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DISCRETE APPROXIMATION OF INTEGRAL EQUATIONS IN THE SPACE OF ESSENTIALLY BOUNDED FUNCTIONS

(Presented by G. Vainikko)

1. Introduction. Consider the numerical solution of the Fredholm integral equation of the second kind

$$x(s)=\int_D K(s,t)x(t)\sigma(dt)+f(s), \quad s\in D, \tag{1}$$

where $x, f\in L^\infty(D,\Sigma,\sigma)=L^\infty(D)$ — the space of essentially bounded functions $y(s)$ with norm

$$\|y\|=\operatorname{vraisup}_{s\in D}|y(s)|.$$

Here σ is a finite regular nonnegative measure on a closed bounded set $D\subset R^r$, Σ is the Borel sigma field on D and $|\cdot|$ denotes the Euclidean norm of a vector.

In [1] the integral operator $kx=\int K(s,t)x(\mathrm{t})\sigma(\mathrm{dt})$ is assumed to be a compact operator from $L^\infty(D)$ to $C(D)$ — the space of continuous functions. Then, assuming that $f\in C(D)$ and P_n is an orthogonal projector of $L^\infty(D)$ onto n-dimensional subspace of $C(D)$, we can replace the operator equation

$$x=kx+f \tag{2}$$

by the approximate equation

$$x_n=P_nkx_n+P_nf, \tag{3}$$

where $\|P_nx-x\|\to 0,\ n\to\infty\ \ \forall x\in C(D)$ and $\|k-P_nk\|\to 0,\ n\to\infty$.

Then one can consider the difference between the solutions of problems (1) and (3) (see [1]):

$$\|x-x_n\|\leqslant\|(I-P_nk)^{-1}\|\|x-P_nx\| \tag{4}$$

along with the rate of convergence.

In this paper we consider approximate solution of (1) when $k:L^\infty(D)\to L^\infty(D)$ and/or $f\in L^\infty(D)$. We replace the equation (1) with a system of linear equations in R^{rn}, $n\in N=\{1,2,3,\dots\}$. In order to get estimates, analogous to (4), we have to introduce a system of (linear) connection operators $P=\{p_n\}$, $p_n:L^\infty(D)\to R^{rn}$, such that

$$\|p_nx\|\to\|x\|, \quad n\to\infty \quad \forall x\in L^\infty(D). \tag{5}$$

Condition (5) guarantees uniqueness of any discretely converging sequences of elements, enabling thus the discrete approximation of the equation (1) in the space of essentially bounded functions $L^\infty(D)$.

In the next section some notions about discrete convergence of elements and mappings, necessary for this paper, are introduced. In Section 3 conditions are presented which guarantee convergence of the method, also the rate of convergence is estimated.

2. Discrete convergence and auxiliary result. Let us restrict the measure σ:

A1) $\sigma\{s \mid |s-t|=\text{const}\}=0 \quad \forall t \in D$.

For the approximate solution of the equation (1) we start from a quadrature formula

$$\int_D h(s)\sigma(ds)=\sum_{i=1}^{n} h(s_{in})m_{in}+\varphi_n(h) \qquad (n\in N), \tag{6}$$

where $m_{in}>0$, $s_{in}\in D$, $i=1,\dots,n$, and $\varphi_n(h)$ is the remaining part of the quadrature formula (6). Quadrature process (6) converges if $\varphi_n(h)\to 0$ $(n\in N)$ for every continuous function $h\in C(D)$.

The convergent quadrature process (6) defines a system of partitions of D, $D=\bigcup_{i=1}^{n} A_{in}$, $\mathcal{A}_n=\{A_{1n},\dots,A_{nn}\}$, $n\in N$, with properties [2]:

1) $A_{in}\in\Sigma$; 2) $\sigma(A_{in})>0$; 3) $A_{in}\cap A_{jn}=\varnothing$, $i\neq j$;

4) $s_{in}\in A_{in}$; 5) $\max\limits_{1\leqslant i\leqslant n}\dfrac{m_{in}}{\sigma(A_{in})}\to 1$, $n\to\infty$;

6) $\operatorname{diam} A_{in}\to 0$, $n\to\infty$; 7) $\sigma(A^0_{in})=\sigma(\bar{A}_{in})$

(here $\operatorname{diam} A=\sup\limits_{s,t\in A}|s-t|$ and A^0 and $\bar{A}$ denote interior and closure of a set A, respectively).

Collection of sets $\{\mathcal{A}_n\}$ $(n\in N)$ with properties 1)—3), 7) constitutes an algebra $\Sigma_0\subset\Sigma$ (see [2]).

Suppose that

A2) $f\in L^\infty(D)$;

A3) $K(s,\cdot)$ is Σ_0 — measurable $\forall s\in D$, $K(\cdot,t)$ is Σ — measurable $\forall t\in D$, $\sup\limits_{s,t\in D}|K(s,t)|\leqslant M<\infty$.

Replace the equation (1) by a system

$$x_{in}=\sum_{j=1}^{n}\frac{1}{\sigma(A_{in})}\int_{A_{in}}K(s,t_{jn})\sigma(ds)x_{jn}m_{jn}+\frac{1}{\sigma(A_{in})}\int_{A_{in}}f(s)\sigma(ds), \tag{7}$$

$$i=1,\dots,n,$$

where $x_{in}\in R^r$ and sets A_{in}, $i=1,\dots,n$, $n\in N$, satisfy properties 1)—7).

Systems of linear equations (7) are defined in R^{rn}, the initial equations (1) are defined in $L^\infty(D)$, so it is necessary to introduce a system of connection operators $\mathfrak{P}=\{p_n\}$ between these spaces. In L^p-spaces, $1\leqslant p\leqslant\infty$, these connection operators should be defined in a piecewise integral form:

$$(p_n x)_{in}=\frac{1}{\sigma(A_{in})}\int_{A_{in}}x(t)\sigma(dt), \quad i=1,\dots,n, \quad n\in N, \tag{8}$$

where the sets A_{in}, $i=1,\dots,n$, $n\in N$, satisfy properties 1)—7). For the system $\mathfrak{P}=\{p_n\}$ of connection operators (8) condition (5) is satisfied [3].

Let us introduce some notions from the theory of discrete convergence of elements and operators necessary for this paper. These definitions are taken mainly from the survey paper [4].

Let E, E_n $(n\in N)$ be real Banach spaces and let $\mathfrak{P}=\{p_n\}$ be a system of linear connection operators $p_n: E\to E_n$ $(n\in N)$ such that $\|p_n x\|_n\to\|x\|$ as $n\to\infty$ $\forall x\in E$.

Definition 1. *A sequence* $\{x_n\}$ *of elements with* $x_n \in E_n$ *P-converges (or converges discretely) to* $x \in E$ *if* $\|x_n - p_n x\|_n \to 0$ *as* $n \to \infty$. We denote this convergence simply by $x_n \to x$.

Definition 2. *A sequence* $\{T_n\}$ *of operators,* $T_n : E_n \to E_n$, $\mathfrak{P}$*-converges (or converges discretely) to an operator* $T : E \to E$ *whenever*

$$x_n \to x \Rightarrow \|T_n x_n - p_n (Tx)\|_n \to 0.$$

We denote this convergence simply by $T_n \to T$.

Definition 3. *A sequence* $\{B_n\}$ *of operators,* $B_n : E_n \to E_n$, *converges compactly to an operator* $B : E \to E$ *if* $B_n \to B$ *and the following compactness condition is fulfilled:*

$$x_n \in E_n, \quad \|x_n\| \leqslant \text{const} \Rightarrow \{B_n x_n\} \text{ is } \mathfrak{P}\text{-compact}.$$

Remark 1. For $T_n \in (E_n, E_n)$, $T \in \mathfrak{L}\mathfrak{L}(E, E)$ the convergence $T_n \to T$ is equivalent to the two following conditions:

(a) $\|T_n\| \leqslant \text{const} \quad (n \in N)$,

(b) $\|T_n p_n x - p_n (Tx)\|_n \to 0, \quad n \to \infty \quad \forall x \in E$.

Let us introduce another partition $\{\mathfrak{B}_k\}$ of D, $\mathfrak{B}_k = \{S_{1k}, \ldots, S_{kk}\}$ $(k \in N)$ with properties

S1) $D = \bigcup_{j=1}^{k} S_{jk}$; S2) $S_{jk} \in \Sigma$, $j = 1, \ldots, k$;

S3) $S_{jk} \cap S_{ik} = \varnothing$, $i \neq j$; S4) $\mathfrak{B}_k \subset \mathfrak{B}_{k+1}$

(the last inclusion means that every set from $\mathfrak{B}_k$, except sets with σ-measure zero, can be presented as a sum of sets from $\mathfrak{B}_{k+1}$).

Let P_k denote the following projector from $L^\infty(D)$ onto the subspace $L^\infty_k(D)$ of simple functions with k values (see, e. g. [5], IV 8, 17):

$$(P_k z)(s) = \frac{1}{\sigma(S_{jk})} \sum_{j=1}^{k} \int_{S_{jk}} z(s) \sigma(ds) \chi_{S_{jk}}(s), \tag{9}$$

where

$$\chi_C(s) = \begin{cases} 1, & s \in C, \\ 0, & s \notin C. \end{cases}$$

Let

A4) $$\underset{s \in D}{\text{vraisup}} \max_{1 \leqslant j \leqslant n} \left| K(s, t_{jn}) - \frac{1}{\sigma(A_{jn})} \int_{A_{jn}} K(s, t) \sigma(\text{dt}) \right| \to 0, \quad n \to \infty.$$

Denote by k_n the following operator from R^{rn} to R^{rn}:

$$(k_n x_n)_{in} = \sum_{j=1}^{n} \frac{1}{\sigma(A_{in})} \int_{A_{in}} K(s, t_{jn}) \sigma(ds) x_{jn} m_{jn}, \quad i = 1, \ldots, n.$$

Let

A5) $$\underset{s \in D}{\text{vraisup}} \sup_{t \in D} \left| \sum_{i=1}^{k} \frac{1}{\sigma(S_{ik})} \int_{S_{ik}} K(s, t) \sigma(ds) \chi_{S_{ik}}(s) - K(s, t) \right| \to 0$$

as $k \to \infty$.

Remark 2. Under condition A5) the operator $k : L^\infty(D) \to L^\infty(D)$ is completely continuous.

Proposition 1. Let conditions A3) — A5) be fulfilled and let the quadrature process (6) converge. Then $k_n \to k$ compactly.

Proof. First of all, let us show that if $x_n \to x$ then $k_n x_n \to kx$, $n \to \infty$. This convergence is equivalent to the following two conditions (see Remark 1):

1a) $\|k_n\| \leqslant \text{const} \quad (n \in N)$,

1b) $\|k_n p_n x - p_n(kx)\| \to 0, \quad n \to \infty \quad \forall x \in L^\infty(D)$.

Consider the condition 1a): $\|k_n\| = \max\limits_{\|x_n\| \leqslant 1} \|k_n x_n\| =$

$$= \max_{\|x_n\| \leqslant 1} \max_{1 \leqslant i \leqslant n} \left| \frac{1}{\sigma(A_{in})} \sum_{j=1}^{n} \int_{A_{in}} K(s, t_{jn}) \sigma(ds) x_{jn} m_{jn} \right| \leqslant$$

$$\leqslant \text{const} \max_{1 \leqslant i \leqslant n} \frac{1}{\sigma(A_{in})} \int_{A_{in}} \sum_{j=1}^{n} |K(s, t_{jn})| m_{jn} \sigma(ds) \leqslant$$

$$\leqslant \text{const} \operatorname{vraisup}_{s \in D} \sum_{j=1}^{n} |K(s, t_{jn})| m_{jn} \leqslant \text{const}$$

since $\sum\limits_{j=1}^{n} m_{jn} \leqslant \text{const} \quad (n \in N)$ (boundedness of the last sum is guaranteed by the convergence (6)).

Consider the condition 1b): $\|k_n p_n x - p_n(kx)\| =$

$$= \max_{1 \leqslant i \leqslant n} \left| \frac{1}{\sigma(A_{in})} \sum_{j=1}^{n} \int_{A_{in}} K(s, t_{jn}) \sigma(ds) \frac{1}{\sigma(A_{jn})} \int_{A_{jn}} x(t) \sigma(\mathrm{d}t) m_{jn} - \right.$$

$$\left. - \frac{1}{\sigma(A_{in})} \int_{A_{in}} \int_{D} K(s, t) x(t) \sigma(\mathrm{d}t) \sigma(ds) \right| \leqslant$$

$$\leqslant \max_{1 \leqslant i \leqslant n} \frac{1}{\sigma(A_{in})} \int_{A_{in}} \left[\sum_{j=1}^{n} \left| K(s, t_{jn}) (p_n x)_{jn} - \right.\right.$$

$$\left.\left. - \frac{1}{\sigma(A_{jn})} \int_{A_{jn}} K(s, t) \sigma(\mathrm{d}t) (p_n x)_{jn} \right| m_{jn} \right] \sigma(ds) +$$

$$+ \max_{1 \leqslant i \leqslant n} \frac{1}{\sigma(A_{in})} \int_{A_{in}} \sum_{j=1}^{n} \left| \int_{A_{jn}} K(s, t) \left[(p_n x)_{jn} \chi_{A_{jn}}(t) \frac{m_{jn}}{\sigma(A_{jn})} - \right.\right.$$

$$\left.\left. - x(t) \right] \sigma(\mathrm{d}t) \right| \sigma(ds) \leqslant \max_{1 \leqslant i \leqslant n} \frac{1}{\sigma(A_{in})} \left[\int_{A_{in}} \max_{1 \leqslant j \leqslant n} \left| K(s, t_{jn}) - \right.\right.$$

$$\left.\left. - \frac{1}{\sigma(A_{jn})} \int_{A_{jn}} K(s, t) \sigma(\mathrm{d}t) \right| \sum_{j=1}^{n} |(p_n x)_{jn}| m_{jn} \right] \sigma(ds) +$$

$$+ M \max_{1 \leqslant i \leqslant n} \frac{1}{\sigma(A_{in})} \int_{A_{in}} \sum_{j=1}^{n} \left| \int_{A_{jn}} \left[(p_n x)_{jn} \chi_{A_{jn}}(t) \frac{m_{jn}}{\sigma(A_{jn})} - \right.\right.$$

$$\left.\left. - x(t) \right] \sigma(\mathrm{d}t) \right| \sigma(ds) \leqslant \text{const} \max_{1 \leqslant i \leqslant n} \frac{1}{\sigma(A_{in})} \int_{A_{in}} \max_{1 \leqslant j \leqslant n} \left| K(s, t_{jn}) - \right.$$

$$\left. - \frac{1}{\sigma(A_{jn})} \int_{A_{jn}} K(s, t) \sigma(dt) \right| \sigma(ds) +$$

$$+ M \sum_{j=1}^{n} \int_{A_{jn}} \left| (p_n x)_{jn} \frac{m_{jn}}{\sigma(A_{jn})} \chi_{A_{jn}}(t) - x(t) \right| \sigma(\mathrm{d}t) \leqslant$$

$$\leqslant \text{const}\ \underset{s\in D}{\text{vraisup}}\ \max_{1\leqslant j\leqslant n} \left| K(s,t_{jn}) - \frac{1}{\sigma(A_{jn})}\int_{A_{jn}} K(s,t)\,\sigma(\mathrm{dt})\right| +$$

$$+M\sum_{j=1}^{n}\int_{A_{jn}}\left|(p_n x)_{jn}\frac{m_{jn}}{\sigma(A_{jn})}\chi_{A_{jn}}(t) - x(t)\right|\sigma(\mathrm{dt}).$$

It is clear that (condition A4) the first part in the last sum tends to zero as $n\to\infty$. Consider the second part of the sum. Take a continuous function $x_c(t)$, $x_c\in C(D)$, and n so large that

a) $\int_D |x_c(t)-x(t)|\,\sigma(\mathrm{dt})\leqslant\varepsilon$ and

b) $\displaystyle\sum_{j=1}^{n}\frac{m_{jn}}{\sigma(A_{jn})}\int_{A_{jn}}\left|(p_n x)_{jn}\chi_{A_{jn}}(t) - (p_n x_c)_{jn}\chi_{A_{jn}}(t)\right|\sigma(\mathrm{dt})\leqslant 2\varepsilon.$

The inequality b) holds for n sufficiently large since

$$\sum_{j=1}^{n}\frac{m_{jn}}{\sigma(A_{jn})}\int_{A_{jn}}\left|(p_n x)_{jn}\chi_{A_{jn}}(t) - (p_n x_c)_{jn}\chi_{A_{jn}}(t)\right|\sigma(\mathrm{dt})\leqslant$$

$$\leqslant \text{const}\sum_{j=1}^{n}\int_{A_{jn}}\left|(p_n(x-x_c))_{jn}\right|\chi_{A_{jn}}(t)\,\sigma(\mathrm{dt})\leqslant$$

$$\leqslant \text{const}\int_D |x(t)-x_c(t)|\,\sigma(\mathrm{dt})+\varepsilon \quad\text{as}\quad n\geqslant n_0$$

(for a small $\varepsilon>0$ one can find an index n_0 such that for all $n\geqslant n_0$

$$\left|\sum_{j=1}^{n}|(p_n(x-x_c))_{jn}|\,\sigma(A_{jn}) - \int |x(t)-x_c(t)|\,\sigma(\mathrm{dt})\right|\leqslant\varepsilon$$

(discrete convergence in $L^1(\sigma)$)).
Taking n_1 so large that for all $n\geqslant n_1$ the inequality

$$\sum_{j=1}^{n}\int_{A_{jn}}\left|(p_n x_c)_{jn}\frac{m_{jn}}{\sigma(A_{jn})}\chi_{A_{jn}}(t) - x_c(t)\right|\sigma(\mathrm{dt})\leqslant\varepsilon$$

holds, we have finally for $n\geqslant\max\{n_0,n_1\}$, that

$$\sum_{j=1}^{n}\int_{A_{jn}}\left|(p_n x)_{jn}\frac{m_{jn}}{\sigma(A_{jn})}\chi_{A_{jn}}(t) - x(t)\right|\sigma(\mathrm{dt})\leqslant$$

$$\leqslant\sum_{j=1}^{n}\frac{m_{jn}}{\sigma(A_{jn})}\int_{A_{jn}}\left|(p_n x)_{jn}\chi_{A_{jn}}(t) - (p_n x_c)_{jn}\chi_{A_{jn}}(t)\right|\sigma(\mathrm{dt})+$$

$$+\sum_{j=1}^{n}\int_{A_{jn}}\left|(p_n x_c)_{jn}\frac{m_{jn}}{\sigma(A_{jn})}\chi_{A_{jn}}(t) - x_c(t)\right|\sigma(\mathrm{dt})+$$

$$+\int_D |x_c(t)-x(t)|\,\sigma(\mathrm{dt})\leqslant 4\varepsilon.$$

Now it is clear that both components in the last sum tend to zero as $n\to\infty$. Consequently, if $x_n\to x$, then $k_n x_n\to kx$, $n\to\infty$.

Let a bounded sequence $\{x_n\}$, $\|x_n\|\leqslant\text{const}$ $(n\in N)$, be given. Consider the sequence $\{k_n x_n\}$.

Define the function $z_n(s)=\sum_{j=1}^{n} K(s,t_{jn})x_{jn}m_{jn}$ $(n\in N)$ and consider the difference $\operatorname{vraisup}_{s\in D}|(P_k z_n)(s)-z_n(s)|\leqslant$

$$\leqslant \operatorname{vraisup}_{s\in D}\left|\sum_{j=1}^{n}\left[\sum_{i=1}^{k}\frac{1}{\sigma(S_{ik})}\int_{S_{ik}}K(s,t_{jn})\sigma(ds)\chi_{S_{ik}}(s)-K(s,t_{jn})\right]x_{jn}m_{jn}\right|\leqslant$$

$$\leqslant \operatorname{vraisup}_{s\in S}\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{k}\frac{1}{\sigma(S_{ik})}\int_{S_{ik}}K(s,t_{jn})\sigma(ds)\chi_{S_{ik}}(s)-\right.$$

$$\left.-K(s,t_{jn})\right|\sum_{j=1}^{n}|x_{jn}|m_{jn}.$$

Assumption A5) guarantees now that the last expression tends to zero as $k\to\infty$ uniformly by $n\in N$.

Let us show that $\{z_n\}$ contains a fundamental subsequence. Take a small $\varepsilon>0$ and any $m\in N$. Consider the difference $\|z_n-z_{n+m}\|\leqslant\leqslant\|z_n-P_kz_n\|+\|P_kz_n-P_kz_{n+m}\|+\|P_kz_{n+m}-z_{n+m}\|$. It is clear that there exists $K(\varepsilon)$ such that $\|z_n-P_kz_n\|\leqslant\varepsilon$, $\|z_{n+m}-P_kz_{n+m}\|\leqslant\varepsilon$ $\forall k\geqslant K(\varepsilon)$, $n\in N$. Since the range space of mappings P_k, $k\in N$, is finite-dimensional, P_k maps a bounded set into compact. Thus there exists a fundamental subsequence (denote it also by $\{P_kz_n-P_kz_{n+m}\}$) such that $\|P_kz_n-P_kz_{n+m}\|\leqslant\varepsilon$ $\forall n\geqslant N(\varepsilon)$. Consequently, for a fundamental subsequence $\{z_n-z_{n+m}\}$ we have $\|z_n-z_{n+m}\|\leqslant 3\varepsilon$ as $n\geqslant N(\varepsilon)$. Since $L^\infty(D)$ is complete, a fundamental subsequence converges to an element from $L^\infty(D)$. Consequently a subsequence of $\{p_nz_n\}$ converges discretely to an element of $L^\infty(D)$ [4]. Q.E.D.

3. Convergence of the method. Denote by p_nf the vector $\frac{1}{\sigma(A_{in})}\int_{A_{in}}f(s)\sigma(ds)$, $i=1,\ldots,n$.

Theorem 1. *Let conditions* A1)—A5) *be fulfilled and let the homogeneous equation*

$$x(s)=\int_D K(s,t)x(t)\sigma(\mathrm{d}t) \tag{10}$$

have only trivial solution $x=0$. *Let quadrature process* (6) *converge. Then the equation* (1) *has a unique solution* $x(s)$ *in* $L^\infty(D)$ *and the system of equations (7) has a unique solution for a sufficiently large n. The discrete convergence* $x_n\to x$, $n\to\infty$, *holds with an estimate*

$$c_1\varepsilon_n\leqslant\|x_n-p_nx\|\leqslant c_2\varepsilon_n,$$

where $c_1, c_2>0$ *and*

$$\varepsilon_n=\max_{1\leqslant i\leqslant n}\left|\sum_{j=1}^{n}\frac{1}{\sigma(A_{in})}\int_{A_{in}}K(s,t_{jn})\sigma(ds)\frac{1}{\sigma(A_{jn})}\int_{A_{jn}}x(t)\sigma(dt)m_{jn}-\right.$$

$$\left.-\frac{1}{\sigma(A_{in})}\int_{A_{in}}\int_D K(s,t)x(t)\sigma(dt)\sigma(ds)\right|.$$

Proof. By proposition 1 the sequence of operators $\{k_n\}$ converges to operator k compactly. Due to condition (10) and complete continuity of k the operator $I-k$ is invertible. Compact convergence $k_n\to k$ $(n\in N)$ guarantees for $n\geqslant n_2$ the existence and boundedness of $(I_n-k_n)^{-1}$, $\|(I_n-k_n)^{-1}\|\leqslant$const, where I_n is the identity operator in R^{rn}. Then from discrete convergences $p_n f \to f$, $(I_n - k_n)^{-1} \to (I - k)^{-1}$ $(n \in N)$ we receive that the convergence

$$x_n = (I_n - k_n)^{-1} p_n f \to (I - k)^{-1} f = x \qquad (n \in N)$$

holds and the error estimate

$$c_1 \varepsilon_n \leqslant \|x_n - p_n x\| \leqslant c_2 \varepsilon_n$$

follows, where $c_1 = \inf_{n \geqslant 1} 1/\|I_n - k_n\|$, $c_2 = \sup_{n \geqslant n_2} \|(I_n - k_n)^{-1}\|$ and ε_n was determined in the formulation of the theorem. Q. E. D.

Remark 3. Let the kernel K and the function f be bounded and Σ_0 — measurable. Then one can approximate the equation (1) by a more simple linear system as compared to the system (7):

$$x_{in} = \sum_{j=1}^{n} K(s_{in}, t_{jn}) x_{jn} m_{jn} + f(s_{in}), \qquad i = 1, \ldots, n,$$

where discretization points s_{in}, t_{jn}, $i, j = 1, \ldots, n$, are taken from the quadrature formula (6).

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INTEGRAALVÕRRANDITE DISKREETNE APROKSIMATSIOON OLULISELT TÕKESTATUD FUNKTSIOONIDE RUUMIS

Teist liiki lineaarne Fredholmi integraalvõrrand oluliselt tõkestatud funktsioonide ruumis on asendatud lineaarse võrrandisüsteemiga lõplikumõõtmelises ruumis. Kasutades diskreetse aproksimatsiooni meetodit, on tõestatud sellise ligikaudse lahendamise meetodi koonduvus ja hinnatud viimase kiirust.

Рихо ЛЕПП

ДИСКРЕТНАЯ АППРОКСИМАЦИЯ ИНТЕГРАЛЬНЫХ УРАВНЕНИЙ В ПРОСТРАНСТВЕ СУЩЕСТВЕННО ОГРАНИЧЕННЫХ ФУНКЦИЙ

Линейное интегральное уравнение Фредгольма второго рода заменено системой линейных алгебраических уравнений в конечномерном пространстве. Используя метод дискретной аппроксимации, показана сходимость такого метода аппроксимации и оценена скорость сходимости.