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A GENERAL METHOD FOR COMPUTING ONE-CARRIER INJECTION AND EXTRACTION CURRENT-VOLTAGE CHARACTERISTICS IN SOLIDS

1. Introduction

The conduction in metal-semiinsulator-metal layer structures is undoubtedly determined not only by the bulk properties of the semiinsulator, or, to put in another way, of the high-resistivity extrinsic semiconductor which our discussion is confined to, but also by the interaction of the latter with the metal electrodes. For this interaction the semiinsulator would contain the space charge. There may be two regions of appreciable space charge, located near the metal electrodes. Alternatively all the semiinsulator layer may be embraced by a single space-charge region if the contact regions overlap, as, in any event, is the case when the bias exceeds a certain value. Of course, the overlapping may take place in thermal equilibrium as well. The conductivity in the space-charge region(s) increases or decreases with respect to that in the bulk semiinsulator, the sign and amount of the change depending on the electrical characteristics of the semiinsulator and the metals, the structure geometry, and the bias. This is the reason why varied conduction values make their appearance.

Turning even to the thermal equilibrium case, the comprehensive analysis of the situation in the structures with imperfection semiinsulators has received very little attention in literature [1–3]. This seems to be due to complications of a mathematical nature. Indeed, the problem is generally solvable only with the aid of electronic computers, and often not without difficulties.

A more arduous task is the adequate treatment of the problem in the current-carrying case. If the external electric field is applied to the structure, the excess space charge appears in the semiinsulator layer. This is accompanied either by an increased or decreased bias-dependent layer conduction. In the former case we speak of the space-charge-limited current (SCLC) flow and the one-carrier injection, in the latter case of the current flow in a blocking structure and the one-carrier extraction.

Analytic treatment of the steady-state SCLC in imperfection semiinsulators has been mainly carried out for one-terminal, semiinfinite devices with free-carrier diffusion effects being entirely neglected. Such a simplification of the theory allows trapping effects to be taken into account with ease, leading to important results [4]. Unfortunately, the derived solutions cannot adequately describe the physical situation near the contacts, and it remains open as to what extent the contact regions influence the shape of the SCLC current-voltage characteristics. In

a few numerical analyses of the SCLC flow in imperfection semiinsulators diffusion is taken into account [5–12]. However, the solutions found are related only to some special cases and cannot describe diffusion effects in full. Here the computational difficulties make themselves felt to a greater extent [8, 10, 13].

As for the blocking structures with the extrinsic semiinsulators, the contribution of trapping and recombination has not been cleared up so far. To the best of our knowledge, the only solution of the problem of a two-terminal device, in which the space-charge extraction takes place, has been given in an important paper by J. R. Macdonald [14]. This author analyzes numerically the electron current flow in blocking structure when donors in semiinsulator are fully ionized and compensating acceptors are absent. Again the computational difficulties are referred to.

As the SCLC and current in blocking semiinsulator structures are the particular cases of the current flow in metal-semiinsulator-metal structures, they give easily way to their uniform treatment. Hence a general numerical method enabling such uniform treatment could be developed. The universal computer program, when available, should permit to carry out numerous integrations referred to different physical situations, the way being indispensable for comprehensive analysis of the conduction in metal-semiinsulator-metal structures. The program would be of use also in handling observed data and for experimental design. The method in view must overcome computing difficulties which are due to the strong nonlinearity of the boundary-value problem and to the extremely rapid variations of the normal dependent variables, i. e. the free-carrier concentration and the electric field strength, near the contacts.

In the present paper we propose a general method for computing the one-carrier steady-state currents in extrinsic compensated semiinsulators provided with two unequal metal contacts. We are interested primarily in the evalution of the current-voltage characteristics, but the free-carrier concentration and electric field strength distributions are also our computing output functions. The latter functions are of particular use in the zero-bias case.

An application of a more simple version of the program and some general ideas are described in our earlier paper [15]. The results of calculations will be presented elsewhere.

2. Formulation and background

We consider a homogeneous semiinsulator layer of a thickness l with an infinite cross-section and plane, parallel metal electrodes at either end forming a diode structure with abrupt junctions. We assume that the semiinsulator contains two sets of sharply defined levels in the gap, one set being originated by acceptors and the other one by donors. It is also assumed, without loss of generality, that the unipolar layer current is carried by electrons, the left contact being the cathode.

The current-flow equation along the x-axis may be written as

$$j = e\mu n(x) E(x) - eD \frac{dn(x)}{dx}, \tag{1}$$

where $j \geqslant 0$ and E are the current density and the electric field strength, respectively, defined as opposite in sign to the conventional quantities, e is the magnitude of the electron charge, μ the carrier mobility, assumed as constant, n the free-carrier concentration, and D is the diffusion

constant. The latter is related to μ by the Einstein relation $eD = \mu kT$, where k is the Boltzmann constant and T the absolute temperature.

The role of space-charge is introduced by the Poisson equation

$$\frac{dE(x)}{dx} = \frac{\varrho(x)}{\varepsilon}, \qquad \varrho(x) = e[n(x) + n_a(x) - N_d + n_d(x)], \tag{2}$$

where ϱ is the space-charge density, ε the static dielectric constant, n_a the filled (negatively charged) acceptor concentration, N_d the donor concentration, and n_d is the filled (neutral) donor concentration.

Assuming that there is a thermodynamic equilibrium between the free and localized electrons, the filled center concentrations may be written as

$$n_a(x) = \frac{N_a n(x)}{N_{ca} + n(x)}, \qquad n_d(x) = \frac{N_d n(x)}{N_{cd} + n(x)}, \tag{3}$$

where N_a is the acceptor concentration, $N_{ca} = N_c \exp(-W_a/kT)$, and $N_{cd} = N_c \exp(-W_d/kT)$, while $N_c = 2(2\pi m^* kT/h^2)^{3/2}$ is the effective density of states in the conduction band (m^* being the effective mass of the electron and h the Planck constant), and W_a and W_d are the acceptor and donor depths, respectively.

We will solve these equations using the following boundary conditions

$$n_L = N_c \exp\left(-\frac{e\Phi_L}{kT}\right), \qquad n_R = N_c \exp\left(-\frac{e\Phi_R}{kT}\right), \tag{4}$$

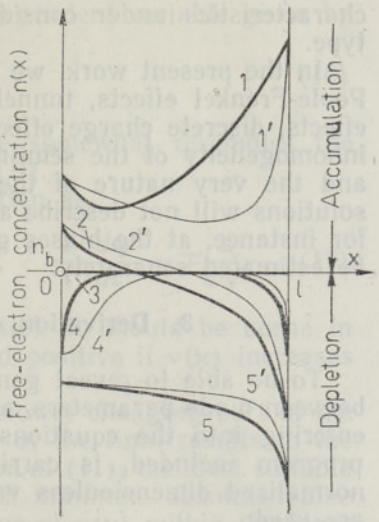
where n_L and n_R are the free-electron concentrations, and Φ_L and Φ_R the metal-semiinsulator barrier potentials at the left and the right end of the layer, respectively. Thus we always assume the thermodynamic equilibrium to exist at the boundaries. In addition, we consider the barrier potentials to be expressed simply through the differences between the metal work functions and the semiinsulator electron affinity.

Sure enough, the shape of the $n(x)$ curve in thermal equilibrium depends on the relations between semiinsulator work function χ_S and the cathode and anode metal ones, χ_L and χ_R, respectively, and the value of l. If $\chi_L = \chi_R = \chi_M$, $n(x)$ is symmetric relative to the semiinsulator layer midplane, having a minimum (when $\chi_M < \chi_S$) or a maximum (when $\chi_M > \chi_S$) there. Thus the free-carrier concentration accumulation regions appear in the former, ohmic-contact case, and the depletion regions in the latter, blocking-contact case (curve *4* in the Figure). Specifically, if $\chi_M = \chi_S$, $\varrho(x) = 0$ everywhere and $n(x)$, not depending on position, equals to the bulk value of the free-carrier concentration n_b. In this particular case, offering no computational difficulties, we speak of neutral contacts (curve *3*).

If $\chi_L \neq \chi_R$, we can distinguish between $n(x)$ curves proceeding from $\chi_L - \chi_S$ and $\chi_R - \chi_S$ signs. If $\operatorname{sign}(\chi_L - \chi_S) = \operatorname{sign}(\chi_R - \chi_S)$ and $|\chi_L - \chi_S| > |\chi_R - \chi_S|$, the extremum is situated to the right of the semiinsulator midplane or vanished, while with the reverse inequality valid the extremum moves to the left. Two corresponding prototype curves are drawn in the Figure. Curves *1* and *5* represent here the cases with remained and vanished extremum, respectively.

If $\operatorname{sign}(\chi_L - \chi_S) \neq \operatorname{sign}(\chi_R - \chi_S)$, one of the possibilities is curve *2* in the Figure. Now the $n(x)$ curve crosses the horizontal line $n(x) = n_b$ and has an inflection point.

It has virtually become common use to characterize the contacts as the "ohmic" or "blocking" ones for electrons when at the semiinsulator boundaries $n(x) > n_b$, i. e. $\varrho(x) > 0$, or $n(x) < n_b$, i. e. $\varrho(x) < 0$, respectively.



Some prototype variations of $\log n$ with position.
The thick and thin lines show equilibrium and non-equilibrium states, respectively. Cathode is at the left.

Further we will make some observations with regard to the expected shape of the non-equilibrium $n(x)$ curves and the current-voltage characteristics. By the way, it is obligatory to have a knowledge of the electrostatic potential φ for the computation of the current-voltage characteristics to be carried out. By the definition,

$$\frac{d\varphi(x)}{dx} = -E(x), \tag{5}$$

where, in our instance, φ is opposite in sign to the conventional quantity.

In our approach the current-voltage relations are to exhibit three distinct regions. At low and very high biases they are to be linear, in the former case due to small disturbances of the equilibrium charge distribution by external fields, in the latter case due to the electron concentration not varying with the position over the most part of the layer and being equal to its value at the boundary which the electrons are flowing away from [12, 14, 16, 17]. The intermediate region of the current-voltage curves is characterized by highly nonlinear behavior. It is this intermediate region that we are mainly interested in when solving our problem.

As we have fixed the $n(x)$ values at the boundaries, the extremum of the $n(x)$ curve, if it altogether exists, moves to the anode with the bias (curve *1'* in Figure), sometimes being able to vanish at high biases. Further, the non-equilibrium $n(x)$ curve can have an inflection point if it exists already in equilibrium (curve *2'*) as well as in a number of the other cases, e. g., when $\varrho(x) < 0$ and $n_L \leqslant n_R$ (curve *4'*), or when $\varrho(x) > 0$, $n_L > n_R$ and the $n(x)$ minimum has vanished. In the cases corresponding to $n_L > n_R$ when $\varrho(x) < 0$ or with $n_L < n_R$ when $\varrho(x) > 0$, the $n(x)$ curve being concave or convex upwards without any extremum in equilibrium, retains its behavior under bias (curve *5'*), as well.

We will return to the question in section 5 where we classify all possible cases. Note that in many instances the one-carrier injection and extraction take place simultaneously (curves *1'* and *4'* in Figure). Besides, when the "ohmic" or "blocking" contacts, defined as above, are available, not only does injection or, respectively, extraction prevail, but also the opposite can be true (curves *1'* and *5'*). Thus, the current-voltage

characteristics under consideration are generally of injection-extraction type.

In the present work, we do not consider the Richardson-Schottky and Poole-Frenkel effects, tunneling, impact ionization and other hot-carrier effects, discrete charge effects, two-carrier conduction, thermal runaway, inhomogeneity of the semiinsulator, finite cross-section of the structure, and the very nature of the metal-semiinsulator contact. Therefore, our solutions will not describe adequately the physical situation in the layer, for instance, at the biases greater than some definite value which could be estimated separately.

3. Derivation of the reduced set of equations

To be able to reveal general trends and relations, such as trade-offs between diode parameters, and to minimize the number of the parameters entering into the equations, the further treatment, the writing of the program included, is carried out in the terms of a set of suitably normalized dimensionless variables. The following normalized quantities are used:

$$\nu=\frac{n}{N^*}, \quad \eta=\frac{E}{E^*}, \quad \phi=\frac{\varphi}{\varphi^*}, \quad \varkappa=\frac{E^*x}{\varphi^*}, \quad \iota=\frac{j}{e\mu N^*E^*}, \quad \alpha=\frac{N_a}{N^*}$$

$$\delta=\frac{N_d}{N^*}, \quad \sigma=\frac{N_{cd}}{N^*}, \tag{6}$$

where $N^*=N_a+N_{ca}+N_d+N_{cd}$, $E^*=(kTN^*/\varepsilon)^{1/2}$ and $\varphi^*=kT/e$.

Equations (1), (2) and (5) now become

$$\frac{d\nu}{d\varkappa}=\nu\eta-\iota, \quad \frac{d\eta}{d\varkappa}=\nu\psi(\nu), \quad \frac{d\phi}{d\varkappa}=-\eta, \tag{7}$$

where $\psi(\nu)=1+\alpha/(1-\alpha-\delta-\sigma+\nu)-\delta\sigma/[\nu(\sigma+\nu)]$ is the normalized space-charge density per free electron. Observe that the inequality $\alpha+\delta+\sigma<1$ holds.

After changing the variables, the possibility to simulate, if necessary, actual diodes has been recreated through optional normalizing and denormalizing routines made available at input and output.

From (7) we readily obtain

$$\frac{d}{d\varkappa}\left(\frac{d}{d\varkappa}\ln\nu+\frac{\iota}{\nu}\right)=\nu\psi(\nu). \tag{8}$$

This strongly nonlinear equation, subject to given boundary conditions, determines a two-point boundary-value problem to be solved.

In order to facilitate the calculations we have gone the way of reducing of this boundary-value problem to the initial-value one.

For a proper treatment of the problem we take as an independent variable the quantity z defined by

$$\nu=\nu_0\pm\frac{z^2}{2}, \tag{9}$$

where ν_0 is the free-carrier concentration in the point $z=0$, the starting point of the integration to the both boundaries. In (9) and throughout the paper the upper sign refers to the semiinsulator region where $\nu(\varkappa)$ increases, and the lower sign to the region where $\nu(\varkappa)$ decreases, when moving away from the point $z=0$ in the direction of the integration.

Further, instead of η we introduce a new dependent variable given by

$$w = \eta - \frac{\iota}{v}. \tag{10}$$

It is worth mentioning that the last change somewhat resembles the one used by G. G. Roberts and L. M. Marks [13].

The set of equations (7) can now be rewritten as

$$\frac{d\varkappa}{dz} = \pm\frac{z}{wv}, \quad \frac{dw}{dz} = \pm\frac{z}{w}\left[\psi(v) + \frac{\iota w}{v^2}\right], \quad \frac{1}{\iota}\frac{d\phi'}{dz} = \mp\frac{z}{wv^2}, \tag{11}$$

where $\phi' = \phi - \ln v$ is the quasi-Fermi potential. It should be borne in mind that w is negative if $v(\varkappa)$ decreases, and positive if $v(\varkappa)$ increases with $\varkappa$.

The quantity v_0 is to be taken in the most slowly changing part of the $v(\varkappa)$ curve, thus avoiding the high sensitivity of the final results to the initial conditions. As seen from the first equation of (11), the most suitable point to start the integration from is that with minimum absolute value of w. Hence we take v_0 to be the extreme value of $v(\varkappa)$ within the layer if there exists such an extremum point with $w = 0$, or the value of $v(\varkappa)$ at that boundary where $|w|$ is minimal. When $w(z) \neq 0$ and the minimum absolute value of w is present somewhere inside the layer, i. e., the point with $dw/dz = 0$ is present there, then this is the point the integration should be started form.

4. Zero-bias case

To solve the problem as an initial-value one we should simultaneously know both v_0 and η_0 (or w_0), the value of η (or w) in the point $z = 0$. Let us consider first the relation between λ, the dimensionless thickness of the layer, η_0 and v_0, which holds in the zero-bias case.

Integrating the second equation of (11), we obtain

$$\frac{w^2}{2} - \frac{\eta_0^2}{2} = F(v) - F(v_0), \tag{12}$$

where $F(v) = v + \alpha \ln(1 - \alpha - \delta - \sigma + v) + \delta \ln[1 + \sigma/v]$. Hence the first equation of (11) gives

$$\lambda = \pm\int_{v_0}^{v_L} \frac{dv}{v\sqrt{2[F(v) - F(v_0)] + \eta_0^2}} \pm \int_{v_0}^{v_R} \frac{dv}{v\sqrt{2[F(v) - F(v_0)] + \eta_0^2}}. \tag{13}$$

It is obvious that η_0 can be evaluated from (13) using an iterative procedure when v_0 is known, and vice versa.

If $\alpha = 0$ and $\delta = 0$, the integrals involved in (13) are readily taken (cf. [18]), giving for the running point

$$\varkappa = \pm\int_{v_0}^{v} \frac{dv}{v\sqrt{2(v - c)}} = \pm\sqrt{2}\left[\operatorname{Act}\left(\frac{v}{c}\right) - \operatorname{Act}\left(\frac{v_0}{c}\right)\right], \tag{14}$$

where $c = v_0 - \eta_0^2/2$ and

$$\operatorname{Act}\left(\frac{v}{c}\right)=\begin{cases}\dfrac{1}{\sqrt{-c}}\ln\left|\dfrac{\sqrt{v-c}-\sqrt{-c}}{\sqrt{v-c}+\sqrt{-c}}\right| & \text{if} \quad c<0,\\ -\dfrac{1}{\sqrt{v}} & \text{if} \quad c=0,\\ \dfrac{1}{\sqrt{c}}\operatorname{tg}^{-1}\sqrt{\dfrac{v-c}{c}} & \text{if} \quad c>0.\end{cases}$$

In the case of large arguments $\operatorname{Act}\left(\frac{v}{c}\right)$ reduces to

$$\operatorname{Act}\left(\frac{v}{c}\right)=-(v-c)^{-1/2}+\frac{c}{3}(v-c)^{-3/2}+\frac{\pi}{2}c^{-1/2}\delta_{1,\operatorname{sign} c}, \tag{15}$$

where δ is the Kroneker delta-symbol.

Let us now distinguish two cases in equilibrium when v_0 or η_0 has to be taken as a previously unknown parameter. If the boundary values of $v(x)$ are equal, $v_L = v_R$, the free-carrier density has an extremum in the midpoint of the layer, and we can evaluate v_0 using (13) and the known value $\eta_0 = 0$. However, if the boundary values of $v(x)$ differ, but $\operatorname{sign}\psi(v_L) = \operatorname{sign}\psi(v_R)$, we determine first whether the extremum point exists within the layer. Taking $v_0 = \min(v_L, v_R)$ if $\psi(v) > 0$, or $v_0 = \max(v_L, v_R)$ if $\psi(v) < 0$, and $\eta_0 = 0$, we calculate the corresponding λ using (13). If it turns out that the computed value of λ is greater than the given thickness of the layer, there is apparently no extremum within the layer. In such a case v_0 should be kept equal to the boundary value chosen as stated above, with η_0 being evaluated from (13). On the contrary, if the computed value of λ is less than the given thickness of the layer, an extremum exists within the layer, and we can proceed as in the case if $v_L = v_R$. If $\psi(v_L)$ and $\psi(v_R)$ have different signs, we take $v_0 = v_b$, the value in the point where space charge changes its sign, determined by the equation $\psi(v) = 0$, and evaluate η_0 from (13).

The evaluation of v_0 or η_0 is carried out by means of an iterative procedure based on calculating the variations δv_0 or $\delta\eta_0$, respectively, given by

$$\frac{\partial\lambda}{\partial v_0}\delta v_0=\delta\lambda, \qquad \frac{\partial\lambda}{\partial\eta_0}\delta\eta_0=\delta\lambda, \tag{16}$$

and exploiting the *regula falsi* scheme to obtain subsequent approximations.

5. Finite-bias case

We next consider how the solution of (11) is found in the case of finite biases. Using the equation

$$\frac{w^2}{2}-\frac{w_0^2}{2}-F(v)+F(v_0)=\iota\int_{v_0}^{v}\frac{w\,dv}{v^2}, \tag{17}$$

we can estimate the critical current at which the equilibrium charge distribution will be appreciably disturbed to obtain

$$\iota_0=\frac{v_0}{\lambda}. \tag{18}$$

In deriving (18) we have replaced the integral in (17) by the greater quantity λ and substituted the left-hand side of the equation by its approximation v_0. It has been practically shown that this estimation is sufficient to start with calculations.

In order to evaluate current-voltage characteristics, we give in succession new values to v_0 or to one of the parameters η_0 or w_0 and, calculating corresponding variations $\delta\iota$ by

$$\frac{\partial\lambda}{\partial\iota}\delta\iota=\delta\lambda, \tag{19}$$

adjust the current until the desired λ is reached and thus the specified boundary conditions of $v(x)$ met. The classification of the cases involved together with the values of the variables to be used in the starting point of the integration is given in the Table.

As already stated, the parameter values in the starting point of the integration should be chosen so that this point would fall into the layer region where w has its minimum absolute value. Since the location of this region can be essentially changed with the bias, it sometimes proves to be useful to shift the origin during the evaluation of the current-voltage characteristic and to switch over from one previously unknown parameter to another. We carry out such a change when, for the new-type starting point, in comparison with the old-type one, the inequality $w_{new} < w_{old}$ holds.

Classification of cases.

$v(x)$ distributions at relevant biases are plotted as thick lines, the thin lines being $v(x)$ distributions at low biases.

Extremum of $v(x)$ is present at low biases								Extremum of $v(x)$ is absent at any biases						
v_L-v_R	$\varphi(v_L)$	$\varphi(v_R)$	Bias	v_0	η_0	w_0	log $v(x)$	v_L-v_R	$\varphi(v_L)$	$\varphi(v_R)$	Bias	v_0	η_0	log $v(x)$
>0	>0	>0	low	v_{min}	—	0		>0	>0	≥0	low	v_R	η_R	
			medium	v_R	η_R	—					high	v_L	η_L	
			high	v_L	η_L	—		<0	≥0	>0	any	v_L	η_L	
0	≥0	≥0	any	v_{min}	—	0		<0	<0	≤0	low	v_R	η_R	
<0	>0	>0	any	v_{min}	—	0					high	v_L	η_L	
<0	<0	<0	low	v_{max}	—	0		>0	≤0	<0	any	v_L	η_L	
			medium	$v_{L,R}$	$\eta_{L,R}$	—		>0	>0	<0	low	v_b	η_b	
			high	v_L	η_L	—					high	v_L	η_L	
0	≤0	≤0	any	v_{max}	—	0		<0	<0	>0	low	v_b	η_b	
>0	<0	<0	any	v_{max}	—	0					high	v_L	η_L	

Whenever switching over from one previously unknown parameter to the other, the known values of the latter, corresponding to a few preceding points of the current-voltage characteristic, have to be stored because all the points of the current-voltage characteristic are found by making first the linear extrapolation of $\iota = \iota(\nu_0)$ or $\iota = \iota(\eta_0)$ [or $\iota = \iota(w_0)$] and carrying out the adjustment in the *regula falsi* scheme thereafter.

In order to proceed when there is no extremum of $\nu(\varkappa)$ within the layer, and at the same time $\nu(\varkappa)$ varies rapidly near both boundaries, the starting point of the integration is placed in the point inside the layer where w has its minimum absolute value.* In this point $w = w_l = -\nu_l^2\psi(\nu_l)/\iota$ and both ν_l and η_l are unknown parameters.

The values of ϕ' are found as computational by-product. Observe that in the starting point of the integration we can always take $\phi' = 0$.

At very high biases we have, for almost all the layer, the asymptotic expression $w \to 0$ which gives $\eta_L = \iota/\nu_L$ and, consequently, $\iota = (\phi_L' - \phi_R')\nu_L/\lambda$.

The calculations have shown that the iterative procedure described converges rapidly when the non-linear part of the current-voltage relations is computed in scores of points characterized by equal differences in $\log \nu_0$ if there exists the extremum of $\nu(\varkappa)$ within the layer, and by equal differences in $\log \eta_0$ if the extremum of $\nu(\varkappa)$ within the layer does not exist.

In some cases with the extremum of $\nu(\varkappa)$ present, when ν_{extr} goes at low biases backwards as compared to the general trend (e. g., curve *1'* in Figure), η_{extr} should be treated as the known parameter until $d\nu_{extr}/d\iota$ changes its sign.

Integration through the layer has been carried out by the linear extrapolation with equal differences in $\log z$, the number of steps being s. Such a choice of the step-size guarantees an adequate computation of the $\varkappa(z)$ curves in their slowly changing parts. Otherwise too large step-sizes in $\varkappa$ will occur, which may essentially lower the accuracy of the numerical results. Thus, in the starting point of the step-by-step integration, where we take $z = z_1$ and $\nu = \nu_1$, the following condition is to be fulfilled

$$z_1 \nu_0^{-1/2} = \left(2\left|\frac{\nu_1}{\nu_0} - 1\right|\right)^{1/2} \ll 1. \tag{20}$$

The value of z for the subsequent integration step N is found as $z_N = a_z z_{N-1}$, where $a_z = [(\nu_B - \nu_0)/(\nu_1 - \nu_0)]^{1/2s}$. Here ν_B is the boundary value of $\nu(\varkappa)$ that one meets integrating in given direction. Several hundreds of steps and the value of $z_1\nu_0^{-1/2}$ equal to about 0.01 proved to ensure an adequate treatment of the current-voltage characteristics.

6. Conclusion

A numerical iterative method for the solution of one-dimensional basic one-carrier steady-state transport equations describing the conduction in extrinsic compensated semiinsulators provided with two ideal metal contacts is proposed. The model used is of quite a general character: space-charge effects, diffusion, trapping and recombination of carriers, and unequality of the contacts are all taken into account. Attention is paid to the achievement of an algorithm sufficiently sound and efficient

* The inflection point of $\nu(\varkappa)$ determined by $d^2\nu/d\varkappa^2 = 0$ can likewise be the origin in these cases. Then $w^2 = w_i^2 = -\nu_i\psi(\nu_i) - \iota w_i/\nu_i$.

to cope with the serious difficulties present in the numerical investigation of the problem. Classification of the different situations covered by the model is presented. The method has been developed with the object of analyzing the problem "exactly", which would allow the accuracy of more simplified solutions to be assessed and further improvements of the theory to be outlined. It should be possible, without substantial complications in computing, to elaborate the model to include effects such as field-dependent boundary values and some other parameters, and involved level spectra in the gap.

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ÜLDINE MEETOD MONOPOLAARSE INJEKTSIOON- JA EKSTRAKTSIOONVOOLU VOLTAMPERTUNNUSJOONTE ARVUTAMISEKS TAHKETES KEHADES

Esitatakse efektiivne numbriline meetod monopolaarse ruumlaengujuhtivuse uurimiseks struktuurides metall—poolisolaator—metall.

А. САПАР, А. РОЗЕНТАЛЬ

ОБЩИЙ МЕТОД ДЛЯ РАСЧЕТА ВОЛЬТ-АМПЕРНЫХ ХАРАКТЕРИСТИК ТОКА МОНОПОЛЯРНОЙ ИНЖЕКЦИИ И ЭКСТРАКЦИИ В ТВЕРДЫХ ТЕЛАХ

Предложен эффективный метод численного решения одномерной задачи о монополярной электропроводности в компенсированных высокоомных полупроводниках, снабженных двумя неодинаковыми идеальными металлическими контактами, при учете диффузии, прилипания и рекомбинации носителей, а также влияния объемного заряда. Дается классификация соответствующих ситуаций, возникающих в полупроводнике.