in* U, *being domain and codomain of* C, *so that the outputs of the closed-loop system*

$$y^{P \circ C_w}(t; x_0, z_0, w(0), \dots, w(t-1), v(0), \dots, v(t-1)), \; t \le t_F$$

do not depend on disturbances $w(t)$ *for every* $x_0 \in X^0$, *all* $v(t) \in V^0$, *all* $w(t) \in W^0$, *and for some finite* t_F.

3. INVERSION ALGORITHM FOR SYSTEMS WITH DISTURBANCES. INVERTIBILITY INDICES

The inversion algorithm for discrete-time nonlinear systems without disturbances has been introduced in [5] and given in more general and simple form by Kotta and Nijmeijer [2]. For the system with disturbances two versions of the inversion algorithm can be given [7]. The first version accomplishes inversion with respect to both types of inputs – the controls and the disturbances – whereas the other version considers disturbances as system parameters and accomplishes inversion with respect to the controls only. Note that the idea of using two versions of the inversion algorithm was at first exploited in the continuous-time case by Moog, Perdon and Conte [8]. In this paper we shall not repeat these two versions of the inversion algorithm and in the following we shall use the notations of [7].

We can apply the inversion algorithm not necessarily in a unique way [7]. There exist, in general, different reorderings (permutations) of output components $\hat{y}_k(t+k+1)$ at step $k+1$, $k \geq 0$, so that the first $\rho_{uw,k+1}$ ($\rho_{u,k}$) rows of the matrix $\partial\left[A_k^T, a_{k+1}^T\right]^T / \partial(u,w)$ $\left(\partial\left[A_k^T, a_{k+1}^T\right]^T / \partial(u)\right)$ are linearly independent. Different permutations of output components, that is, different selections of $\tilde{y}_{k+1}(t+k+1)$ in each step $k+1$, $k \geq 0$, result in different functions $A_{k+1}(\cdot)$ and $\psi_{k+1}(\cdot)$.

In the inversion algorithm, certain constant rank assumptions have been imposed to ensure that the algorithm can be applied around a given equilibrium point. We shall summarize these conditions in the definition of regularity of an equilibrium point (associated with the inversion algorithm).

D e f i n i t i o n 3.1. *We call the equilibrium point* (x^0, u^0, w^0) *of the system* (2.1) *regular with respect to the inversion algorithm if for some specific application s of the inversion algorithm the constant rank assumptions of the algorithm are satisfied. We call* (x^0, u^0, w^0) *strongly regular if the constant rank assumptions of the algorithm hold for each application of the algorithm.*

Using the inversion algorithm around the regular equilibrium point (x^0, u^0, w^0) of the system (2.1), we obtain a sequence of integers

$$0 \leq \rho_{uw,1} \leq \ldots \leq \rho_{uw,k} \leq \ldots \leq p$$

$$(0 \leq \rho_{u,1} \leq \ldots \leq \rho_{u,k} \leq \ldots \leq p).$$

Let $\rho^*_{uw} = \max\{\rho_{uw,k}, k \geq 1\}$ $(\rho^*_u = \max\{\rho_{u,k}, k \geq 1\})$ and let α be defined as the smallest $k \in N$ such that $\rho_{uw,k} = \rho^*_u$ $(\rho_{u,k} = \rho^*_u)$.

Though the result of application of the inversion algorithm apparently depends on the choice of admissible permutations made at each step of the algorithm, it can be proved in analogy with the case without input disturbances [2] that the integers $\rho_{uw,1}, \dots, \rho_{uw,k}, \dots, (\rho_{u,1}, \dots, \rho_{u,k}, \dots)$ do not depend on the particular permutation of the components of $\hat{y}_k(t+k+1)$. Thus, around a strongly regular equilibrium point of the system, these integers define some structural properties of the system. On the analogy with [9], we call the integers $\rho_{uw,k}$, $k \geq 1$ ($\rho_{u,k}$, $k \geq 1$) the invertibility indices of the system (2.1) with respect to both inputs (with respect to the controls).

Moreover, on the analogy with the case without disturbances [3] it can be proved that around a regular equilibrium point the inversion algorithm terminates at most n steps, that is

$$\rho^*_{uw} = \rho_{uw,n} \quad (\rho^*_u = \rho_{u,n}).$$

4. PROBLEM SOLUTION: THE CASE OF UNMEASURABLE DISTURBANCES

In [3] necessary and sufficient conditions for local solvability of the DDDPud via regular compensator are derived in terms of certain functions appearing in the inversion algorithm. By a little modification of the proof in [3], the result can be formulated as in Theorem 4.1.

T h e o r e m 4.1. *Apply the inversion algorithm to P described by equations* (2.1) *with respect to the control u around a strongly regular equilibrium point* (x^0, u^0, w^0). *The DDDPud for system P is locally solvable around* (x^0, u^0, w^0) *via regular dynamic state feedback C if and only if for* $1 \leq k \leq n$

$$\frac{\partial}{\partial w} a^P_k (x, u, w, \{\tilde{y}_i(t+j+1),\ 1 \leq i \leq k,\ i \leq j \leq k\}) = 0. \qquad (4.1)$$

Now we are going to translate necessary and sufficient conditions for the solvability of the DDDPud via regular compensator, formulated by Theorem 4.1 in terms of the inversion algorithm, into system-intrinsic and algorithm-independent conditions stated in terms of the invertibility indices of the so-called auxiliary system formed from the original system by adding one-step backward shift (delay) operator into the control loop. So, the auxiliary system P_a with the states $(x(t), u(t))$ and the inputs $u_a(t)$ is defined as follows:

$$\begin{aligned} x(t+1) &= f(x(t), u(t), w(t)), \\ u(t+1) &= u_a(t), \qquad (4.2) \\ y(t) &= h(x(t)). \end{aligned}$$

The equilibrium point of P_a is $(x^0, u^0, u^0{}_a, w^0)$ with $u^0{}_a = u^0$. The idea of delaying the inputs, as is done by the introduction of the auxiliary system P_a, was already employed for continuous-time systems in [1].

We need the following lemma.

L e m m a 4.2. [7] *Apply the inversion algorithm to P described by equation* (2.1) *with respect to the control u around a strongly regular equilibrium point* (x^0, u^0, w^0). *Let the point* (x^0, u^0, w^0) *be such that* $(x^0, u^0, u^0{}_a, w^0)$ *with* $u^0{}_a = u^0$ *is a strongly regular equilibrium point of* P_a *with respect to both versions of the inversion algorithm. Then at every step of the algorithm* $\partial a_k^P(t)/\partial w = 0$ *if and only if* $\rho_{u_a w,k}(P_a) = \rho_{u_a,k}(P_a)$ *for all* $k \geq 1$.

R e m a r k 4.3. The continuous-time counterpart of Lemma 4.2 is given in [1].

Now we are ready to formulate the main result of this section.

T h e o r e m 4.4. *Consider the system P described by equations* (2.1) *around a strongly regular equilibrium point* (x^0, u^0, w^0) *with respect to the inversion algorithm with regard to the control. Let the point* (x^0, u^0, w^0) *be such that* $(x^0, u^0, u^0{}_a, w^0)$ *with* $u^0{}_a = u^0$ *will be a strongly regular equilibrium point of* P_a *with respect to both versions of the inversion algorithm. Then the regular DDDPud for P will be locally solvable around* (x^0, u^0, w^0) *via regular dynamic feedback C if and only if for all* $1 \leq k \leq n+m$

$$\rho_{u_a w,k}(P_a) = \rho_{u_a,k}(P_a). \tag{4.3}$$

P r o o f . The proof of Theorem 4.4 follows easily from Theorem 4.1 and Lemma 4.2.

T h e o r e m 4.5. *Consider the system P described by equations* (2.1) *around a strongly regular equilibrium point* (x^0, u^0, w^0) *with respect to the inversion algorithm with regard to the control. Let the point* (x^0, u^0, w^0) *be such that* $(x^0, u^0, u^0{}_a, w^0)$ *with* $u^0{}_a = u^0$ *will be a strongly regular equilibrium point of* P_a *with respect to both versions of the inversion algorithm. Then the DDDPud for P will be locally solvable if and only if* (4.3) *holds.*

P r o o f . Sufficiency. As in Theorem 4.1.

Necessity. Let us assume that there exists a dynamic state feedback of the form (2.2) that locally around a strongly regular equilibrium point (x^0, u^0, w^0) of (2.1) solves the DDDPud for (2.1). Apply the first step of the inversion algorithm to P with respect to the control u only, considering disturbances w as parameters:

$$y(t+1) = h(f(x(t), u(t), w(t))). \tag{4.4}$$

If we replace $u(t)$ in (4.4) by the output of C, we can assert that equation (4.4) does not depend on w any more since C defined by (2.2) solves the DDDPud for (2.1). This means that either

$$\frac{\partial h(f(x, u, w))}{\partial w} = 0 \tag{4.5}$$

everywhere around the point (x^0, u^0, w^0) or, if not, the compensator C defined by (2.2) will guarantee the equality (4.5). The latter is impossible around the strongly regular equilibrium point where by the definition $\partial h(f(x, u, w)) / \partial w$ is either equal to zero everywhere or different from zero everywhere. This means that if $\partial h(f(x, u, w)) / \partial w \neq 0$, we cannot make it ever equal to zero by the suitable choice of the compensator. This implies that (4.5) holds which, by Lemma 4.2, means that also (4.3) holds for $k = 0$. Applying this argument repeatedly, we can show that (4.3) holds for $k = 0, 1, ..., n - 1$.

R e m a r k 4.6. Note that Theorem 4.5 does not require the regularity of the compensator. So, the solvability conditions for the DDDPud via regular or nonregular dynamic state feedback around the strongly regular equilibrium point are the same. Of course, around the nonregular equilibrium point the conditions (4.1) are not necessary for the solvability of the DDDPud. Sometimes, if the conditions (4.1) are not satisfied, a nonregular compensator can still be found that imposes the constraint (4.1) and solves the DDDPud. For a continuous-time system this aspect of the DDDPud has been studied in [10]. Unfortunately, it has not been stated clearly in [10], that the possibility to guarantee (4.1) by the proper choice of a nonregular compensator is not possible around the regular equilibrium point of the system.

5. PROBLEM SOLUTION: THE CASE OF MEASURABLE DISTURBANCES

In this section we shall consider the case of measurable disturbances. We shall prove the following Theorem.

T h e o r e m 5.1. *Apply the inversion algorithm to* (2.1) *with respect to the control u around a strongly regular equilibrium point* (x^0, u^0, w^0). *The DDDPmd for the system* (2.1) *is locally solvable around* (x^0, u^0, w^0) *via regular dynamic state feedback* C_w *if and only if for* $1 \leq k \leq n$

$$\frac{\partial}{\partial w}\hat{y}_k(x, w, \{\tilde{y}_i(t+j), \quad 1 \leq i \leq k, \quad i \leq j \leq k\}) = 0. \tag{5.1}$$

P r o o f . Sufficiency. Notice at first that if (5.1) holds for $1 \leq k \leq n$, then it holds for every $k \geq 0$. It follows from the fact that the inversion algorithm terminates at most n steps. If (5.1) holds, then applying the second version of the inversion algorithm to (2.1) gives at the nth step

$$\tilde{Y}_n = A_n(x(t), u(t), w(t), \{\tilde{y}_i(t+j), 1 \le i \le n-1, i+1 \le j \le n\}),$$
$$\hat{y}_n(t+n) = \hat{y}_n(x(t), \{\tilde{y}_i(t+j), 1 \le i \le n, i \le j \le n\}), \tag{5.2}$$

where $\tilde{Y}_n = \left[\tilde{y}_1^{\mathrm{T}}(t+1), \tilde{y}_2^{\mathrm{T}}(t+2), \dots, \tilde{y}_n^{\mathrm{T}}(t+n)\right]^{\mathrm{T}}$ and the matrix $\partial A_n(t)/\partial u$ has full row rank $\rho_{u,n}$ on a neighbourhood O_n of (x^0, u^0, w^0). In the proof we shall denote $\rho_{u,k}$ by ρ_k for simplicity. For $i = 1, 2, ..., \rho_n$, let $t + \gamma_i$ be the smallest time instant and $t + \varepsilon_i$ be the greatest time instant in which y_i appears in (5.2). Then we can rewrite (5.2) as

$$\left[y_{\rho_{k-1}+1}(t+k), \dots, y_{\rho_k}(t+k)\right]^{\mathrm{T}} = \tilde{a}_k(x(t), u(t), w(t), \{y_i(t+j),$$
$$1 \le i \le \rho_{k-1}, \gamma_i + 1 \le j \le \min(k, \varepsilon_i)\}), \quad k = 1, 2, \dots, n. \tag{5.3}$$

After a possible permutation of inputs we may assume that the Jacobian matrix of the right-hand side of (5.3) with respect to $u^1 = (u_1, ..., u_{\rho_n})^{\mathrm{T}}$ around the point $(x^0, u^0, y^0, ..., y^0, w^0)$ has full row rank ρ_n. Therefore, equation (5.3) can be solved for $u^1(t)$ uniquely around the point $(x^0, u^0, y^0, ..., y^0, w^0)$ by applying the Implicit Function Theorem. Define $u^2 = (u_{\rho_n+1}, ..., u_m)^{\mathrm{T}}$. Then, from (5.3), we obtain

$$u^1(t) = \varphi(x(t), \{y_i(t+j), 1 \le i \le \rho_n, \gamma_i \le j \le \varepsilon_i\}, u^2(t), w(t)), \tag{5.4}$$

which is such that for $k = 1, 2, ..., n$

$$\left[y_{\rho_{k-1}+1}(t+k), \dots, y_{\rho_k}(t+k)\right]^{\mathrm{T}} \equiv$$
$$\equiv \tilde{a}_k(x(t), \varphi(\cdot), \{\tilde{y}_i(t+j), \quad 1 \le i \le \rho_{k-1},$$
$$\gamma_i + 1 \le j \le \min(k, \varepsilon_i)\}, w(t)). \tag{5.5}$$

Notice that $\varphi: V_1 \to V_2$ is analytic for some (possible small) neighbourhoods V_1 and V_2 of $(x^0, y^0, ..., y^0, u^{20}, w^0)$ in $X^0 \times (Y^0)^n \times U^{20} \times W^0$ and of u^{10} in U^{10}. This implies that (5.5) will hold as long as $(x(t), \{y_i(t+j), 1 \le i \le \rho_n, \gamma_i \le j \le \varepsilon_i\}, u^2(t), w(t)) \in V_1$ and, defined by (5.4), $u^1(t) \in V_2$. Of course, the identity (5.5) is lost if we leave the neighbourhoods V_1 *resp.* V_2.

Now construct the compensator for (2.1) in the following way. Let $z_i = (z_{i1}, ..., z_{i,\varepsilon_i - \gamma_i})^{\mathrm{T}}$, $i = 1, ..., \rho_n$ be a vector of dimension $\varepsilon_i - \gamma_i$, v^2 a vector of dimension $m - \rho_n$, and consider the system

$$z_i(t+1) = A_i z_i(t) + B_i v_i(t), i = 1, \dots, \rho_n,$$
$$u^1(t) = \varphi(x(t), \{z_{ij}(t), 1 \le j \le \varepsilon_i - \gamma_i, v_i(t), 1 \le i \le \rho_n\},$$
$$v^2(t), w(t)), \tag{5.6}$$
$$u^2(t) = v^2(t)$$

with the controls $v^1 = (v_1, ..., v_{\rho_n})^T$ and v^2, the disturbances w, the outputs $u = (u^{1,T}, u^{2,T})^T$ and (A_i, B_i) in Brunovsky canonical form

$$A_i = \begin{bmatrix} 0 & I_{\varepsilon_i - \gamma_i - 1} \\ 0 & 0 \end{bmatrix}, \quad B_i = (0 \ldots 0\, 1)^T.$$

It can be shown that the compensator (5.6) is regular on a neighbourhood of (x^0, u^0, y^0, w^0). We omit the proof which is quite analogous to the case with unmeasurable disturbances [3, Lemma 3].

Now, it is easy to see that the compensator (5.6) with arbitrary initial state, applied to (2.1) locally around (x^0, u^0, w^0), yields for $i = 1, ..., \rho_n$

$$y_i(\gamma_i + j - 1) = z_{ij}(0), \quad j = 1, ..., \varepsilon_i - \gamma_i,$$

$$y_i(t + \varepsilon_i) = v_i(t), \quad 0 \le t \le t_F,$$

which of course are independent from $w(t)$. Moreover, $y_i(j)$ for $1 \le i \le \rho_n$, $0 \le j \le \gamma_i - 1$ and for $\rho_n + 1 \le i \le p$, $j \ge 0$, being the components of $\hat{y}_k(k)$, do not depend on $w(t)$ by assumption. Hence, the compensator (5.6) solves the DDDPmd locally.

Necessity. Let us assume that there exists a regular dynamic feedback control defined by (2.5) for (2.1) that locally around the equilibrium point (x^0, u^0, w^0) solves the DDDPmd. Furthermore, assume that (5.1) does not hold for $k = 1$, that is

$$\frac{\partial}{\partial w}\hat{y}_1(x, w, \tilde{y}_1(t+1)) \neq 0.$$

Then, at the first step of the inversion algorithm, we have that $\hat{y}_1(t+1)$ explicitly depends on w:

$$\hat{y}_1(t+1) = \hat{y}_1(x(t), w(t), \tilde{y}_1(t+1)). \tag{5.7}$$

Since (2.5) solves the DDDPmd for (2.1) this w-dependence should disappear, if we use the compensator (2.5). Since (5.7) does not depend on the control, the only possibility is that (2.5) must be such that it imposes the constraint

$$\frac{\partial}{\partial w}\hat{y}_1(x, w, \tilde{y}_1(t+1)) = \xi_1(x, w, \tilde{a}_1(x, u)) = 0.$$

But this would imply the nonregularity of the compensator. So, (5.1) must hold for $k = 1$. Following the same way, we prove the theorem.

Note that the result of Theorem 5.1 provides us with a constructive procedure for solving the DDDPmd via regular compensator. Namely, we proceed by applying the second version of the inversion algorithm to (2.1), checking at every step $1 \le k \le n$ if (5.1) holds. If (5.1) does not hold for some k, we conclude that the DDDPmd is not solvable via regular compensator. If (5.1) does hold for $1 \le k \le n$, then the DDDPmd can be solved by means of the regular compensator (5.6) with arbitrary initial state.

Now we are going to translate necessary and sufficient conditions for the solvability of the DDDPmd formulated by Theorem 5.1 in terms of the inversion algorithm into system-intrinsic and algorithm-independent conditions. For this purpose, we need the following lemma.

L e m m a 5.2. [7] *Apply the inversion algorithm to P with respect to u around an equilibrium point* (x^0, u^0, w^0). *Suppose that the point* (x^0, u^0, w^0) *is strongly regular equilibrium point of P with respect to both versions of the inversion algorithm. Then, at every step of the algorithm,* $\partial \hat{y}_k(t+k)/\partial w = 0$ *if and only if* $\rho_{uw,k}(P) = \rho_{u,k}(P)$ *for all k.*

Now we are ready to formulate the main result of this section.

T h e o r e m 5.3. *Consider the plant P described by equations* (2.1) *around a strongly regular equilibrium point* (x^0, u^0, w^0) *associated with both versions of the inversion algorithm. Then the DDDPmd for system P is locally solvable around* (x^0, u^0, w^0) *via regular dynamic state feedback* C_w *if and only if for* $i \leq k \leq n$

$$\rho_{uw,k}(P) = \rho_{u,k}(P). \tag{5.8}$$

The proof of Theorem 5.3 follows easily from Theorem 5.1 and Lemma 5.2.

T h e o r e m 5.4. *Consider the system P described by equations* (2.1) *around a strongly regular equilibrium point* (x^0, u^0, w^0) *associated with both versions of the inversion algorithm. Then the DDDPmd for system P is locally solvable around* (x^0, u^0, w^0) *if and only if* (5.8) *holds for* $i \leq k \leq n$.

P r o o f. Sufficiency. As in Theorem 5.1.

N e c e s s i t y. Similar to the proof of Theorem 4.5.

6. CONCLUSIONS

We have considered the dynamic disturbance decoupling problem (DDDP) for a discrete-time nonlinear system locally around its equilibrium point. The complete solution of the DDDP, that is the solution without any further assumption on the discrete-time nonlinear system, has been given under certain regularity assumptions of the equilibrium point. The cases of measurable and unmeasurable disturbances have been considered separately. For both cases necessary and sufficient conditions as well as the procedure for the construction of the compensator has been given.

Instrumental in the problem solution are the two versions of the inversion algorithm for discrete-time nonlinear system with input disturbances. Every version of the inversion algorithm produces the finite sequence of uniquely defined integers, the so-called invertibility indices, either with respect to both inputs, controls and disturbances, or with respect to controls, respectively. At first, the necessary and sufficient conditions have been given in terms of certain functions appearing in the inversion algorithm; then the equivalent conditions have been formulated in terms of invertibility indices. Using the vector space technique introduced by Grizzle, [4] it is not difficult to show that the latter conditions are actually system-intrinsic and algorithm-independent conditions stated in terms of a certain structure at infinity either of the original system (for the case of measurable disturbances) or some auxiliary system (for the case of unmeasurable disturbances).

Under the aforementioned regularity assumptions of the equilibrium point it has been shown that the solvability condition for the DDDP via regular or nonregular dynamic state feedback are the same, being in full accordance with the results for linear systems.

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HÄIRINGUTE DÜNAAMILINE KOMPENSEERIMINE DISKREETSETES MITTELINEAARSETES SÜSTEEMIDES: LAHENDUVUSTINGIMUSED SÜSTEEMI STRUKTUURIPARAMEETRITE TERMINITES

Ülle KOTTA

Mittelineaarsete analüütiliste süsteemide klassi tarbeks on uuritud dünaamilise tagasiside kujul esitatava kompensaatori konstrueerimise ülesannet, mis tagaks suletud süsteemi väljundite invariantsuse sisendhäiringute suhtes. Käsitlust on leidnud nii mõõdetavate kui ka mittemõõdetavate häiringute juhud.

Ülesande lahenduvus põhineb pööramisalgoritmi kahel versioonil, mille abil on võimalik häiringutega süsteemi jaoks leida kaks täisarvuliste struktuuriparameetrite lõplikku hulka, nn. pööratavusindeksid ainult juhttoime kui ka mõlema sisendi (s. t. juhttoime ja häiringute) suhtes. Süsteemi tasakaalupunkti ümbruses on leitud ülesande lokaalse lahenduvuse tarvilikud ja piisavad tingimused, mis on formuleeritud pööratavusindeksite terminites. Leitud lahenduvustingimused on üldisemad varasematest, mis on esitatud pööramisalgoritmi rakendamise käigus leitavate funktsioonide omaduste kaudu. Kuigi pööramisalgoritmi rakendamise tulemus ei ole üldjuhul ühene, vaid sõltub teatud valikutest algoritmi igal sammul, on näidatud, et pööratavusindeksite väärtus jääb muutmatuks kõigi võimalike lubatud valikute korral.

ДИНАМИЧЕСКАЯ КОМПЕНСАЦИЯ ВОЗМУЩЕНИЙ В ДИСКРЕТНЫХ НЕЛИНЕЙНЫХ СИСТЕМАХ: РЕШЕНИЕ В ТЕРМИНАХ СТРУКТУРНЫХ ПАРАМЕТРОВ СИСТЕМЫ

Юлле КОТТА

Для дискретных систем со многими входами и выходами изучается задача построения компенсатора в виде динамической обратной связи по состоянию, обеспечивающего инвариантность выхода замкнутой системы по отношению к входным возмущениям. Рассматриваются случаи измеряемых и неизмеряемых возмущений. Решение задачи основывается на двух вариантах алгоритма обращения, с помощью которых для системы с возмущениями можно найти два конечных набора целочисленных структурных параметров системы, так наз. индексы обратимости относительно управления и обоих выходов (т. е. управлений и возмущений). Найдены необходимые и достаточные условия разрешимости задачи локально в окрестности точки равновесия системы, которые сформулированы в терминах индексов обратимости. Найденные условия более общие, чем ранние, которые были представлены в терминах свойств некоторых функций, найденных при применении алгоритма обращения. Хотя результат применения алгоритма обращения в общем случае не единственный (однозначный) и зависит от некоторых выборок на каждом шагу алгоритма, значения индексов при всех возможных выборках не изменяются.