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MODEL MATCHING OF LINEAR-ANALYTIC DISCRETE-TIME SYSTEMS VIA DYNAMIC STATE FEEDBACK

(Presented by Ü. Jaaksoo)

1. Introduction

The model matching problem, i. e, the problem of compensating a system in order to obtain the same input-output behavior as the one of a prespecified model, has attracted a great deal of interest during the last two decades. Most papers consider linear systems and a number of results have been obtained for continuous time nonlinear systems either by tools of differential geometry [1, 2], differential algebra [3], by the so-called structure algorithm [4], or by zeroing the output of the extended system, i. e. considering the model matching problem as the disturbance decoupling problem [5,6].

To the author's knowledge, no papers have been written on the topic of model matching of nonlinear discrete-time systems. However, the related problems of input-output linearization [7] and decoupling [8] have been studied. In these cases the model is respectively linear or decoupled, but otherwise entirely arbitrary. Here lies the main difference from model matching problem in which case the model is prefixed.

This paper deals with the model matching problem for discrete-time linear-analytic systems characterized by rather restrictive conditions. Actually, we consider the same class of systems as in [7, 8]. The justification for considering this class of systems lies in the fact that the problem of model matching has in this case a closed-form solution. The solution of model matching problem is obtained via the right inverse system. A similar approach is followed in [9] to problems of input-output linearization and decoupling.

2. Problem statement

Consider the nonlinear discrete-time system described by equations

$$x(t+1)=x(t)+f_0(x(t))+\sum_{i=1}^{m} f_i(x(t))u_i(t), \quad x(0)=x_0, \tag{1}$$

$$y(t)=h(x(t)),$$

where the state $x(t)\in R^n$, $u(t)=[u_1(t)\ldots u_m(t)]^{\mathrm{T}}$ is the m-dimensional input vector, $y(t)=[y_1(t)\ldots y_p(t)]^{\mathrm{T}}$ is the p-dimensional output vector $(p\leqslant m)$, $f_i: R^n\to R^n$, $i=0, 1, \ldots, m$ and $h: R^n\to R^p$ are analytic functions on R^n.

In this paper we shall study the problem of using dynamic-state feedback in order to make the input-dependent part of the input-output behavior of (1) the same as that of a prespecified nonlinear system described by equations

$$z(t+1)=z(t)+g_0(z(t))+\sum_{i=1}^{p} g_i(z(t))v_i(t), \ z(0)=z_0, \tag{2}$$

$$y^*(t)=h^*(z(t)),$$

where $z(t)\in R^l$, $v(t)=[v_1(t)\dots v_p(t)]^T\in R^p$, $y^*(t)\in R^p$, $g_i: R^l\to R^l$ and $h^*: R^l\to R^p$ are analytic functions on R^l. The dynamic state feedback control law is described by equations of the form

$$\xi(t+1)=\xi(t)+a(\xi(t), x(t))+\sum_{i=1}^{p} b_i(\xi(t), x(t))v_i(t),$$
$$u(t)=c(\xi(t), x(t))+d(\xi(t), x(t))\,v(t) \tag{3}$$

in which $\xi(t)\in R^\nu$ and the entries of $a, b_1, \dots, b_p, c$ and d are real analytic functions defined on a suitable open and dense subset of $R^\nu\times R^n$.

3. Preliminaries

This section briefly describes the tools which will be used in the sequel. The more detailed presentation of this material can be found in [10]. With a reference to the analytic functions $f(x): R^n\to R^n$ and $g(x): R^n\to R^n$, let us introduce the following differential operators:

$$\overset{\otimes\circ}{L_f}=I,$$
$$\overset{\otimes k}{L_f}=\sum_{i_1,\dots,i_k=1}^{n} f_{i_1}\dots f_{i_k}\frac{\partial^k}{\partial x_{i_1}\dots\partial x_{i_k}},\quad k\geqslant 1,$$
$$\Delta_f=\sum_{k\geqslant 0}\frac{1}{k!}\overset{\otimes k}{L_f}, \tag{4}$$
$$\overset{\otimes k}{L_f}\otimes\overset{\otimes r}{L_g}=\sum_{i_1,\dots,j_r=1}^{n} f_{i_1}\dots f_{i_k}g_{j_1}\dots g_{j_r}\frac{\partial^{k+r}}{\partial x_{i_1}\dots\partial x_{j_r}},$$
$$k\geqslant 1,\quad r\geqslant 1,$$

where f_i, g_j, $i, j=1,\dots,n$, and I denote the ith component of f, jth component of g and the identity operator, respectively. The differential operator Δ_f can be used to express the composition of functions. More precisely, if $f: R^n\to R^n$ and $h: R^n\to R^p$ are two analytic functions on R^n, then the composition (denoted by «$\circ$») of h with $I+f$ can be expressed as

$$h\circ(I+f)x=\Delta_f h|_x,$$

where $|_x$ denotes the evaluation at x. More generally, if $f_i: R^n\to R^n$, $i=1,\dots,k$, then

$$h\circ(I+f_k)\circ\dots\circ(I+f_1)x=\Delta_{f_1}\circ\dots\circ\Delta_{f_k}h|_x.$$

The differential operator (4), the so-called operator exponent, is suitable to represent the evolution of the system (1) which involves the compositions of functions. The output evolution in t step can be written as

$$y(t)=\Delta_{f_0+Fu(0)}\circ\dots\circ\Delta_{f_0+Fu(t-1)}h|_{x_0},$$

where $F=[f_1\dots f_m]$.

For each output y_i, $i=1,\dots,p$, let d_i be the delay order defined as the smallest integer such that the following two conditions hold [7]:

1) $\Delta_{f_0}\otimes L_{f_{i_\nu}}\otimes\dots\otimes L_{f_{i_1}}\circ\Delta^r_{f_0}h_i|_x=0,$

$$\forall x,\ \forall r<d_i,\ \forall\nu,\ \forall i_1,\dots,i_\nu\in\{1,\dots,m\};$$

2) $\exists\mu$ and a sequence $i_1,\dots,i_\mu$ such that

$$\Delta_{f_0}\otimes L_{f_{i_\mu}}\otimes\dots\otimes L_{f_{i_1}}\circ\Delta^{d_i}_{f_0}h_i|_x\neq 0$$

$\forall x \in V$, where V is some open subset of R^n. Here $\Delta_f^r = \Delta_f \circ \ldots \circ \Delta_f$ (r times).

It can be shown that $t = d_i + 1$ is the first instant of time at which the ith output is affected by at least one input at time $t = 0$.

In the sequel we shall consider only the systems for which the condition (5) holds:

$$\Delta_{f_0} \otimes L_{f_{i_\nu}} \otimes \ldots \otimes L_{f_{i_1}} \circ \Delta_{f_0}^{d_i}(h_i)\,|_x = 0, \tag{5}$$

$$\forall x \in V, \forall \nu \geqslant 2, i_1, \ldots, i_\nu \in \{1, \ldots, m\}, i = 1, \ldots, p.$$

The class of systems we consider includes bilinear systems and nonlinear systems with linear output function if all orders of delay $d_i = 0$. Of course, this class of systems is quite restrictive but it is characterized by the following fact. Provided the system is right invertible, the problem considered here and the related problems of decoupling, feedback linearization and disturbance decoupling have the closed-form solutions.

4. Right inverse system

The right invertibility problem can be defined in the following way. The system S is right invertible if there exists another system S_R^{-1} such that the input-output map of the composition of S_R^{-1} and S is the identity map,

$$S \circ S_R^{-1} = I_p.$$

It never holds, however, that a system (1) is right invertible in the above sense. Consequently, the notion of the delay right invertibility will be used here. We say that the system S is $(d_1+1, \ldots, d_p+1)$ — delay right invertible, if there exists another system S_R^{-1} such that

$$S \circ S_R^{-1} = \operatorname{diag}\{z^{-d_1-1} \ldots z^{-d_p-1}\} I_p.$$

Here z^{-1} is the delay operator, $z^{-1}y(t) = y(t-1)$. Let us introduce the matrix $A(x) = [a_{ij}(x)]$, $i = 1, \ldots, p$, $j = 1, \ldots, m$ and the vector $\delta(x) = [\delta_k(x)]$, $k = 1, \ldots, p$ defined by

$$a_{ij}(x) = \Delta_{f_0} \otimes L_{f_j} \circ \Delta_{f_0}^{d_i}(h_i)\,|_x,$$

$$\delta_k(x) = \Delta_{f_0}^{d_k+1} h_k\,|_x.$$

Theorem 1 [11]. *Consider the system* (1) *for which the condition* (5) *holds: If the rank of the $p \times m$ matrix $A(x)$ is equal to p in some open and dense subset V of R^n, then the system* (1) *has in V a $(d_1+1, \ldots, d_p+1)$ — delay right inverse, which is defined by equations*

$$\begin{aligned} x(t+1) = {} & x(t) + f_0(x(t)) - F(x(t))A^R(x(t))\delta(x(t)) + \\ & + F(x(t))[I_m - A^R(x(t))A(x(t))]g(x(t)) + \\ & + F(x(t))A^R(x(t))\tilde{u}(t), \end{aligned} \tag{6}$$

$$\begin{aligned} \tilde{y}(t) = {} & -A^R(x(t))\delta(x(t)) + A^R(x(t))\tilde{u}(t) + \\ & + [I_m - A^R(x(t))A(x(t))]g(x(t)), \end{aligned} \tag{7}$$

where $A^R = A^T(AA^T)^{-1}$ and the vector function $g(x)$ is arbitrary. In the sequel we will assume that the assumptions of the Theorem hold.

5. Problem solution via right inverse system

We look for the feedback control law such that the input-output behavior of the closed-loop system coincides with the input-output behavior of the model system (2). Let the input-output maps of the original system and model system be denoted by S and M respectively, and the input-output map of the compensator be denoted by C. In that case we have

$$S \circ C = M.$$

If we choose

$$C = S_R^{-1} \circ \{z^{d_1+1} \ldots z^{d_p+1}\} I \circ M,$$

we attain the desired goal. So, to obtain the required control law, we must feed into the output equation of the right inverse system the appropriate shifts of the outputs of the model system $Y^*(t) = [y^*_1(t+d_1+1) \ldots y^*_p(t+d_p+1)]^{\mathrm{T}}$ and then express $Y^*(t)$ by $z(t)$ and $v(t)$.

Let us feed into the output equation of the right inverse system (7) the appropriate shifts of the outputs of the model system (2)

$$\begin{aligned} u(t) = {} & -A^R(x(t))\delta(x(t)) + [I_m - A^R(x(t))A(x(t))]g(x(t)) + \\ & + A^R(x(t))[y^*_1(t+d_1+1) \ldots y^*_p(t+d_p+I)]^{\mathrm{T}}. \end{aligned} \tag{8}$$

Let us assume that the characteristic numbers of the model system d^*_i are equal or greater than those of the original system, i. e.

$$\Delta_{g_0} \otimes L_{g_{i_\nu}} \otimes \ldots \otimes L_{g_{i_1}} \circ \Delta^r_{g_0} h^*_i|_z = 0, \tag{9}$$

$$\forall z, \forall r < d_i, \ \forall \nu, \ \forall i_1, \ldots, i_\nu \in \{1, \ldots, p\}, \ i = 1, \ldots, p.$$

Moreover, assume that either $d^*_i > d_i$ or, if not, $y^*_i(t+d_i+1)$ is also linear in the input $v(t)$ $(i = 1, \ldots, p)$. Then we obtain from equations of system (2)

$$y^*_i(t+d_i+1) = \Delta^{d_i+1}_{g_0} h^*_i|_z + \sum_{j=1}^{p} \Delta_{g_0} \otimes L_{g_j} \circ \Delta^{d_i}_{g_0} h^*_i|_z v_j(t), \qquad i = 1, \ldots, p. \tag{10}$$

The equations (8) and (10) show that the dynamic feedback

$$\begin{aligned} z(t+1) = {} & z(t) + g_0(z(t)) + \sum_{i=1}^{p} g_i(z(t)) v_i(t), \\ u(t) = {} & -A^R(x(t))\delta(x(t)) + [I_m - A^R(x(t))A(x(t))]g(x(t)) + \\ & + A^R(x(t)) N(z(t)) + A^R(x(t)) M(z(t)) v(t), \end{aligned} \tag{11}$$

where

$$N(z(t)) = [n_i(z)], \quad i = 1, \ldots, p, \quad M(z(t)) = [m_{ij}(z)],$$

$$i, j = 1, \ldots, p,$$

$$n_i(z) = \Delta^{d_i+1}_{g_0} h^*_i|_z,$$

$$m_{ij}(z) = \Delta_{g_0} \otimes L_{g_j} \circ \Delta^{d_i}_{g_0} h^*_i|_z$$

probably solves the model matching problem. We will formulate this result as a Theorem.

Theorem 2. *Consider the system* (1) *for which the condition* (5) *holds and the* rank *of the* $p\times m$ *matrix* $A(x)$ *is equal to* p *in some open and dense subset* V *of* R^n. *If the delay orders of the model system* (2) *are equal or greater than those of the original system and in case of their equality also for model* (2) *condition* (5) *holds, then the model matching problem can be solved by dynamic state feedback given by equations* (11).

The proof of Theorem 2 is given in the Appendix.

Notation 1. If the characteristic numbers of the model system are equal to those of the original system, yet the condition (5) does not hold for the model (2), then the model matching problem is still solvable, only the dynamic feedback becomes more complicated. Namely, it will be a polynomial in $v_j(t)$, $j=1,\ldots,p$.

Notation 2. From the condition

$$y(t+i)=y^*(t+i)$$

it follows

$$Q(z(t))=H(x(t)), \tag{12}$$

where

$$Q^{\mathrm{T}}(z)=[h_1^*,\ \Delta_{g_0}h_1^*,\ \ldots,\ \Delta_{g_0}^{d_1}h_1^*,\ \ldots,\ h_p^*,\ \Delta_{g_0}h_p^*,\ \ldots,\ \Delta_{g_0}^{d_p}h_p^*],$$

$$H^{\mathrm{T}}(x)=(h_1,\ \Delta_{f_0}h_1,\ \ldots,\ \Delta_{f_0}^{d_1}h_1,\ \ldots,\ h_p,\ \Delta f_0 h_p,\ \ldots,\ \Delta_{f_0}^{d_p}h_p].$$

Let us denote $\sum_{i=1}^{p}(d_i+1)$ by μ. If the $\mu\times l$ matrix $\partial Q(z)/\partial z$ has rank μ, then, from the equations (12), one can recover μ components of z, denoted by z_1 and expressed as functions of x, and of the remaining $l-\mu$ components, denoted by z_2: $z_1=\varphi(x,z_2)$.

In that case the dynamics of compensator (11) can be replaced by reduced order dynamics

$$z_2(t+1)=z_2(t)+\bar g_0(z_2(t),\ x(t))+\sum_{i=1}^{p}\bar g_i(z_2(t),\ x(t))v_i(t).$$

This compensator is equivalent to compensator (11), provided $z_1(0)=\varphi(x(0),\ z_2(0))$.

If $l=\mu$, then $z_1=z=\psi(x)$ and the static state feedback law

$$u(t)=-A^R(x(t))\delta(x(t))+[I_m-A^R(x(t))A(x(t))]g(x(t))+$$
$$+A^R(x(t))N(\psi(x))+A^R(x(t))M(\psi(x))v(t)$$

solves the model matching problem. Note that in the last case the delay orders of the model must coincide with those of the system, otherwise $l>\mu$.

Notation 3. Our paper can be interpreted as a generalization of the results of [7], where the linear model is considered. Moreover, they assume that $d_i^*=d_i$ and $l=\mu$. In any other sense their model is arbitrary.

Monaco and Normand-Cyrot [7] do not assume the full row rank of the matrix $A(x)$. Instead, they require the solvability of the system of equations

$$A(x)[\alpha(x)\ \vdots\ \beta(x)]=[\delta(x)\ \vdots\ M], \tag{13}$$

where M is a constant matrix whose elements are defined by the model. The device for computing the solution of this system is the generalized inverse $A^-(x)$ of the matrix $A(x)$. The equation (13) has a solution if and only if the consistency condition for the equation (13)

$$[I_p-A(x)A^-(x)][\delta(x)\ \vdots\ M]=0$$

is satisfied [12]. If the matrix $A(x)$ has a full row rank then $A^R = A^T(AA^T)^{-1}$ serves as A^-, and the consistency condition is automatically satisfied. So, the assumption of Monaco and Normand-Cyrot [7] is less restrictive than ours, and it extends the possibilities if the model is arbitrary. But it does not contribute anything essential to the case of fixed models: for systems which have not a full row rank matrix $A(x)$, the amount of models which can be matched is very small. This is one of the reasons we have chosen the assumption of a full row rank. The other reason is that our paper can also be interpreted as a generalization of the results on input-output decoupling [8]. In this paper the assumption of the full row rank is essential.

6. An example

Consider the process which produces single-cell protein from yeast grown on methanol. This process is described by the following equations [13]

$$x_1(t+1) = x_1(t) + H\mu_m \frac{x_2(t)}{K+x_2(t)} x_1(t) - Hx_1(t)u_1(t),$$

$$x_2(t+1) = x_2(t) - \frac{H}{R}\mu_m \frac{x_2(t)}{K+x_2(t)} x_1(t) - Hx_2(t)u_1(t) + Hu_2(t),$$

$$y_1(t) = x_1(t),$$

$$y_2(t) = x_2(t).$$

The delay orders of the system are $d_1 = 0$, $d_2 = 0$. The condition (5) is satisfied and the matrix $A(x)$ is given by

$$A(x) = \begin{bmatrix} -Hx_1 & 0 \\ -Hx_2 & H \end{bmatrix}$$

which is nonsingular in the subspace $V = R^2 - \{x/x_1 \neq 0\}$. Note that by x_1 is denoted the biomass concentration and, therefore, we may assume that it is not equal to zero. The inverse system is given by equations

$$x(t+1) = x(t) + [y_1(t+1),\ y_2(t+1)]^T,$$

$$u_1(t) = \mu_m x_2(t)/(K+x_2(t)) - y_1(t+1)/(Hx_1(t)),$$

$$u_2(t) = \mu_m x_2(t)(Rx_2(t)+x_1(t))/R(K+x_2(t)) - $$
$$- x_2(t)y_1(t+1)/(Hx_1(t)) - y_2(t+1)/H.$$

In case of the model

$$z_1(t+1) = z_1(t) + v_1(t),$$

$$z_2(t+1) = z_2(t) + z_2(t)v_1(t) + v_2(t),$$

$$y_1^*(t) = z_1(t),$$

$$y_2^*(t) = z_2(t)$$

the delay orders of the model are $d_1^* = d_2^* = 0$, $\mu = l = 2$ and the static state feedback

$$u_1(t) = \mu_m x_2(t)/(K+x_2(t)) - 1/H - v_1(t)/(Hx_1(t)),$$

$$u_2(t) = \mu_m x_2(t)(Rx_2(t)+x_1(t))/(R(K+x_2(t))) - x_2(t)/H -$$
$$- (x_2(t)/(Hx_1(t)) + x_2(t)/H)v_1(t) - v_2(t)/H$$

solves the model-matching problem.

In case of the model

$$\begin{aligned}
&z_1(t+1)=z_2(t),\\
&z_2(t+1)=z_1(t)+v_1(t),\\
&z_3(t+1)=z_4(t),\\
&z_4(t+1)=z_3(t)+v_2(t),\\
&y_1^*(t)=z_1(t),\\
&y_2^*(t)=z_3(t),
\end{aligned}$$

the delay orders of the model are $d_1^*=d_2^*=1$ and the dynamic feedback

$$z_2(t+1)=x_1(t)+v_1(t),$$

$$z_4(t+1)=x_3(t)+v_2(t),$$

$$u(t)=\mu_m\frac{x_2(t)}{K+x_2(t)}\begin{bmatrix}1\\ x_2(t)+\dfrac{x_1(t)}{R}\end{bmatrix}-\frac{1}{H}\begin{bmatrix}\dfrac{x_2(t)}{x_1(t)}\\ \dfrac{x_2(t)}{x_1(t)}z_2(t)+z_4(t)\end{bmatrix}$$

solves the model-matching problem.

7. Conclusions

Here we have tackled the problem of compensating a multivariable linear-analytic discrete-time system in order to obtain the same input-output behavior as the one of the prespecified linear-analytic discrete-time model. The considered systems are assumed to be
1) $(d_1+1, \ldots, d_p+1)$ — delay right invertible,
2) such that $y_i(t+d_i+1)$, $i=1, \ldots, p$ are linear in the input $u(t)$.

For this subclass, the given problem is solvable by dynamic state feedback which is linear in the new input if the orders of delay of the model system are equal or greater than those of the original system and in case of their equality $y_i^*(t+d_i+1)$ is also linear in the input $v(t)$.

It turns out that the concept of right inverse plays a central role in the solution of this problem. In fact, the required feedback law can be given by the output equation of the right inverse system if we feed into it the appropriate shifts of outputs of the prespecified model system coupled with the equations of the model.

If the second assumption does not hold, either for original system (1) or for model system (2) in case $d_i=d_i^*$, the problem is still solvable, but in that case the feedback is polynomial in the new input. However, we have not considered these cases here.

Appendix: Proof of Theorem 2

The closed-loop system (1), (11) is also the linear-analytic system

$$\begin{aligned}
&\tilde{x}(t+1)=\tilde{x}(t)+\tilde{f}_0(\tilde{x}(t))+\tilde{F}(\tilde{x}(t))v(t),\\
&y(t)=\tilde{h}(\tilde{x}(t)),
\end{aligned}\tag{14}$$

where

$$\tilde{x}^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t), z^{\mathrm{T}}(t)],$$

$$\tilde{f}_0(\tilde{x})=\begin{bmatrix}f_0(x)+\sum\limits_{i=1}^{m}f_i(x)\alpha_i(x,z)\\ g_0(x)\end{bmatrix},$$

$$F(\tilde{x})=[\tilde{f}_1(\tilde{x})\ldots\tilde{f}_p(\tilde{x})]=\begin{bmatrix}\sum_{i=1}^{m}f_i(x)\beta_{i1}(x,z)\ldots\sum_{i=1}^{m}f_i(x)\beta_{ip}(x,z)\\ g_1(z)\ldots\ldots\ldots\ldots g_p(z)\end{bmatrix},$$

$$\tilde{h}(\tilde{x})=h(x),$$

$$[\alpha_1(x,z)\ldots\alpha_m(x,z)]^{\mathrm{T}}=-A^R(x)\delta(x)+A^R(x)N(z)+ +[I_m-A^R(x)A(x)]g(x), \tag{15}$$

$$\begin{bmatrix}\beta_{11}(x,z)&\ldots&\beta_{1p}(x,z)\\ \ldots&\ldots&\ldots\\ \beta_{m1}(x,z)&\ldots&\beta_{mp}(x,z)\end{bmatrix}=A^R(x)M(z). \tag{16}$$

The input-output behavior of the linear-analytic systems (2) and (14) can be described by Volterra series [10]

$$y(t+1)=w_0(t+1)+\sum_{i_1=1}^{m}\sum_{\tau_1=0}^{t}w_{i_1}(t+1,\tau_1)u_{i_1}(\tau_1)+\ldots$$

$$\ldots+\sum_{i_1,\ldots,i_k=1}^{m}\sum_{\tau_1=0}^{t}\sum_{\tau_2=\tau_1}^{t}\ldots\sum_{\tau_k=\tau_{k-1}}^{t}w_{i_1\cdots i_k}(t+1,\tau_1,\ldots,\tau_k)\prod_{j=1}^{k}u_{i_j}(\tau_j)+\ldots,$$

where Volterra kernels of the model system (2) are

$$w_0(t+1)=\Delta_{g_0}^{t+1}h^*\big|_{z_0},$$

$$w_{i_1}(t+1,\tau_1)=\Delta_{g_0}^{\tau_1}\circ\Delta_{g_0}\otimes L_{g_{i_1}}\circ\Delta_{g_0}^{t-\tau_1}h^*\big|_{z_0},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$w_{i_1\cdots i_k}(t+1,\tau_1,\ldots,\tau_k)=\Delta_{g_0}^{\tau}\circ\Delta_{g_0}\otimes L_{g_{i_1}}\circ\Delta_{g_0}^{\tau_2-\tau_1-1}\circ$$

$$\circ\Delta_{g_0}\otimes L_{g_{i_2}}\circ\Delta_{g_0}^{\tau_3-\tau_2-1}\circ\ldots\circ\Delta_{g_0}\otimes L_{g_{i_k}}\circ\Delta_{g_0}^{t-\tau_k}h^*\big|_{z_0},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

For compact expression of the kernels, the following notation has been used (here Δ_{f}^{-1} has not been defined and carries no meaning)

$$\Delta_{g_0}\otimes L_{g_{i_1}}\circ\Delta_{g_0}^{-1}\circ\Delta_{g_0}\otimes L_{g_{i_2}}\circ\Delta_{g_0}^{-1}\circ\ldots\circ\Delta_{g_0}^{-1}\circ\Delta_{g_0}\otimes L_{g_{i_r}}=$$

$$=\frac{1}{r!}\Delta_{g_0}\otimes L_{g_{i_1}}\otimes L_{g_{i_2}}\otimes\ldots\otimes L_{g_{i_r}},\quad r\geqslant 2. \tag{17}$$

Volterra kernels of the closed-loop system (14) are

$$w_0(t+1)=\Delta_{\tilde{f}_0}^{t+1}\tilde{h}\big|_{\tilde{x}_0},$$

$$w_{i_1}(t+1,\tau_1)=\Delta_{\tilde{f}_0}^{\tau_1}\circ\Delta_{\tilde{f}_0}\otimes L_{\tilde{f}_{i_1}}\circ\Delta_{\tilde{f}_0}^{t-\tau_1}\tilde{h}\big|_{\tilde{x}_0},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$w_{i_1\cdots i_k}(t+1,\tau_1,\ldots,\tau_k)=\Delta_{\tilde{f}_0}^{\tau_1}\circ\Delta_{\tilde{f}_0}\otimes L_{\tilde{f}_{i_1}}\circ\Delta_{\tilde{f}_0}^{\tau_2-\tau_1-1}\circ$$

$$\circ\Delta_{\tilde{f}_0}\otimes L_{\tilde{f}_{i_2}}\circ\Delta_{\tilde{f}_0}^{\tau_3-\tau_2-1}\circ\ldots\circ\Delta_{\tilde{f}_0}\otimes L_{\tilde{f}_{i_k}}\circ\Delta_{\tilde{f}_0}^{t-\tau_k}\tilde{h}\big|_{\tilde{x}_0},$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Let us denote by $w_{i_1\cdots i_k}^{j}$ the jth component of the kernel $w_{i_1\cdots i_k}$.

We shall show that Volterra kernels of closed-loop system (14) will coincide with Volterra kernels of model system (2), except $w_0^i(t+1)$ for $t<d_i$.

Let us first observe that

$$\Delta_{\tilde f_0}\tilde h_i=\Delta_{f_0+\Sigma f_l\alpha_l}h_i=$$

$$=\sum_{s\geqslant 0}\sum_{\substack{n_1,\dots,n_m\geqslant 0\\ n_1+\dots+n_m=s}}\Delta_{f_0}\otimes L_{f_1}^{\otimes n_1}\otimes\ldots\otimes L_{f_m}^{\otimes n_m}h_i\frac{\alpha_1^{n_1}\ldots\alpha_m^{n_m}}{n_1!\ldots n_m!}.$$

If $d_i>0$, then

$$\Delta_{\tilde f_0}\tilde h_i=\Delta_{f_0}h_i.$$

Similarly,

$$\Delta_{\tilde f_0}\tilde h_i=\Delta_{f_0}^2 h_i,\ \ldots,\ \Delta_{\tilde f_0}^{d_i}\tilde h_i=\Delta_{f_0}^{d_i}h_i,$$

but

$$\Delta_{\tilde f_0}^{d_i+1}\tilde h_i=\Delta_{\tilde f_0}\circ\Delta_{f_0}^{d_i}h_i=\Delta_{f_0}^{d_i+1}h_i+\sum_{j=1}^{m}\alpha_j\Delta_{f_0}\otimes L_{f_j}\circ\Delta_{f_0}^{d_i}h_i$$

because of the assumption (5). Taking into account (15), we get

$$\Delta_{\tilde f_0}^{d_i+1}\tilde h_i=\Delta_{f_0}^{d_i+1}h_i-\Delta_{f_0}^{d_i+1}h_i+n_i(z)=\Delta_{g_0}^{d_i+1}h^*.$$

Furthermore,

$$\Delta_{\tilde f_0}^{d_i+2}\tilde h_i=\Delta_{\tilde f_0}\circ\Delta_{g_0}^{d_i+1}h_i^*=\Delta_{g_0}\circ\Delta_{g_0}^{d_i+1}h_i^*=\Delta_{g_0}^{d_i+2}h_i^*,$$

and so on.
Therefore,

$$\Delta_{\tilde f_0}^{t}\tilde h_i=\begin{cases}\Delta_{f_0}^t h_i, & \text{if}\quad t\leqslant d_i\\ \Delta_{g_0}^t h_i^*, & \text{if}\quad t>d_i.\end{cases}$$

Next, we shall show that

$$\Delta_{\tilde f_0}\otimes L_{\tilde f_{j_1}}\otimes\ldots\otimes L_{\tilde f_{j_k}}\circ\Delta_{\tilde f_0}^{t}\tilde h_i=\Delta_{g_0}\otimes L_{g_{j_1}}\otimes\ldots\otimes L_{g_{j_k}}\circ\Delta_{g_0}^t h_i^*.\qquad(18)$$

Really, if $t\leqslant d_i$, then

$$\Delta_{\tilde f_0}\otimes L_{\tilde f_{j_1}}\otimes\ldots\otimes L_{\tilde f_{j_k}}\circ\Delta_{f_0}^t\tilde h_i=$$

$$=\Delta_{f_0+\Sigma f_i\alpha_i}\otimes L_{\Sigma f_i\beta_{ij_1}}\otimes\ldots\otimes L_{\Sigma f_i\beta_{ij_k}}\circ\Delta_{f_0}^t h_i.$$

If $t<d_i$, the last expression is equal to zero because of the definition d_i. If $t=d_i$ and $k\geqslant 2$, then this expression is equal to zero because of the assumption (5). In both these cases

$$\Delta_{g_0}\otimes L_{g_{j_1}}\otimes\ldots\otimes L_{g_{j_k}}\circ\Delta_{g_0}^t h_i^*=0$$

because of the assumptions about the model system made in the Theorem 2.

If $t=d_i$ and $k=1$, this expression is equal to

$$\sum_{r=1}^{m} \beta_{rj} \Delta_{f_0} \otimes L_{f_r} \circ \Delta_{f_0}^{d_i} h_i = m_{ij}(z) = \Delta_{g_0} \otimes L_{g_j} \circ \Delta_{g_0}^{d_i} h_i^*$$

because of the assumption (5) and the equality (16). If $t>d_i$, then

$$\Delta_{\tilde{f}_0} \otimes L_{\tilde{f}_{j_1}} \otimes \ldots \otimes L_{\tilde{f}_{j_k}} \circ \overset{t}{\Delta}_{\tilde{f}_0} \tilde{h}_i = \Delta_{\tilde{f}_0} \otimes L_{\tilde{f}_{j_1}} \otimes \ldots \otimes L_{\tilde{f}_{j_k}} \circ \Delta^t_{g_0} h_i^* =$$

$$= \Delta_{g_0} \otimes L_{g_{j_1}} \otimes \ldots \otimes L_{g_{j_k}} \circ \Delta^t_{g_0} h_i^*.$$

Now, consider the ith component of the kth-order kernel $w^i_{j_1 \cdots j_k}$ $(t+1, \tau_1, \ldots, \tau_k)$ of the closed-loop system (14) (i and k arbitrary). If

$$\tau_k = \tau_{k-1} = \ldots = \tau_{k-r} > \tau_{k-r-1}, \quad r \geqslant 0,$$

then by (17) and (18)

$$w^i_{j_1 \cdots j_k}(t+1, \tau_1, \tau_2, \ldots, \tau_k) =$$

$$= \overset{\tau_1}{\Delta}_{\tilde{f}_0} \circ \Delta_{\tilde{f}_0} \otimes L_{\tilde{f}_{j_1}} \circ \overset{\tau_2-\tau_1-1}{\Delta}_{f_0} \circ \Delta_{\tilde{f}_0} \otimes L_{\tilde{f}_{j_2}} \circ \ldots \circ \overset{\tau_{k-r}-\tau_{k-r-1}-1}{\Delta}_{\tilde{f}_0} \circ$$

$$\circ \Delta_{g_0} \otimes L_{g_{j_{k-r}}} \otimes \ldots \otimes L_{g_{j_k}} \circ \overset{t-\tau_k}{\Delta}_{g_0} h_i^* =$$

$$= \overset{\tau_1}{\Delta}_{g_0} \circ \Delta_{g_0} \otimes L_{g_{j_1}} \circ \overset{\tau_2-\tau_1-1}{\Delta}_{g_0} \circ \Delta_{g_0} \otimes L_{g_{j_2}} \circ \ldots \circ \overset{\tau_{k-r}-\tau_{k-r-1}-1}{\Delta}_{g_0} \circ$$

$$\circ \Delta_{g_0} \otimes L_{g_{j_{k-r}}} \otimes \ldots \otimes L_{g_{j_k}} \circ \overset{t-\tau_k}{\Delta}_{g_0} h_i^*,$$

which coincides with the ith component of the kth-order kernel of the model (2).

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DISKREETSETE LINEAAR-ANALÜÜTILISTE SÜSTEEMIDE SOBITAMINE DÜNAAMILISE TAGASISIDEGA OLEKU JÄRGI

On vaadeldud mitme sisendi ja mitme väljundiga diskreetse lineaar-analüütilise süsteemi kompenseerimise ülesannet eesmärgiga saavutada kompenseeritud süsteemi sisend—väljund-kujutise kokkulangevus fikseeritud mudelsüsteemi sisend—väljund-kujutisega. On leitud nimetatud ülesande lahendamise piisavad tingimused $(d_1+1, \ldots, d_p+1)$ nihkega paremalt pööratavate süsteemide klassi jaoks ja nende tingimuste täidetuse korral tuletatud oleku järgi kompensaatori võrrandid dünaamilise tagasiside kujul. Otsitav tagasiside on määratud mudeli dünaamika võrrandite ja parempoolse pöördsüsteemi väljundvõrrandiga, kui viimase sisenditeks võtta mudelsüsteemi väljundid sobivate, mittelineaarse süsteemi struktuuri poolt määratud nihetega.

Юлле КОТТА

СОГЛАСОВАНИЕ ЛИНЕЙНО-АНАЛИТИЧЕСКИХ СИСТЕМ ДИСКРЕТНОГО ВРЕМЕНИ С ПОМОЩЬЮ ДИНАМИЧЕСКОЙ ПО СОСТОЯНИЮ ОБРАТНОЙ СВЯЗИ

Рассматривается задача построения компенсатора, обеспечивающего совпадение вход—выход отображений замкнутой системы и заранее фиксированной линейно-аналитической модели. Для класса с $(d_1+1, \ldots, d_p+1)$ — сдвигом обратимых справа линейно-аналитических систем со многими входами и выходами предполагаются достаточные условия разрешимости рассматриваемой задачи и при выполнении их найдены уравнения компенсатора в виде динамической по состоянию обратной связи. Искомое управление определяется уравнениями динамики модели и выходным уравнением правой обратной системы, если подставить в качестве входов в это уравнение подходящие, определенные структурой нелинейной системы сдвиги выходов модели.