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CANAL SUBMANIFOLDS WITH PARALLEL MEAN CURVATURE VECTOR

Ülo LUMISTE

Tartu Ülikool (Tartu University). Vanemuise 46, EE-2400 Tartu, Eesti (Estonia)

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Abstract. All m-dimensional envelopes M^m of one-parameter families of m-spheres $S^m(R)$, whose mean curvature vector is parallel with respect to the normal connection of M^m in a Euclidean space E^n, are classified and geometrically described by means of the characteristic spheres $S^{m-1}(r)$, their radius r and line of centres.

Key words: canal submanifolds, constant mean curvature.

1. INTRODUCTION

Submanifolds M^m in Euclidean space E^n, satisfying a certain geometric condition, can often be considered as integral submanifolds of some system of partial differential equations. Of special interest among them are such submanifolds M^m which are generated by a one-parameter family of some simple symmetric $(m-1)$-dimensional submanifolds, e.g. of spheres $S^{m-1}(r)$. For such M^m the system of partial differential equations reduces to a system of ordinary differential equations. This approach was initiated as a general programme in [1] and has been developed in several papers. For example, the spherical Bernstein problem, stated by Chern [2] and investigated for some low dimensions by Hsiang [3, 4], was recently solved for all even dimensions by Tomter [5]. It was shown that for every natural p in $S^{2p}(1) \subset E^{2p+1}$ there exists a minimally embedded hypersurface which is diffeomorphic to $(2p-1)$-sphere but different from the equator $S^{2p-1}(1) \subset S^{2p}(1)$.

The first classic examples of such an approach are the investigations of the surfaces of revolution $M^2 \subset E^3$ with constant Gaussian or mean curvature respectively by Minding [6] and Delaunay (C. Sturm) [7]. Delaunay's result was recently generalized to the case of hypersurfaces $M^{n-1} \subset E^n$ in [8], where not only rotationally invariant M^{n-1} are considered but also the more general M^{n-1} with constant mean curvature, invariant with respect to some subgroup of $SO(n)$, the principal orbits of which have codimension 2.

The purpose of the present paper is to generalize Delaunay's result to the case of higher dimension and codimension. In the role of rotationally invariant surface $M^2 \subset E^3$ we consider a more general canal submanifold $M^m \subset E^n$. The condition of the mean curvature constancy is replaced by a more general condition: the mean curvature vector H must be parallel in normal connection $\nabla^\perp$, i.e., $\nabla^\perp H = 0$ must be satisfied.

Here a submanifold $M^m \subset E^n$ is called a canal submanifold if there exists in E^n a one-parameter family of m-spheres $S^m(R)$ so that M^m is the envelope of this family; such M^m consists of characteristic $(m-1)$-spheres $S^{m-1}(r)$ along which M^m touches the family spheres $S^m(R)$.

THEOREM. *Let M^m be a canal submanifold with parallel mean curvature vector H in a Euclidean space E^n. Then M^m lies in its first osculating space $E^{m_1} \subset E^n$ of the dimension $m_1 = m+1$ or $m_1 = m+2$. Its characteristic spheres $S^{m-1}(r)$ are in parallel m-planes, their centres lie on a line l in a 2-plane totally orthogonal to these m-planes.*

Let $r = r(\sigma)$ give the radius r as a function of the arc length parameter σ on the line l.

Only the following possibilities occur:

(i) *If $H \equiv 0$, then l is a straight line, M^m is a minimal hypersurface of revolution in some $E^{m+1} \subset E^n$ and $r(\sigma)$ is given by*

$$r^{m-1}\frac{d\sigma}{ds} = c, \tag{1.1}$$

where $ds = \sqrt{1+\left(\frac{dr}{d\sigma}\right)^2}\, d\sigma$ *and* $c = \text{const}$.

(ii) *If $H \neq 0$ and l is a straight line, then M^m is a hypersurface of revolution with constant mean curvature $\|H\| = a \neq 0$ in some $E^{m+1} \subset E^n$ and $r(\sigma)$ is given by*

$$r^m - ma^{-1}r^{m-1}\frac{d\sigma}{ds} + b = 0, \quad b = \text{const}. \tag{1.2}$$

(iii) *If $H \neq 0$ and $r = \text{const}$, then l is a circle $S^1(r_1)$ and M^m is the product $S^1(r_1) \times S^{m-1}(r)$ in some $S^{m+1}(\sqrt{r_1^2 + r^2}) \subset E^{m+2} \subset E^n$.*

(iv) *If $H \neq 0$, l is not a straight line and r is not a constant, then for every arc of l, on which $r(\sigma)$ is decreasing,*

$$\sigma = -\int \sqrt{\frac{m^2 r^{2(m-1)}}{m^2b^2 + a^2(r^m + c)^2} - 1}\, dr, \tag{1.3}$$

where a, b and c are constants, $a = \|H\|$, $b > 0$, and the curvature of the curve l is

$$k = \frac{ma^3[r^{m-1}(r^m+c)]^2}{b[m^2b^2 + a^2(r^m+c)^2]^{3/2}}. \tag{1.4}$$

Remarks. 1. The canal submanifold $M^m \subset E^n$ with $\nabla^\perp H = 0$ in case (iv) is given by the following geometric construction. At first the function $r = r(\sigma)$ is to be determined as the inverse function of (1.3). After substitution of this function (1.4) gives the natural equation $k = k(\sigma)$ of the plane curve l, determining this curve up to a motion. It remains to add spheres $S^{m-1}(r)$ in parallel m-planes, totally orthogonal to the 2-plane of l, with centres on l and radii $r(\sigma)$. These spheres $S^{m-1}(r)$ generate the desired M^m in some $E^{m+2} \subset E^n$.

2. The first part of the Theorem reduces case (i) for $m = 2$ to the classic result of Meusnier: (1.1) is then satisfied by $r = c \operatorname{ch}\frac{\sigma}{c}$. For $m > 2$ the minimal hypersurfaces of revolution are considered by do Carmo and Dajczer [9].

3. The same argumentation reduces case (ii) for $m = 2$ to the classic result of Delaunay (and C. Sturm) [7]. Then Eq. (1.2) is that of Sturm and the meridian curve is described, due to Delaunay, by a focus of a nonparabolic conic section rolling without slip on the axis. (Parabol gives a catenary, i. e. the previous case (i)).

For $m>2$ the hypersurfaces of revolution with constant mean curvature are considered by Hsiang and Yu [8]; see above.

4. Case (iii) is a particular case of submanifolds $M^m \subset E^n$ with parallel mean curvature vector H and some other conditions leading to the products of spheres, considered by Yano and Ishihara [10], Erbacher [11], and Smyth [12] (see also [13], Ch. 4, § 7).

2. ADAPTED FRAME BUNDLE

For the bundle of orthonormal frames $\{x; e_1, \dots, e_n\}$ in a Euclidean space E^n there hold the following formulae:

$$dx = e_I \omega^I, \quad de_I = e_J \omega^J_I, \quad \omega^J_I + \omega^I_J = 0,$$

and the following structure equations:

$$d\omega^I = \omega^J \wedge \omega^I_J, \quad d\omega^I_J = \omega^K_J \wedge \omega^I_K.$$

Here the point $x \in E^n$ is identified with its radius vector by a fixed origin and $I, J, \dots$ run $\{1, \dots, n\}$.

Let $M^m \subset E^n$ be a canal submanifold. The frame can be adapted to it so that $x \in M^m$, $e_1 \in T_x M^m$ and $e_2, \dots, e_m$ are tangent to the characteristic sphere $S^{m-1}(r)$. It follows that $e_a \in T_x M^m$, too; $a, b, \dots$ ... run $\{2, \dots, m\}$. Then $\omega^\alpha = 0$ and thus $\omega^\alpha_i = h^\alpha_{ij} \omega^j$, $h^\alpha_{ij} = h^\alpha_{ji}$; $i, j, \dots$ ... run $\{1, \dots, m\}$; $\alpha, \beta, \dots$ run $\{m+1, \dots, n\}$.

If x moves in $S^{m-1}(r)$ then $\omega^1 = 0$ and $\omega^1_a = \varkappa \omega^a$, $\omega^\alpha_a = k^\alpha \omega^a$, because each point $x \in S^{m-1}(r)$ is totally umbilical for $S^{m-1}(r)$. As e_1 is tangent to the family m-sphere $S^m(R)$, touching M^m along $S^{m-1}(r)$, and orthogonal to $T_x S^{m-1}(r)$ in $T_x S^m(R) = T_x M^m$, so either $\omega^1 = 0 \Rightarrow de_1 = 0$, or there exists λ such that $\omega^1 = 0$ implies $d(x + \lambda e_1) = 0$. In the first case $\varkappa = 0$ and $h^\alpha_{1a} = 0$. In the second case $\varkappa \neq 0$ and from

$$d(x + \lambda e_1) = e_1 \omega^1 + e_a \omega^a + e_1 d\lambda + \lambda [e_a(-\varkappa \omega^a) + h^\alpha_{1i} \omega^i e_\alpha]$$

it follows that $\lambda = \varkappa^{-1}$ and $h^\alpha_{1a} = 0$, too.

Hence, the canal $M^m \subset E^n$ is in its adapted frame bundle characterized by

$$\omega^\alpha = 0, \quad \omega^1_a = \lambda_a \omega^1 + \varkappa \omega^a, \quad \omega^\alpha_a = k^\alpha \omega^a, \quad \omega^\alpha_1 = l^\alpha \omega^1. \tag{2.1}$$

Now $\omega^1 = 0 \Rightarrow dx = e_a \omega^a$, $de_a = e_b \omega^b_a + (\varkappa e_1 + e_\alpha k^\alpha) \omega^a$, thus $\varkappa e_1 + e_\alpha k^\alpha$ is the normal vector of $S^{m-1}(r)$ and $\varkappa^2 + \sum_\alpha (k^\alpha)^2 = r^{-2} \neq 0$. Moreover, $e_\alpha k^\alpha$ is the normal vector of $S^m(R)$ and $\sum_\alpha (k^\alpha)^2 = R^{-2} \neq 0$.

Using in (2.1) the exterior differentiation we obtain

$$(d\lambda_a - \lambda_b \omega^b_a) \wedge \omega^1 + [d\varkappa - (\varkappa^2 + \sum_\alpha k^\alpha l^\alpha) \omega^1] \wedge \omega^a = 0, \tag{2.2}$$

$$[dk^\alpha + k^\beta \omega^\alpha_\beta + \varkappa (l^\alpha - k^\alpha) \omega^1] \wedge \omega^a = 0, \tag{2.3}$$

$$[dl^\alpha + l^\beta \omega^\alpha_\beta + (l^\alpha - k^\alpha) \lambda_a \omega^a] \wedge \omega^1 = 0. \tag{2.4}$$

Relation (2.4) gives

$$dl^\alpha = -l^\beta \omega^\alpha_\beta + (k^\alpha - l^\alpha) \lambda_a \omega^a + L^\alpha \omega^1. \tag{2.5}$$

Relation (2.3) yields

$$dk^\alpha = -k^\beta \omega_\beta^\alpha + \varkappa (k^\alpha - l^\alpha)\,\omega^1, \tag{2.6}$$

if $m \geqslant 3$, and

$$dk^\alpha = -k^\beta \omega_\beta^\alpha + \varkappa (k^\alpha - l^\alpha)\,\omega^1 + K^\alpha \omega^2, \tag{2.7}$$

if $m=2$.

In the last case for the characteristic circle $S^1(r)$ we have that $\omega^1=0$ yields $dx=e_2\omega^2$, $de_2=(\varkappa e_1+e_\alpha k^\alpha)\,\omega^2$ and it must be $d[\varkappa^2+\sum_\alpha (k^\alpha)^2]=0$; moreover, $d(\varkappa e_1+e_\alpha k^\alpha)$ must lie in the linear hull of e_α and $\varkappa e_1+e_\alpha k^\alpha$. On the other hand, $\omega^1=0$ yields, due to (2.2) and (2.7), $d\varkappa=\varkappa_2\omega^2$,

$$d(\varkappa e_1+e_\alpha k^\alpha)=e_1\varkappa_2\omega^2+\varkappa(-\varkappa e_2\omega^2)+[e_2\textstyle\sum (k^\alpha)^2+e_\alpha K^\alpha]\,\omega^2,$$

$d[\varkappa^2+\sum_\alpha (k^\alpha)^2]=2(\varkappa\varkappa_2+\sum k^\alpha K^\alpha)\,\omega^2$, so the vector $\varkappa_2 e_1+e_\alpha K^\alpha$ must be collinear to $\varkappa e_1+e_\alpha k^\alpha$, i. e. $K^\alpha=\varkappa^{-1}\varkappa_2 k^\alpha$ and $0=\varkappa^{-1}\varkappa_2[\varkappa^2+\sum_\alpha (k^\alpha)^2]$. Thus $\varkappa_2=0$, $K^\alpha=0$ and (2.6) is true also by $m=2$.

The mean curvature vector of the canal M^m is $H=H^\alpha e_\alpha$, where

$$H^\alpha=\frac{1}{m}[(m-1)k^\alpha+l^\alpha]. \tag{2.8}$$

From (2.5) and (2.6) it follows that

$$dH^\alpha=-H^\beta\omega_\beta^\alpha+[(m-1)\varkappa(k^\alpha-l^\alpha)+L^\alpha]\,\omega^1+(k^\alpha-l^\alpha)\,\lambda_a\omega^a.$$

3. CANAL SUBMANIFOLD WITH $\nabla^\perp H=0$

In this case $dH^\alpha=-H^\beta\omega_\beta^\alpha$ and thus

$$L^\alpha=-(m-1)\varkappa(k^\alpha-l^\alpha),\quad \lambda_a(k^\alpha-l^\alpha)=0.$$

Let $k^\alpha=l^\alpha$. Then every point $x\in M^m$ is umbilical and M^m is a sphere $S^m(R)$, i.e a special case of (ii) in the Theorem.

Let $\lambda_a=0$. Then from (2.2)

$$d\varkappa=(\varkappa^2+\textstyle\sum k^\alpha l^\alpha)\,\omega^1$$

($\varkappa_2=0$ is to be used, if $m=2$) and

$$dl^\alpha=-l^\beta\omega_\beta^\alpha-(m-1)\varkappa(k^\alpha-l^\alpha)\,\omega^1.$$

Denoting $k=e_\alpha k^\alpha$, $l=e_\alpha l^\alpha$ we have

$$dx=e_1\omega^1+e_a\omega^a,$$

$$de_1=-\varkappa e_a\omega^a+l\omega^1,\quad de_a=e_b\omega_a^b+(\varkappa e_1+k)\,\omega^a, \tag{3.1}$$

$$d\varkappa=(\varkappa^2+\langle k,l\rangle)\,\omega^1, \tag{3.2}$$

$$dk=-\langle k,l\rangle e_1\omega^1-k^2e_a\omega^a+\varkappa(k-l)\,\omega^1, \tag{3.3}$$

$$dl=-l^2e_1\omega^1-\langle k,l\rangle e_a\omega^a-(m-1)\varkappa(k-l)\,\omega^1. \tag{3.4}$$

It is seen that m_1-plane through $x\in M^m$, spanned on $e_1, e_2, \ldots, e_m, k, l$, is absolutely invariant and M^m is contained in it; $m_1\leqslant m+2$. As $k^2=\sum_\alpha (k^\alpha)^2=R^{-2}\neq 0$, the normal part $\{e_{m+1},\ldots e_n\}$ of the frame basis can be adapted so that $k=\mu e_{m+1}$, $\mu>0$ and $l=\nu e_{m+1}+\tau e_{m+2}$, $\tau\geqslant 0$.

On the other hand, $d\omega^1=\omega^a \wedge \omega^1_a=\omega^a \wedge (\varkappa\omega^a)=0$, thus $\omega^1=ds$, where s is the arc length parameter on the orthogonal trajectory of characteristic spheres $S^{m-1}(r)$, because $\omega^a=0 \Rightarrow dx=e_1\,ds$. It now follows from (3.2)—(3.4) that

$$d\varkappa=(\varkappa^2+\mu\nu)\,ds,$$
$$d\mu=\varkappa(\mu-\nu)\,ds, \quad \mu\omega^{m+2}_{m+1}=-\varkappa\tau\,ds,$$
$$d\nu=\tau\omega^{m+2}_{m+1}-(m-1)\varkappa(\mu-\nu)\,ds,$$
$$d\tau=-\nu\omega^{m+2}_{m+1}+(m-1)\varkappa\tau\,ds.$$

It is seen that $\varkappa$, μ, ν, τ are functions of s only and

$$\dot{\varkappa}=\varkappa^2+\mu\nu, \tag{3.5}$$

$$\dot{\mu}=\varkappa(\mu-\nu), \tag{3.6}$$

$$\dot{\nu}=-\varkappa\mu^{-1}\tau^2-(m-1)\varkappa(\mu-\nu), \tag{3.7}$$

$$\dot{\tau}=\varkappa\mu^{-1}\nu\tau+(m-1)\varkappa\tau, \tag{3.8}$$

i.e. these functions satisfy a normal autonomous system of ordinary differential equations.

4. PROOF OF THE THEOREM

The assertion that the characteristic spheres $S^{m-1}(r)$ lie in parallel m-planes follows from (3.1) and

$$d(\varkappa e_1+k)=-(\varkappa^2+k^2)e_a\omega^a+\varkappa(\varkappa e_1+k)\omega^1.$$

Here $r^2=(\varkappa^2+k^2)^{-1}=(\varkappa^2+\mu^2)^{-1}$ and

$$\dot{r}=-\varkappa r. \tag{4.1}$$

As the normal unit vector of $S^{m-1}(r)$ is

$$n=r(\varkappa e_1+\mu e_{m+1})$$

and $dn=-r^{-1}e_a\omega^a$, for the radius vector $z=x+rn$ of the centre z of $S^{m-1}(r)$ we have

$$dz=f\,ds=t\,d\sigma,$$

where $f=e_1-\varkappa rn=(1-\varkappa^2r^2)e_1-\varkappa r^2\mu e_{m+1}$, $f^2=r^{-2}\mu^{-2}$ and $t=r^{-1}\mu^{-1}f$, $d\sigma=r\mu\,ds$ denote, respectively, the unit tangent vector and the differential of the arc length parameter of the central curve l described by z. Further,

$$dt=r^{-2}\mu^{-2}\tau e_{m+2}\,d\sigma, \quad de_{m+2}=-r^{-2}\mu^{-2}\tau t\,d\sigma. \tag{4.2}$$

The remaining part of the first assertion of the Theorem now follows from the fact that $\langle n, f\rangle=r[\varkappa(1-\varkappa^2r^2)-\varkappa r^2\mu]=r\varkappa[1-(\varkappa^2+\mu^2)r^2]=0$.

(i) Let $H=0$. Then $(m-1)\mu+\nu=\tau=0$ due to (2.8), thus $dt=0$ and l is a straight line. Hence M^m is a minimal hypersurface of revolution. Since $\left(r^{m-1}\dfrac{d\sigma}{ds}\right)^{\cdot}=(r^m\mu)^{\cdot}=mr^{m-1}(-\varkappa r)\mu+r^m\varkappa m\mu=0$, we have (1.1).

(ii) Let $H \neq 0$ and l be a straight line, i.e. in (4.2) we have $\tau = 0$. Then $(m-1)\dot{\mu} + \dot{\nu} = 0$ (see (3.7) and (3.8)) and thus $(m-1)\mu + \nu = a = \text{const} \neq 0$. Now

$$\left(r^m - ma^{-1}r^{m-1}\frac{d\sigma}{ds}\right)^{\cdot} = [r^m(1 - ma^{-1}\mu)]^{\cdot} =$$

$$= mr^{m-1}(-\varkappa r)(1 - ma^{-1}\mu) + r^m(-ma^{-1})\varkappa(m\mu - a) = 0,$$

hence (1.2) holds.

(iii) If $H \neq 0$, l is not a straight line and $r = \text{const}$, then $\varkappa = 0$ due to (4.1), $\nu = 0$ due to (3.5), $\mu = \text{const}$ and $\tau = \text{const}$ due to (3.6) and (3.8). From (4.2) it follows that l has the curvature $k = r^{-2}\mu^{-2}\tau = \text{const} \neq 0$, thus l is a circle and M^m is $S^1(r_1) \times S^{m-1}(r)$, where $r_1 = k^{-1}$.

(iv) If $H \neq 0$, l is not a straight line, and r is not a constant, then $\tau > 0$ and directing e_1 (and thus the direction of growth of s) so that r is decreasing (this can be done, of course, between two neighbouring critical values of r) we have $\dot{r} < 0$ and $\varkappa > 0$ due to (4.1).

Denoting $\varrho = (m-1)\mu + \nu$ we obtain from (3.6)—(3.8) that

$$\dot{\varrho} = -\varkappa\mu^{-1}\tau^2, \qquad \dot{\tau} = \varkappa\mu^{-1}\tau\varrho,$$

thus $(\varrho^2 + \tau^2)^{\cdot} = 0$ and $\|H\|^2 = \varrho^2 + \tau^2 = a^2 = \text{const} \neq 0$. This together with $\varkappa^2 + \mu^2 = r^{-1}$ gives that there exist φ and ψ, $0 < \varphi < \frac{\pi}{2}$, $0 < \psi < \pi$, so that

$$r\varkappa = \cos\varphi, \quad r\mu = \sin\varphi, \quad \varrho = a\cos\psi, \quad \tau = a\sin\psi, \quad a > 0;$$

here φ is the angle between n and e_1, directed so that $\sigma = \sigma(s)$ is increasing, i.e. $d\sigma : ds = \sin\varphi > 0$, and ψ is the angle between k and H. Using now Eqs. (4.1), (3.5) and (3.6) we get $\dot{\varphi} = -\nu$ and analogically $\dot{\psi} = a\cot\varphi\sin\psi$. So the system (3.5)—(3.8) reduces to

$$\dot{r} = -\cos\varphi, \tag{4.3}$$

$$\dot{\varphi} = (m-1)r^{-1}\sin\varphi - a\cos\psi, \tag{4.4}$$

$$\dot{\psi} = a\cot\varphi\sin\psi \tag{4.5}$$

and the curvature of the central curve l is

$$k = a\sin\psi\sin^{-2}\varphi. \tag{4.6}$$

The straightforward computation gives $(r^{m-1}\sin\varphi\sin\psi)^{\cdot} = 0$, thus $r^{m-1}\sin\varphi\sin\psi = b = \text{const} \neq 0$. Now (4.5) reduces to $\dot{\psi} = a(-\dot{r})\sin^{-1}\varphi\sin\psi$ or to $\dot{\psi} = -ab^{-1}r^{m-1}\dot{r}\sin^2\psi$. Hence

$$\frac{d\psi}{\sin^2\psi} = -\frac{a}{b}r^{m-1}\,dr$$

and

$$\cot\psi = \frac{a}{mb}(r^m + c),$$

where $c = \text{const}$. By means of these two independent first integrals of the system (4.3)—(4.5) it is possible to eliminate φ and ψ from this system and (4.6), using also $d\sigma = \sin\varphi\,ds$. This procedure leads easily to (1.3) and (1.4).

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KANAL-ALAMMUUTKONNAD PARALLEELSE KESKMISE KÕVERUSE VEKTORIGA

Ülo LUMISTE

Eukleidilises ruumis E^n on vaadeldud alammuutkondi M^m, mis on m-sfääride $S^m(R)$ üheparameetriliste parvede mähkijad ning mille keskmise kõveruse vektor H on paralleelne normaalseostuses. On tõestatud, et iga selline M^m on oma $(m+1)$- või $(m+2)$-mõõtmelises kooldumisruumis ning tema karakteristlikud sfäärid $S^{m-1}(r)$ on paralleelsetes m-tasandites, kusjuures nende keskpunktide joon l on kas sirge, ringjoon või asub 2-tasandis, mis on risti nende m-tasanditega. Kõikidel juhtudel on antud seosed raadiuse r leidmiseks joone l loomuliku parameetri funktsioonina ning kõverjoone l puhul ka selle joone kõveruse avaldis raadiuse r kaudu.

КАНАЛОВЫ ПОДМНОГООБРАЗИЯ С ПАРАЛЛЕЛЬНЫМ ВЕКТОРОМ СРЕДНЕЙ КРИВИЗНЫ

Юло ЛУМИСТЕ

Рассматриваются подмногообразия M^m в евклидовом пространстве E^n, являющиеся огибающими однопараметрических семейств m-сфер $S^m(R)$ и вектор средней кривизны H которых параллелен в нормальной связности. Доказывается, что каждое такое M^m лежит в своем $(m+1)$- или $(m+2)$-мерном соприкасающемся пространстве и его характеристические сферы $S^{m-1}(r)$ лежат в параллельных m-плоскостях, причем линия l их центров является либо прямой, окружностью или линией в 2-плоскости, ортогональной к этим m-плоскостям. Для всех случаев даются соотношения для нахождения радиуса r как функции натурального параметра линии l, а в случае криволинейной l также выражение ее кривизны через r.