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ON THE TRANSFORMATION OF TWO-DIMENSIONAL SYSTEM TO CANONICAL FORM

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Юлле Котта. О ПРЕОБРАЗОВАНИИ ДВУМЕРНОЙ СИСТЕМЫ В КАНОНИЧЕСКУЮ ФОРМУ

(Presented by N. Alumäe)

1. Introduction. During recent years, several authors have proposed different state-space models for two-dimensional causal systems [1–4]. It is shown that Roesser's model is the most general since the others can be imbedded in this model [5]. In [6] a canonical form is presented for a class of Roesser's model such that the denominator of the transfer function can be expressed by the product of two polynomials with a variable; the model is assumed to be separately locally controllable. We consider another canonical form obtained by duality. This is for the system which is separately locally observable. This canonical form has certain practical advantages when applied to identification.

2. System description and its equivalent system. Consider the Roesser's model of the two-dimensional system

$$\begin{bmatrix} x^h(i+1,\ j) \\ x^v(i,\ j+1) \end{bmatrix} = \begin{bmatrix} F_1 & F_2 \\ F_3 & F_4 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \begin{bmatrix} g_1 \\ g_2 \end{bmatrix} u(i,j),$$

$$y(i,j) = [h_1\ h_2] \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}, \quad i,j \geqslant 0, \tag{1}$$

where $x^h(i,j)$ is an $n\times 1$ horizontal state vector, $x^v(i,j)$ is an $m\times 1$ vertical state vector, $u(i,j)$ is a scalar input, $y(i,j)$ is a scalar output, and F_1, F_2, F_3, F_4, g_1, g_2, h_1, h_2 are real matrices of proper dimensions. Let us define the state variable $x(i,j)$ at the point (i,j) by

$$x(i,j) = \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}.$$

In [1] Roesser introduced the following class of similarity transformations

$$\begin{bmatrix} \tilde{x}^h(i,j) \\ \tilde{x}^v(i,j) \end{bmatrix} = \begin{bmatrix} T_1 & 0 \\ 0 & T_4 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix},$$

where T_1 is an $n \times n$ constant nonsingular matrix, T_4 is an $m \times m$ constant nonsingular matrix, and 0 is a null matrix of appropriate dimensions. Performing the coordinate transformation from the state vector $x(i,j)$ to a new state vector $\tilde{x}(i,j)$, we obtain

$$\begin{bmatrix} \tilde{x}^h(i+1,j) \\ \tilde{x}^v(i,j+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} \tilde{x}^h(i,j) \\ \tilde{x}^v(i,j) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} u(i,j),$$

$$y(i,j) = [c_1 \ c_2] \begin{bmatrix} \tilde{x}^h(i,j) \\ \tilde{x}^v(i,j) \end{bmatrix}, \quad i,j \geqslant 0. \tag{2}$$

The systems (1) and (2) are related by

$$A_1 = T_1 F_1 T_1^{-1}, \quad b_1 = T_1 g_1,$$

$$A_2 = T_1 F_2 T_4^{-1}, \quad b_2 = T_4 g_2,$$

$$A_3 = T_4 F_3 T_1^{-1}, \quad c_1 = h_1 T_1^{-1},$$

$$A_4 = T_4 F_4 T_4^{-1}, \quad c_2 = h_2 T_4^{-1}.$$

The system (2) is equivalent to the system (1) in the sense that their transfer functions are equal.

3. A canonical form for a class of Roesser's model. In this section, a canonical form is presented for the system (1) with $F_2=0$. For such a system the denominator of the transfer function can be expressed by the product of two polynomials with a variable [6]. We assume here that the system (1) with $F_2=0$ is separately locally observable. The canonical form is constructed by transforming the state variable into a new coordinate system in which the system equation takes a particular form.

Definition. The system (1) is separately locally observable if, for $x^h(0,j)$, $j \geqslant 1$, and $x^v(i,0)$, $i \geqslant 1$, there are no nonzero initial horizontal state $x^h(0,0)$ and vertical state $x^v(0,0)$ such that for zero input $u(i,j) = 0$, $i,j \geqslant 0$, the output is also identically zero, $y(i,j)=0$, $i,j \geqslant 0$.

As shown in [7], the system (1) with $F_2=0$ is separately locally observable if and only if

$$\operatorname{rank} [\underset{n(m+1)\times n}{H_m^T}, \ (H_m F_1)^T, \ \ldots, \ (H_m F_1^{n-1})^T]^T = n, \tag{3a}$$

$$\operatorname{rank} [\underset{m\times n}{h_2^T}, \ (h_2 F_4)^T, \ \ldots, \ (h_2 F_4^{m-1})^T]^T = m, \tag{3b}$$

where

$$\underset{n\times(m+1)}{H_m} = [h_1^T, \ (h_2 F_3)^T, \ (h_2 F_4 F_3)^T, \ \ldots, \ (h_2 F_4^{m-1} F_3)^T]^T. \tag{4}$$

We define T_1 as

$$T_1 = [H_m^T e_1, \ \ldots, \ (F_1^T)^{n_1-1} H_m^T e_1, \ \ldots, \ (F_1^T)^{n_r-1} H_m^T e_r]^T,$$

where e_i, $i=1, \ldots, m+1$, denotes an $(m+1)\times 1$ unit vector whose i-th element is equal to 1 and the vectors of T_1 are selected in the following order:

$$H_m^T e_1, \ H_m^T e_2, \ \ldots, \ H_m^T e_{m+1}, \ F_1^T H_m^T e_1, \ \ldots, \ F_1^T H_m^T e_{m+1}, \ \ldots. \tag{5}$$

The essential constraint is to retain a vector $(F_1^T)^s H_m^T e_v$ if and only if it

is linearly indenpendent from all previously selected vectors. Since (3a) holds, it is always possible to select n linearly independent vectors from the sequence (5).
Then we obtain

$$A_1=T_1F_1T_1^{-1}=[A_{ij}], \quad i,j=1, \ldots, r,$$

$$c_1=h_1T_1^{-1}=[1\ 0\ \ldots\ 0],$$

where

$$\underset{n_i\times n_i}{A_{ii}}=\begin{bmatrix} 0 & I_{n_i-1} \\ a_1^{ii} \ldots & a_{n_i}^{ii} \end{bmatrix}, \quad \underset{n_i\times n_j}{A_{ij}}=\begin{bmatrix} 0 & & & \\ a_1^{ij} \ldots a_{n_{ij}}^{ij} & 0 & \ldots & 0 \end{bmatrix},$$

$$n_{ij}=\begin{cases} \min(n_i+1, n_j), & j<i; \\ \min(n_i, n_j), & j\geqslant i. \end{cases}$$

By letting

$$T_4=[h_2^T, (h_2F_4)^T, \ldots, (h_2F_4^{m-1})^T]^T \tag{6}$$

we obtain

$$A_4=T_4F_4T_4^{-1}=\begin{bmatrix} 0 & I_{m-1} \\ a_1^4 \ldots & a_m^4 \end{bmatrix},$$

$$c_2=h_2T_4^{-1}=[1\ 0\ \ldots\ 0].$$

Next we define the rows of the matrix A_3 as a_k^T, $k=1, \ldots, m$. Substituting (6) into $A_3=T_4F_3T_1^{-1}$ and using (4) we have

$$\begin{bmatrix} a_1^T \\ \cdot \\ \\ \cdot \\ a_m^T \end{bmatrix} T_1=\begin{bmatrix} e_2^T \\ \cdot \\ \cdot \\ \cdot \\ e_{m+1}^T \end{bmatrix} H_m.$$

Therefore, if $e_k^TH_m$, $k=2, \ldots, m+1$, is in the rows of T_1, a_{k-1} becomes a unit vector whose $(n_1+\ldots+n_{k-1}+1)$-th element is equal to 1, otherwise $a_k^{T_1}=(a_1^k \ldots a_n^k)$. Note that vectors b_1 and b_2 can not be given by simple forms.

Substituting A_1, A_3, A_4, b_1, b_2, c_1, c_2 found above and $A_2=0$ into (2), we obtain a canonical form for the system (1) with $F_2=0$.

REFERENCES

1. Roesser, R. P., IEEE Trans. Automat. Contr., **20**, № 1, 1—10 (1975).
2. Givone, D., Roesser, R. P., IEEE Trans. Comput., **22**, № 7, 673—678 (1976).
3. Fornasini, E., Marchesini, G., IEEE Trans. Automat. Contr., **21**, № 4, 484—492 (1976).
4. Attasi, S., In: System Identification. Advances and Case Studies, Academic Press, New York, 1976.
5. Kung, S.-Y., Lévy, B. C., Morf, M., Kailath, T., Proc. IEEE, **65**, № 6, 945—961 (1977).
6. Hinamoto, T., IEEE Trans. Circuits and Syst., **27**, № 8, 710—712 (1980).
7. Hinamoto, T., IEEE Trans. Circuits and Syst., **27**, № 1, 36—44 (1980).

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