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A REPRODUCING FORMULA FOR VECTOR VALUED FUNCTIONS OVER A CARTAN DOMAIN

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Abstract. A reproducing formula is proved for holomorphic differential forms over a Cartan domain. Analogous formulae prevail also for tensor fields.

Key words: reproducing kernel, quadratic map, Bergman operator, Hankel form, symbol.

INTRODUCTION

Some years ago we wrote a paper [1] where we gave reproducing formulae for holomorphic tensor fields over the unit ball in $\mathbf{C}^d$ (i. e. the "Rudin ball" [2]). In this paper we wish to do something similar in the case of a general Cartan domain.

In [1] we wrote: "We hope that we, if we live long enough, will be able to come to this question on a future occasion." However, this is not yet the generalization that we had in mind when we wrote those lines. The difference is that here we consider just sections of the (weighted) tangent or cotangent bundle (or their tensor products), while in [1] we had more sophisticated vector bundles in mind.

Let us remark that results of the type obtained here are of interest, inasmuch as holomorphic tensor fields arise as "symbols" of Hankel forms (cf. [3]).

What concerns the theory of symmetric domains, we shall rely on Loos's beautiful book [4]. That is, we shall prefer the Jordan approach.

1. REPRODUCING VECTOR VALUED FUNCTIONS

Let $\mathfrak{D}$ be a Cartan domain, that is, an irreducible bounded symmetric domain in its Harish-Chandra realization. We denote by V the containing finite dimensional vector space over the complex numbers $\mathbf{C}$, often identified with $\mathbf{C}^d$, where d is the dimension of $\mathfrak{D}$. Let further $Q(w):\overline{V}\rightarrow \rightarrow V (w\in V)$ be the quadratic map and set for $z, w, \zeta \in V$

$$\{z, \overline{w}, \zeta\} = D(z, \overline{w})\zeta = Q(z, w)\overline{w} = Q(z+\zeta)\overline{w} - Q(z)\overline{w} - Q(\zeta)\overline{w}.$$

Similarly, the *Bergman operator* B is defined by

$$B(z,\bar{w})\zeta=\zeta-D(z,\bar{w})\zeta+Q(z)Q(\bar{w})\zeta.$$

As is well known, the *Bergman kernel* K of $\mathfrak{D}$ is proportional to (or, if properly normalized, equal to) the inverse of its determinant,

$$K(z,\bar{w})=c\cdot\det B^{-1}(z,\bar{w}).$$

Let further $d\iota$ be the *Bergman* (or *invariant*) *measure*,

$$d\iota(z)=K(z,\bar{z})\,d\varepsilon(z),$$

where $d\varepsilon$ is the Hermitean measure on V induced by the *Bergman metric* on $\mathfrak{D}$. Recall also that the latter is given by the expression

$$\|v\|_z^2=(B(z,\bar{z}^{-1})v,\bar{v}).$$

The following reproducing formula

$$f(w)=c\int_{\mathfrak{D}} K^{\frac{\nu}{p}}(z,\bar{w})f(z)K^{-\frac{\nu}{p}}(z,\bar{z})\,d\iota(z), \tag{1.1}$$

where f is any *conjugate* analytic function, p is the *genus* of $\mathfrak{D}$, ν a sufficiently large parameter and c a normalization constant (depending on ν), is well known and not hard to prove. In this note we shall generalize (1.1) to the case of vector valued (V-valued) functions.

Theorem. *We have*

$$f(w)=c_1\int_{\mathfrak{D}} K^{\frac{\nu}{p}}(z,\bar{w})B(z,\bar{w})^{-1}B(z,\bar{z})f(z)K^{-\frac{\nu}{p}}(z,\bar{z})\,d\iota(z), \tag{1.2}$$

where f is any conjugate analytic V-valued function, ν a sufficiently large real number and c_1 another normalization constant. (The value of the constant c_1 will be discussed in Section 3.)

Proof. *Step 1* (reduction to the case $w=0$). Let $\mathbf{G}=\operatorname{Aut}\mathfrak{D}$ be the group of biholomorphic automorphisms of $\mathfrak{D}$. If $\varphi\in\mathbf{G}$, then ([4], p. 1.2 and p. 2.10 respectively)

$$K(\varphi(z),\overline{\varphi(w)})\det\varphi'(z)\,\overline{\det\varphi'(w)}=K(z,\bar{w}); \tag{1.3}$$

$$B(\varphi(z),\overline{\varphi(w)})=\varphi'(z)B(z,\bar{w})\varphi'(w)^{*}. \tag{1.4}$$

For any $w\in\mathfrak{D}$, let $\sigma_w\in\mathbf{G}$ be the symmetry that interchanges 0 and w, $\sigma_w(0)=w$, $\sigma_w(w)=0$, $\sigma_w^2=\mathrm{id}$. Taking $\varphi=\sigma_w$ in (1.3) and (1.4) we obtain (with $K(z,0)=1$, $B(z,0)=\mathrm{id}$; this is the preferred normalization for the time being)

$$K(z,\bar{w})=\det\sigma_w'(z)\cdot\overline{\det\sigma_w'(w)}; \tag{1.5}$$

$$B(z,\bar{w})=\sigma_w'(z)^{-1}\cdot\sigma_w'(w)^{-1*}. \tag{1.6}$$

Assume now that (1.2) holds for $z=0$, i.e.

$$f(0)=c_1\int_{\mathfrak{D}} B(z,\bar{z})f(z)K^{-\frac{\nu}{p}}(z,\bar{z})\,d\iota(z). \tag{1.7}$$

Given f we put

$$f_1(z)=(\sigma'_w(z))^*f(\sigma_w(z))\,\overline{(\det\sigma'_w(z))}^{\frac{\nu}{p}}$$

and apply (1.7) with f replaced by f_1. This is still a conjugate analytic function. We obtain

$$(\sigma'_w(0))^*f(w)\,\overline{(\det\sigma'_w(0))}^{\frac{\nu}{p}}=$$

$$=c_1\int_{\mathfrak{D}}B(z,\bar z)(\sigma'_w(z))^*f(\sigma_w(z))\,\overline{(\det\sigma'_w(z))}^{\frac{\nu}{p}}K^{-\frac{\nu}{p}}(z,\bar z)\,d\iota(z).$$

Let us make the substitution $z=\varphi_w(\zeta)$. Then we find

$$(\sigma'_w(0))^*f(w)\,\overline{(\det\sigma'_w(0))}^{\frac{\nu}{p}}=c_1\int_{\mathfrak{D}}B(\sigma_w(\zeta),\overline{\sigma_w(\zeta)})\,\sigma'_w(\sigma_w(\zeta))^*f(\zeta)\times$$

$$\times\overline{(\det\sigma'_w(\sigma_w(\zeta)))}^{\frac{\nu}{p}}K^{-\frac{\nu}{p}}(\sigma_w(\zeta),\overline{\sigma_w(\zeta)})\,d\iota(\zeta),$$

where we have used $\sigma_w(\sigma_w(\zeta))=\zeta$. The latter relation gives also $\sigma'_w(\sigma_w(\zeta))\sigma'_w(\zeta)=1$ or, setting $\zeta=w$, $\sigma'_w(0)\sigma'_w(w)=1$. Therefore we can also write

$$f(w)=c_1\int_{\mathfrak{D}}\sigma'_w(0)^{*-1}B(\sigma_w(\zeta),\overline{\sigma_w(\zeta)})\,\sigma'_w(\zeta)^{*-1}f(\zeta)\times$$

$$\times\overline{(\det\sigma'_w(0))}^{-\frac{\nu}{p}}\overline{(\det\sigma'_w(\zeta))}^{\frac{\nu}{p}}K^{-\frac{\nu}{p}}(\sigma_w(\zeta),\sigma_w(\zeta))\,d\iota(\zeta)$$

or using (1.3) and (1.4)

$$f(w)=c_1\int_{\mathfrak{D}}\sigma'_w(0)^{*-1}\sigma'_w(\zeta)B(\zeta,\bar\zeta)f(\zeta)\times$$

$$\times\overline{(\det\sigma'_w(0))}^{-\frac{\nu}{p}}\,\overline{\det\sigma'_w(\zeta)}^{\frac{\nu}{p}}K^{-\frac{\nu}{p}}(\zeta,\bar\zeta)\,d\iota(\zeta)$$

or, finally, using (1.5) and (1.6) and once more $\sigma'_w(\sigma_w(\zeta))^{-1}=\sigma'_w(\zeta)$

$$f(w)=c_1\int_{\mathfrak{D}}B(\zeta,\bar w)^{-1}B(\zeta,\bar\zeta)f(\zeta)K^{\frac{\nu}{p}}(\zeta,w)K^{-\frac{\nu}{p}}(\zeta,\bar\zeta)\,d\iota(\zeta),$$

which is the same as (1.2).

Step 2 (proof of (1.7), i.e. (1.2) with $w=0$). Explicitly:

$$f(0)=c_1\int_{\mathfrak{D}}(1-D(z,\bar z)+Q(z)Q(\bar z))f(z)\,d\mu(z)=$$

$$=c_1\left(\int_{\mathfrak{D}}f(z)\,d\mu(z)-\int_{\mathfrak{D}}D(z,\bar z)f(z)\,d\mu(z)+\int_{\mathfrak{D}}Q(z)Q(\bar z)f(z)\,d\mu(z)\right),\tag{1.8}$$

where we have put

$$d\mu(z)=K^{-\frac{\nu}{p}}(z,\bar z)\,d\iota(z)=K^{1-\frac{\nu}{p}}(z,\bar z)\,d\varepsilon(z).$$

Let us choose linear coordinates $z^1, \ldots, z^d$ in V. Then f has the power series expansion

$$\bar{f}(z) = \sum_J a_J \bar{z}^J$$

where $z^J = (z^1)^{j_1} \ldots (z^d)^{j_d}$ and the coefficients a belong to V. Clearly, the first term in (1.8) depends on $a_0 = f(0)$ only (this is just (1.1) for $z=0$). Let us look at e. g. the second term. It is clear that it is a sum of integrals of the type $\int_{\mathfrak{D}} z^j \bar{z}^k \bar{z}^J \, d\mu(z)$. As monomials of different degree are orthogonal in this metric, it is clear that every such integral vanishes unless $J = 0 = (0, \ldots, 0)$. Thus the second term in (1.8) also depends on $f(0)$ only and, by a similar argument, likewise the third term. This argument establishes (1.8) with the provision that the quantity c_1 might not be a number but a linear operator, $c_1 = \Gamma^{-1}$, where

$$\Gamma = \int_{\mathfrak{D}} B(z, \bar{z}) \, d\mu(z).$$

However, it is clear that the operator Γ commutes with the isotropy group $\mathbf{K}$ of $\mathbf{G}$ at the origin, $\varphi\Gamma = \Gamma\varphi$ for $\varphi \in \mathbf{K}$, so by Schur's lemma we conclude that Γ must be a scalar multiple of the identity operator. ∎

2. DISCUSSION

Why have we formulated our result (the theorem in Section 1) for conjugate analytic functions? It is because it is really a question of a reproducing formula for holomorphic differential forms (of degree one). Let us explain this. We let differential forms transform according to the rule

$$u(z) = \sum_{j=1}^{d} a_j(z) \, dz^j \to \varphi^* u(z) = \sum_{j=1}^{d} a_j(\varphi(z)) \, d\varphi^j(z) \cdot (\det \varphi'(z))^{\frac{\nu}{p}}, \quad (\varphi \in \mathbf{G}) \tag{2.1}$$

that is, with weight ν. Such forms should be viewed as sections of a certain Hermitean holomorphic vector bundle $T'^*\mathfrak{D} \otimes \mathfrak{L}^\nu$. Here we refer to $T^c\mathfrak{D} = T'\mathfrak{D} \dotplus T''\mathfrak{D}$, the splitting of the complexified tangent bundle into its "analytic" and "anti-analytic" parts, a star * indicating that one passes to the dual bundle, while $\mathfrak{L}^\nu$ is a line bundle, a fractional power of the "volume element" bundle (the case $\nu = p$). The natural integral metric for such differential forms u is

$$\|u\|^2 = \int_{\mathfrak{D}} \|u(z)\|_z^2 \, d\iota(z), \tag{2.2}$$

where $\|\cdot\|_z$ is the norm in the bundle $T'^*\mathfrak{D} \otimes \mathfrak{L}^\nu$. (We assume that the parameter ν is sufficiently large, so as to guarantee the convergence of the integral.) It is clear that the metric (2.2) is $\mathbf{G}$-invariant.

Remark. As already stated in the Introduction, one reason for undertaking this research is that symbols of Hankel forms are differential forms which transform like in (2.1). Indeed, the condition for a Hankel form to be in S_2 (Hilbert-Schmidt class) is very likely that its symbol be in a Hilbert space with a norm of the type (2.2). For some more information about this we refer to [3].

Now, every differential form of the above type can be written in the form

$$u(z)=(dz,\overline{f(z)}),$$

where f is a conjugate analytic vector valued function. (This is essentially the Riesz representation lemma.) To the rule (2.1) corresponds

$$f(z)\to\varphi'(z)^*f(\varphi(z))\,\overline{(\det\varphi'(z))}^{\frac{\nu}{p}}.$$

Then the metric (2.2) is expressed by the integral

$$\int_{\mathfrak{D}}(B(z,\bar z)f(z),\overline{f(z)})\,d\mu(z).$$

We conclude that our result (viz. formula (1.2)) *is indeed a reproducing formula for holomorphic differential forms.*

Let us rewrite the said formula in such a way that its covariance becomes manifest. Taking formally the scalar product with dw gives

$$\begin{aligned}u(w)&=(dw,\overline{f(w)})=\\&=c\int_{\mathfrak{D}}(dw,K^{\frac{\nu}{p}}(z,\bar w)B^{-1}(z,\bar w)B(z,\bar z)f(z))\,d\mu(z)=\\&=\int_{\mathfrak{D}}(B(z,\bar z)K^{\frac{\nu}{p}}(z,\bar w)B^{-1}(z,\bar w)dw,f(z))\,d\mu(z).\end{aligned}$$

So if we introduce the "double" form

$$\boxed{\mathcal{K}(z,\bar w)=K^{\frac{\nu}{p}}(z,\bar w)\,(B^{-1}(z,\bar w)dz,d\bar w)}$$

we can write

$$\boxed{u(w)=c_1\int_{\mathfrak{D}}(u(z),\mathcal{K}(z,\bar w))_z\,d\iota(z)}\,.$$

Example. $\mathfrak{D}=$ the $m\times n$ matrix ball $(m\geqslant n)$. Then $Q(w)z=zw^*z$, which yields

$B(z,\bar w)=(1-zw^*)\zeta(1-w^*z)$;

$K(z,\bar w)=\det(1-zw^*)^{-(m+n)}=\det(1-w^*z)^{-(m+n)}$, $\quad d=mn,\quad p=m+n$,

so in this case we have

$$\mathcal{K}(z,\bar w)=c\det(1-w^*z)^{-(\nu+1)}\operatorname{tr}[(1-zw^*)^{-1}dz(1-w^*z)^{-1}dw^*].$$

Sub-example. Specializing to $n=1$ we get the ordinary complex ball ("Rudin ball" [2]), complex vectors in $\mathbf{C}^m$ being identified to column vectors. In this case $1-w^*z$ is a scalar,

$$1-w^*z=1-\langle z,\bar w\rangle,$$

while

$$1-w^*z=1-\langle z,\bar w\rangle\equiv L(\zeta).$$

We claim that

$$L^{-1}(\zeta)=\zeta+\frac{\langle\zeta,\overline{w}\rangle}{1-\langle z,w\rangle}z.$$

Indeed, one finds

$$L(L^{-1}(\zeta))=\zeta+\frac{\langle\zeta,\overline{w}\rangle}{1-\langle z,w\rangle}z-\left(\langle\zeta,\overline{w}\rangle+\frac{\langle\zeta,\overline{w}\rangle}{1-\langle z,w\rangle}\langle z,\overline{w}\rangle z\right)=$$

$$=\zeta+\underbrace{\left(\frac{1}{1-\langle z,\overline{w}\rangle}z-1-\frac{\langle z,\overline{w}\rangle}{1-\langle z,\overline{w}\rangle}z\right)}_{0}\langle\zeta,\overline{w}\rangle\zeta=\zeta.$$

Thus in this case we find

$$\mathcal{K}(z,\overline{w})=c(1-\langle z,\overline{w}\rangle)^{-(\nu+1)}\left\langle dz+\frac{\langle dz,\overline{w}\rangle}{1-\langle z,\overline{w}\rangle}z,d\overline{w}\right\rangle,$$

which is, but for a slight change of notation, Theorem 3_1 in [1]. It gives thus the reproducing formula in the integral metric

$$\|u\|^2=\int_{\mathfrak{D}}(\|u(z)\|^2-|\langle u(z),\overline{z}\rangle|^2)(1-\|z\|^2)\,d\mu(z)$$

with

$$d\mu(z)=(1-\|z\|^2)^{\nu-(d+1)}\,d\varepsilon(z).$$

3. ATTEMPTS TO DETERMINE THE VALUE OF THE CONSTANT c_1

We have to compute the integral

$$c_1=\int_{\mathfrak{D}}\operatorname{tr}B(z,\overline{z})\,d\mu(z).$$

As it is a sum of three terms we can consider these three terms separately.

First term. This is easy because $\int_{\mathfrak{D}}d\mu(z)=1$, if $d\mu$ is normalized.

Second term. This is easy too, because it is known [4] that

$$\operatorname{tr}D(z,\overline{z})=\|z\|^2.$$

Thus we find

$$\int_{\mathfrak{D}}\operatorname{tr}D(z,\overline{z})\,d\mu(z)=\int_{\mathfrak{D}}\|z\|^2\,d\mu(z).$$

As $\|z\|^2=|z^1|^2+\ldots+|z^d|^2$ if we select an orthonormal basis in V and as G acts irreducibly on first-degree polynomials, we can also write

$$\int_{\mathfrak{D}}\operatorname{tr}D(z,\overline{z})\,d\mu(z)=d\int_{\mathfrak{D}}|z^1|^2\,d\mu(z).$$

In order to treat the third term we need some preparations. By the spectral theorem for symmetric domains we can write any $z\in V$ in the form

$$z=\lambda_1\varepsilon_1+\ldots+\lambda_r\varepsilon_r,$$

where $\lambda_1, \ldots, \lambda_r$ are complex numbers and the vectors $\varepsilon_1, \ldots, \varepsilon_r$ are orthogonal tripotents ($Q(\varepsilon_j)\bar{\varepsilon}_j=\varepsilon_j$; $D(\varepsilon_j, \bar{\varepsilon}_k)=0$ $(j\neq k)$ for $j, k=1, \ldots, r$), forming a frame in V. In particular, r is the rank of V. We put

$$|z|_q=(|\lambda^1|^q+\ldots+|\lambda^r|^q)^{\frac{1}{q}} \qquad (0<q\leqslant\infty);$$

$|z|_\infty$ is the spectral norm of V and $\mathfrak{D}$ happens to be its open unit ball, $\mathfrak{D}=\{z: |z|_\infty<1\}$.

Let us invoke the Peirce decomposition for the frame $(\varepsilon_1, \ldots, \varepsilon_r)$:

$$V=\sum_{0\leqslant i<j\leqslant r}^{\oplus} V_{ij},$$

where V_{ij} is the joint eigenspace of the operators $D(\varepsilon_j, \bar{\varepsilon}_j)$ and $D(\varepsilon_k, \bar{\varepsilon}_k)$ with eigenvalues j and k respectively. As our domain $\mathfrak{D}$ (a Cartan domain) is irreducible by hypothesis, we know that

$$\begin{aligned} &\dim V_{ii}=1,\\ &\dim V_{ij}=a \quad \text{if} \quad 1\leqslant i<j\leqslant r,\\ &\dim V_{0j}=b \quad \text{if} \quad 1\leqslant j\leqslant r,\\ &V_{00}=0,\end{aligned}$$

where a and b are the Korányi parameters of $\mathfrak{D}$. Moreover, we have [4]

$$D(z, \bar{z})=(|\lambda_i|^2+|\lambda_j|^2)\cdot\mathrm{id}.$$

It follows that

$$\|z\|^2=\mathrm{tr}\, D(z, \bar{z})=\sum_{i,j}(|\lambda_i|^2+|\lambda_j|^2)\dim V_{ij}=$$
$$=2\sum_{1\leqslant i\leqslant r}|\lambda_i|^2+\sum_{1\leqslant i<j\leqslant r}(|\lambda_i|^2+|\lambda_j|^2)a+\sum_{1\leqslant j\leqslant r}|\lambda_j|^2 b.$$

The middle sum here can be written

$$\sum_i|\lambda_i|^2\sum_{i<j\leqslant r}1+\sum_j|\lambda_j|^2\sum_{1\leqslant i<j}1=$$
$$=\sum_i|\lambda_i|^2(r-i)+\sum_j|\lambda_j|^2(j-1)=$$
$$=\sum_i|\lambda_i|^2(r-i).$$

Thus we have

$$\|z\|^2=\mathrm{tr}\, D(z, \bar{z})=(2+a(r-1)+b)\sum_j|\lambda_j|^2=g\sum_j|\lambda_j|^2=g|z|_2^2,$$

where g is the genus of $\mathfrak{D}$.

Actually, we do not need the preceding calculation so much, but the point is that it can be readily adapted if we replace $D(z, \bar{z})$ by $Q(z)Q(\bar{z})$. *Third term.* We have [4]

$$Q(z)Q(\bar{z})\,|\,V_{ij}=|\lambda_i\lambda_j|^2.$$

Therefore

$$\mathrm{tr}\, Q(z)Q(\bar{z})=\sum_{1\leqslant i\leqslant j\leqslant r}|\lambda_i\lambda_j|^2\cdot\dim V_{ij}=$$
$$=\sum_{1\leqslant i<j\leqslant r}|\lambda_i\lambda_j|^2\cdot a+\sum|\lambda_i|^4\cdot 1.$$

The first sum here can be rewritten as follows:

$$\frac{1}{2}\sum_i |\lambda_i|^2 \sum_{i+1\leqslant r<j} |\lambda_j|^2+\frac{1}{2}\sum_j |\lambda_j|^2 \sum_{1\leqslant i\leqslant j-1} |\lambda_i|^2=$$

$$=\frac{1}{2}\sum_i |\lambda_i|^2 \sum_{j\neq i} |\lambda_j|^2=\frac{1}{2}\left(\sum_i |\lambda_i|^2\right)^2-\frac{1}{2}\sum_i |\lambda_i|^2=$$

$$=\frac{1}{2}\left(|z|_4^2-|z|_4^4\right).$$

Thus we get

$$\operatorname{tr} Q(z)\,Q(\bar z)=\frac{1}{2}|z|_2^4 a+\left(1-\frac{a}{2}\right)|z|_4^4 a.$$

Joining all these results we thus have

$$c=1+d\int_{\mathfrak{D}} |z^1|^2\,d\mu(z)+\int_{\mathfrak{D}}\left[\frac{1}{2}|z|_2^4 a+\left(1-\frac{a}{2}\right)|z|_4^4 a\right]d\mu(z),$$

which is the end result of this Section.

Remark. Arazy [5] has done a similar calculation in the case of the more general integral

$$\int_{\mathfrak{D}} \operatorname{tr} B^{\alpha}(z,\bar z)\,d\mu(z),$$

where α is an auxiliary parameter.

4. THE CASE OF TENSOR FIELDS

The considerations of Sections 1—3 extend *mutatis mutandis* to the case of holomorphic covariant tensor fields (cf. [1]). To fix the ideas let us have a brief look at the case of a tensor of rank 2. Then it is a question of holomorphic sections of the vector bundle $T'^*\mathfrak{D}\otimes T'^*\mathfrak{D}\otimes\mathfrak{J}^{\nu}$. Any such section can be written as

$$u(z)=\sum_{j,k=1}^{n} a_{jk}(z)\,dz^j\otimes dz^k,$$

where the coefficients $a_{jk}(z)$ are holomorphic functions. In terms of conjugate analytic functions $f(z)$ with values in $V\otimes V$ it is then a question of the reproducing formula

$$f(w)=c_2\int_{\mathfrak{D}} K^{\frac{\nu}{p}}(z,\bar w)\left(B(z,\bar w)^{-1}B(z,\bar z)\otimes B(z,\bar w)^{-1}B(z,\bar z)\right)\times$$

$$\times f(z)\,K^{-\frac{\nu}{p}}(z,\bar z)\,d\iota(z), \qquad (4.1)$$

where c_2 is a suitable constant. The proof of (4.1) goes along similar lines as the proof of the Theorem in Section 1 and will be omitted.

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REPRODUTSEERIV VALEM VEKTORVÄÄRTUSTEGA FUNKTSIOONIDELE ÜLE CARTANI PIIRKONNA

Jaak PEETRE

Olgu V d-mõõtmeline vektorruum üle kompleksarvude korpuse $\mathbf{C}$ ja $\mathfrak{D} \subset V$ selle Cartani piirkond, s. t. taandumatu ringikujuline tõkestatud sümmeetriline piirkond. On vaadeldud üle selle piirkonna $\mathfrak{D}$ V-väärtustega antiholomorfseid teatud kaaluga ν integreeruva ruuduga funktsioone ja leitud nende jaoks sobiv reprodutseeriv tuum.

Saadud tulemust on võimalik tõlgendada kui teoreemi teatud Hermite'i holomorfse vektorkihtkonna $T'^*\mathfrak{D} \otimes \mathfrak{L}^\nu$ holomorfsete lõigete kohta. Siinjuures on $T'^*\mathfrak{D}$ kompleks-kopuutuja kihtkond ning $\mathfrak{L}^\nu$ kaaluga ν ruumielementide joonkond.

Üldisemalt võib vaadelda ka tensorkorrutisi, nagu $T'^* D \otimes T'^*D \otimes L^\nu$ jne. Selliste kihtkondade lõiked esinevad teatud Hankeli vormide sümbolitena.

ВОСПРОИЗВОДЯЩАЯ ФОРМУЛА ДЛЯ ВЕКТОРОЗНАЧНЫХ ФУНКЦИЙ НАД КАРТАНОВОЙ ОБЛАСТЬЮ

Яак ПЕЭТРЕ

Пусть V d-мерное векторное пространство над полем комплексных чисел $\mathbf{C}$ и $\mathfrak{D} \subset V$ — его Картанова область, т. е. неприводимая круглая ограниченная симметричная область. Рассматриваются функции над этой областью $\mathfrak{D}$ со значениями в V, антиголоморфные с интегрируемым квадратом, имеющим некоторый вес ν, и находится подходящее для них воспроизводящее ядро.

Этот результат может быть истолкован как теорема о голоморфных сечениях некоторого эрмитового голоморфного расслоения $T'^*\mathfrak{D} \otimes \mathfrak{L}^\nu$. Здесь $T'^*\mathfrak{D}$ — комплексное кокасательное расслоение и $\mathfrak{L}^\nu$ — линейное расслоение пространственных элементов весом ν.

В общем случае так же можно рассматривать и тензорное произведение $T'^* \mathfrak{D} \otimes T'^*\mathfrak{D} \otimes \mathfrak{L}^\nu$ и т. д. Сечения таких расслоений являются символами некоторых Ганкелевых форм.