Proc. Estonian Acad. Sci. Phys. Math., 1990, 39, N 3, 276—281
https://doi.org/10.3176/phys.math.1990.3.12

УДК 624.072

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STATICAL BEHAVIOUR OF STRAIGHT CABLES UNDER PRIMARY AND SECONDARY CROSS LOADING

Attention is focussed on statical behaviour of prestressed straight cables (strings) under cross loading. On the first step of loading the initial form of the cable is a straight line. With secondary loading we have to proceed from the deformed shape and increased inner forces within the cable. It means that the behaviour of a string under secondary loading may be characterized by equations used for a prestressed curved cable (thread). But a member should be introduced for cables taking into account the doubled prestresses, caused by initial prestressing forces as well as by primary cross loads. Apart from that, the influence of support displacements has been analysed.

1. Primary loading

The elongation of a string may be expressed by a geometrical equation as follows

$$\varepsilon=\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2,$$

and using Hooke's law as

$$\varepsilon=\frac{H-H_0}{EA}.$$

After equalizing both values of ε and integration, we may write for symmetrical cross loading of the string (Fig. 1)

$$\int_0^a \frac{du}{dx}\,dx+\frac{1}{2}\int_0^a\left(\frac{dw}{dx}\right)^2 dx=\frac{(H-H_0)a}{EA}, \tag{1}$$

where H_0 and H — the inner force of the string before and after loading, respectively; u — displacement of the string in the direction of axis x; w — ordinate of the string after loading; EA — stiffness of the string in tension.

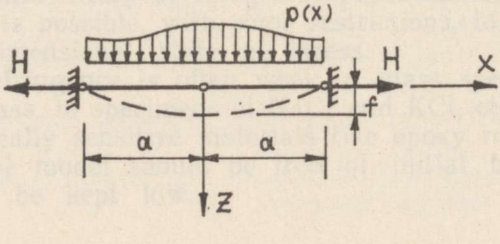

Fig. 1.

The first member of equation (1) represents the horizontal displacement of the supporting point

$$\int_0^a \frac{du}{dx}\, dx = u_{x=a}. \tag{2}$$

Assuming the linear relation between forces and displacements of supports, we may write

$$u_{x=a} = (H - H_0)\, u_1, \tag{3}$$

where u_1 — displacement of the support under the action of unit load. So we may eliminate the horizontal displacement and equation (1) may be written in the form

$$\int_0^a \left(\frac{dw}{dx} \right)^2 dx = \frac{2(H - H_0)}{EA} \left(1 + EA \frac{u_1}{a} \right). \tag{4}$$

From the condition of equilibrium (Fig. 2) $\sum z = 0$ we obtain for vertical loading

$$\frac{d^2 w}{dx^2} = \frac{p_1(x)}{H}. \tag{5}$$

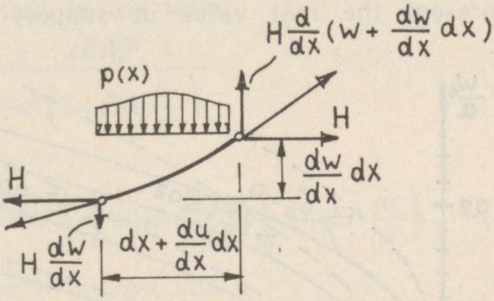

Fig. 2.

After double integration and taking into account the boundary conditions ($x=0$, $w=w_0$; $x=a$, $w=0$) we obtain for the uniformly distributed load

$$w = f\left(1 - \frac{x^2}{a^2} \right), \tag{6}$$

and

$$H = \frac{p_1 a^2}{2f}, \tag{7}$$

where f — deflection of the string.

Now, taking into account equation (4) we may write for the string deflection f

$$\left(\frac{f}{a} \right)^3 + \frac{3H_0}{2EA} \left(1 + EA \frac{u_1}{a} \right) \frac{f}{a} = \frac{3pa}{4EA} \left(1 + EA \frac{u_1}{a} \right). \tag{8}$$

So we have completed the equation of a string by members, taking into account displacement of supports. To illustrate the influence of supports displacements on the deflection of a string, let us determine the unit displacement for a support with vertical pylon and inclined anchor cable (Fig. 3). In view that stiffness of the pylon is much bigger than the anchor cable, we may write for the unit displacement

$$u_1 = \frac{b}{E_a A_a \cos^3 \beta}, \tag{9}$$

where $E_a A_a$ — rigidity of the anchor cable in tension. So, in our case equation (8) obtains the form

$$\left(\frac{f}{a}\right)^3 + \frac{3H_0}{2EA}\left(1+\frac{EAb}{E_a A_a a \cos^3 \beta}\right)\frac{f}{a} = \frac{3pa}{4EA}\left(1+\frac{EAb}{E_a A_a a \cos^3 \beta}\right). \quad (10)$$

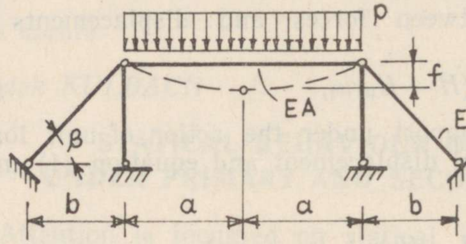

Fig. 3.

The second member after unit in the brackets of formula (9) shows how important displacements of cable supports are. For example, if we have $\beta=45°$, $EA=E_a A_a \cos \beta$ and $b=0.5a$, the value of the member in the brackets will be doubled as compared to the case of rigid supports. A plot of deflections is presented in Fig. 4; Fig. 5 shows a plot of cable forces as functions of load parameters with different values of prestressing forces. Continuous lines demonstrate rigid supports, while broken lines represent the real value of support rigidity.

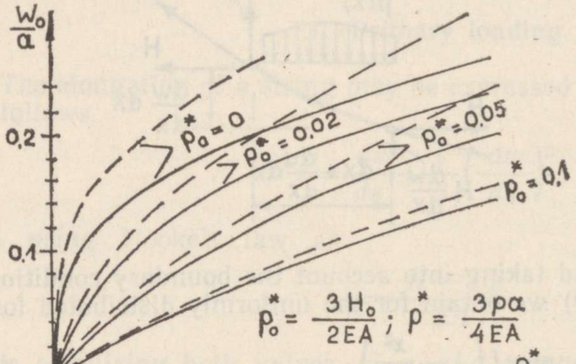

Fig. 4.

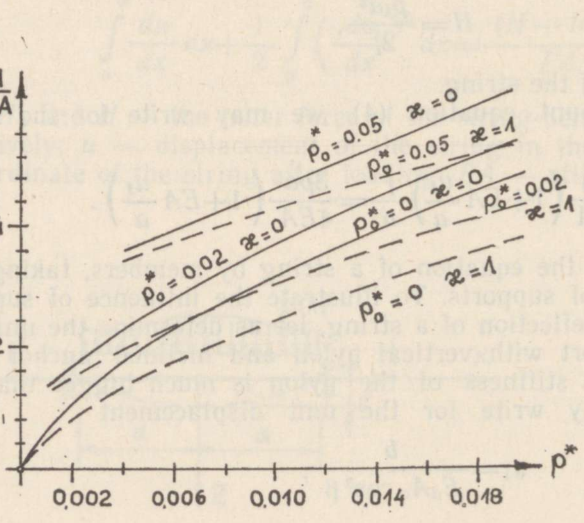

Fig. 5.

Displacements of cable supports may be taken into account in the same way when the strings equation is presented by inner forces. The corresponding equation may be written in the completed form as follows

$$\left(\frac{H}{EA}\right)^3-\frac{H_0}{EA}\left(\frac{H}{EA}\right)^2=\frac{1}{6}\left(\frac{pa}{EA}\right)^2\frac{1}{1+EA\frac{u_1}{a}}. \tag{11}$$

2. Secondary loading

For secondary loading the string must be taken as a doubly prestressed cable, having initial deflection. For a parabolic cable, loaded by additional uniformly distributed load p_2, the additional deflection w_0 may be determined by the cubic equation [1]

$$\left(\frac{w_0}{f}\right)^3+3\left(\frac{w_0}{f}\right)^2+2\frac{w_0}{f}+\frac{3H_1a^2}{2EAf^2}\left(1+EA\frac{u_1}{a}\right)\frac{w_0}{f}=$$

$$=\frac{3p_2a^4}{4EAf^3}\left(1+EA\frac{u_1}{a}\right), \tag{12}$$

where w_0 — additional deflection of the string, H_1 — cable force before the secondary loading.
The initial force may be presented in the form

$$H_1=H_0+\frac{2EAf^2}{3a^2\left(1+EA\frac{u_1}{a}\right)}.$$

Now we obtain from (11)

$$w_0^3+3fw_0^2+3f^2w_0+\frac{3H_0a^2}{2EA}\left(1+EA\frac{u_1}{a}\right)w_0=\frac{3p_2a^4}{4EA}\left(1+EA\frac{u_1}{a}\right). \tag{13}$$

For the first step of loading the string we had

$$f^3+\frac{3H_0a^2}{2EA}\left(1+EA\frac{u_1}{a}\right)f=\frac{3p_1a^4}{4EA}\left(1+EA\frac{u_1}{a}\right). \tag{14}$$

At the same time, applying formula (8) to the total load p_1+p_2, we obtain

$$(f+w_0)^3+\frac{3H_0a^2}{2EA}\left(1+EA\frac{u_1}{a}\right)(f+w_0)=\frac{3(p_1+p_2)a^4}{4EA}\left(1+EA\frac{u_1}{a}\right). \tag{15}$$

It is easy to see that by summing up (13) and (14) we obtain a formula, which coincides with formula (15) for a prestressed string under the action of summary load p_1+p_2.

We may conclude that the cubic equation for a curved cable may be applied for a string under secondary loading if doubled initial forces are taken into account. The same result will be obtained if the primary cross load is neglected, and the equation for curved cable with changed coefficient before the third member of the cable equation (Formula 12) is used.

Applying the equation for a curved cable to the string with uniformly distributed primary load p_1 and with secondary load on the right half of the span p_2, the following cubic equation is obtained for cable deflection

$$\varkappa\left(\frac{w_0}{f}\right)^3+3\varkappa\left(\frac{w_0}{f}\right)^2+\left[3\varkappa+\frac{3H_0a^2}{2EAf^2}\left(1+EA\frac{u_1}{a}\right)\right]\frac{w_0}{f}+\varkappa-1=$$

$$=\frac{3p_2a^4}{8EAf^3}\left(1+EA\frac{u_1}{a}\right), \tag{16}$$

where $\varkappa = 1 + \frac{p_2^2}{4(2p_1+p_2)^2}$ — the load factor, H_0 — the initial inner force of the string before the first step of loading.

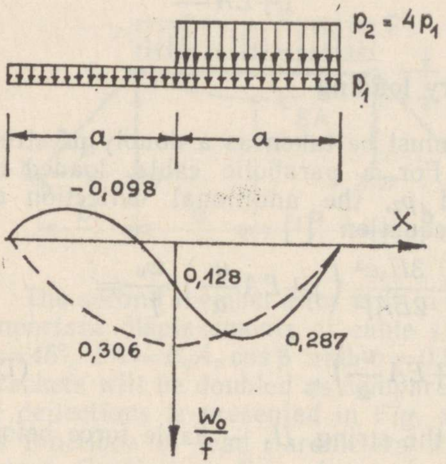

Fig. 6.

On the graphs in Fig. 6 deflections of a cable under the action of loads on the whole span and on the right half of it may be compared.

3. Conclusion

The significance of support displacements on deflections and inner forces of the cable is emphasized by nonlinear analysis of statical behaviour of straight cables. It has been verified that the principle of superposition of the stress-strain state is applicable not only to linear systems, but to geometrically nonlinear structures (including cable systems) as well. It enables to analyse stress-strain state, dividing loads into certain parts and to calculate deflections and inner forces step by step [2]. For example, in many cases it is suitable to divide nonsymmetrical loads into symmetrical and antisymmetrical parts. But it is absolutely necessary for calculating the inner forces and deflections for the secondary loading to proceed from changed initial form and inner forces. It is shown here that the principle of superposition is applicable for prestressed straight cables. In the second step of loading the cable must be observed not as a string, but as a curved thread.

REFERENCES

1. *Кульбах В. Р.* Вопросы статического расчета висячих систем. Таллинн, Изд. Таллиннск., политехн., ин-та, 1970.
2. *Aare, J., Kulbach, V.* // J. Struct. Mech., 1984, 17, № 3.

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Received
Feb. 26, 1990

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SIRGETE TROSSIDE KÄITUMINE ESMA- JA TEISTKORDSE STAATILISE PÕIKKOORMUSE MÕJUL

Artiklis on peatähelepanu pööratud eelpingestatud, põikkoormusega elastsetele sirgetele trossidele (keeltele). Esimesel koormusastmel on trossi algkujuks sirgjoon, kuid trossi läbipainde ja sisejõu määramiseks teise koormusastme rakendamisel tuleb lähtuda kõverast algkujust ja suurenenud eelpingest. Seega ei tule sirget trossi teisel koormusastmel vaadata mitte kui keelt, vaid kui elastset niiti, võttes arvesse kahekordset eelpingejõudu (sh. keele eelpinget ja esimese koormusastme mõjul tekkivat lisajõudu). Paralleelselt nimetatud ülesandega on lahendatud ka keele (niidi) tugede siirete arvestamine. Toodud materjalist selgub, et staatilisel koormamisel saab pingeolukorra superpositsiooni printsiipi laiendada ka geomeetriliselt mittelineaarsetele süsteemidele.

Вальдек КУЛЬБАХ

ПОВЕДЕНИЕ ПРЯМЫХ КАНАТОВ ПОД ДЕЙСТВИЕМ ПЕРВИЧНОЙ И ВТОРИЧНОЙ ПОПЕРЕЧНОЙ НАГРУЗОК

Основное внимание в статье обращено предварительно напряженным упругим канатам (струнам). На первом этапе приложения нагрузки за исходную форму каната принимается прямая линия, на втором этапе следует исходить из искривленной формы каната и увеличенного начального усилия. Таким образом, прямой канат на втором этапе загружения следует рассматривать не как струну, а как упругую нить. При этом следует иметь в виду двойное предварительное напряжение нити, в том числе преднапряжение струны и дополнительное напряжение от нагрузки первого этапа загружения. Параллельно с указанной задачей решена задача учета перемещения опор струны (нити). Из представленного материала также явствует, что в условиях статического загружения возможно распространение принципа суперпозиции напряженного состояния на геометрические нелинейные системы.