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T. VIIK

METHOD OF REGIONAL AVERAGING IN THE RADIATIVE TRANSFER

In paper [1] we used the method of regional averaging to solve analytically the equation of radiative transfer in the case of spherical and scattering stellar atmosphere. It was assumed that the opacity of the atmosphere varies inversely as the n-th power of the geometrical radius r of the layer. Unlike Y. S. Chou and C. L. Tien [2,3] who considered homogeneous medium we could not ignore the members expressing curvature in the higher-order (than the first) moment equations because in the inhomogeneous case when opacity equals to $^3/_2r$ all angular moments diverge.

In the case of real stellar atmosphere the dependence of the opacity on the geometrical radius of the layer is very complicated, and therefore the solution of the problem can be found only numerically, e. g. using difference methods.

However, these methods require powerful computers with a large storage. In the present paper we show how this problem can be solved in a quite simple way.

In a spherical atmosphere the monochromatic angular moments of the intensity J, H, K and L satisfy the following equations:

$$\begin{aligned} &\frac{dH}{d\tau} = 2fH + \beta(J - B), \\ &\frac{dK}{d\tau} = f(3K - J) + H, \\ &\frac{dL}{d\tau} = f(4L - 2H) + K - \frac{1}{3}\beta B - \frac{1}{3}\omega J, \\ &3K - J = 4L - 2H, \end{aligned} \tag{1}$$

where

$$f = [(k_\nu + \sigma_\nu)\varrho r]^{-1}, \quad \beta = \frac{k_\nu}{k_\nu + \sigma_\nu}, \quad \omega = 1 - \beta,$$

and B is the monochromatic Planck function.

The last equation of system (1) is obtained following the approximations of Y. S. Chou and C. L. Tien [3].

The boundary conditions are given as [3]

$$\begin{aligned} &J(\tau_1) + 2H(\tau_1) = B_1, \\ &J(\tau_1) - 3K(\tau_1) = 0, \\ &J(0) - 2H(0) = 0. \end{aligned} \tag{2}$$

where τ_1 is the monochromatic optical thickness of the atmosphere, and $B_1 = B(\tau_1)$.

Now we change the functions as follows:

$$\begin{aligned} x &= J + 2H, \\ y &= J - 3K, \\ z &= J - 2H, \end{aligned} \tag{3}$$

and after substituting relations (3) into (1) we have

$$\begin{aligned} x' &= G_{11}x + G_{12}y + G_{13}z + F_1, \quad & x(\tau_1) &= B_1, \\ y' &= G_{21}x + G_{22}y + G_{23}z + F_2, \quad & y(\tau_1) &= 0, \\ z' &= G_{31}x + G_{32}y + G_{33}z + F_3, \quad & z(0) &= 0, \end{aligned} \tag{4}$$

where

$$\begin{aligned} &G_{31} = \frac{1}{12}(8\omega + 1), \quad G_{32} = \frac{1}{3}(3f + 4), \quad G_{33} = \frac{1}{12}(8\omega - 17), \\ &G_{21} = \frac{1}{3}(3f + 3\beta - 2 + 2\omega), \quad G_{22} = \frac{1}{3}(12f + 4), \\ &G_{23} = G_{21} - 2f, \quad G_{11} = 2f + 2\beta + G_{31}, \\ &G_{12} = G_{32}, \quad G_{13} = -2f + 2\beta + G_{33}, \end{aligned} \tag{5}$$

and

$$\begin{aligned} F_1 &= -\frac{8}{3}\beta B, \\ F_2 &= \frac{1}{4}F_1, \\ F_3 &= -\frac{1}{2}F_1. \end{aligned} \tag{6}$$

Now we reduce the boundary value problem (4) to a common Cauchy problem, following the ideas of G. B. Rybicki and P. D. Usher [4].

Let us define a function ψ by the relation

$$z = \psi + Tx + Ry, \tag{7}$$

where T and R are certain functions which will be defined in the following way:

$$\begin{aligned} R' &= G_{32} + (G_{33} - G_{22})R - G_{12}T - G_{13}TR - G_{23}R^2, \quad & R(0) &= 0, \\ T' &= G_{31} + (G_{33} - G_{11})T - G_{21}R - G_{23}TR - G_{13}T^2, \quad & T(0) &= 0. \end{aligned} \tag{8}$$

Function ψ can now be found from the equation

$$\psi' = (G_{33} - G_{13}T - G_{23}R)\psi - F_1T - F_2R + F_3, \quad \psi(0) = 0, \tag{9}$$

and the functions x and y can be found from equations

$$\begin{aligned} x' &= (G_{11} + G_{13}T)x + (G_{12} + G_{13}R)y + G_{13}\psi + F_1, \quad & x(\tau_1) &= B_1, \\ y' &= (G_{21} + G_{23}T)x + (G_{22} + G_{23}R)y + G_{23}\psi + F_2, \quad & y(\tau_1) &= 0, \end{aligned} \tag{10}$$

respectively.

As it can be seen from equations (8) through (10), the generalized Riccati transformation (7) has converted the linear third-order two-point boundary problem into an equivalent one-point system.

The problem can be solved in the following way. System (8) may be integrated from $\tau = 0$, storing at each step both R and T. Next, equation

(9) may be integrated from $\tau = 0$, storing ψ, since R and T are now known functions. Then equations (10) are integrated from $\tau = \tau_1$, since ψ, R and T are known.

At last, we can find function z from the formula (7), and angular moments J, H, K and L from formulae (3).

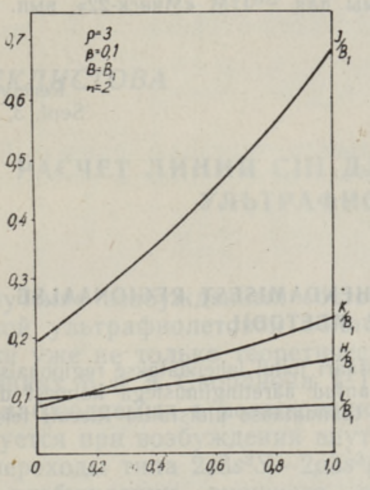

Fig. 1.

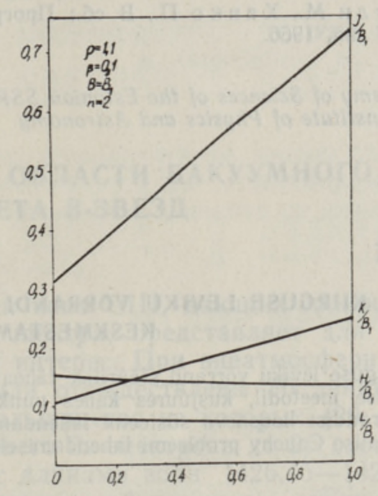

Fig. 2.

This approximate method can be used when we are not interested in an angular distribution of outward or inward intensity. The solution of the equation of radiative transfer in a spherical atmosphere by the method of regional averaging can also be used as a first approximation in the high-accuracy iteration schemes.

The program has been written in algorithmic language MALGOL [5] for solving the problem on the computer Minsk-22 at the Institute of Cybernetics of the Estonian Academy of Sciences. Some examples of solutions with prefixed parameters are given in Figs 1—3, representing the monochromatic angular moments J, H, K and L as functions of optical depth. It is assumed that the ratio of opacity coefficients β and Planck function B_1 are constants throughout the atmosphere. The opacity $\varrho(k_\nu + \sigma_\nu)$ is assumed to be varying inversely as the n-th power of the geometrical radius of the layer. In the figures ϱ means the ratio of outer and inner radii of the atmosphere.

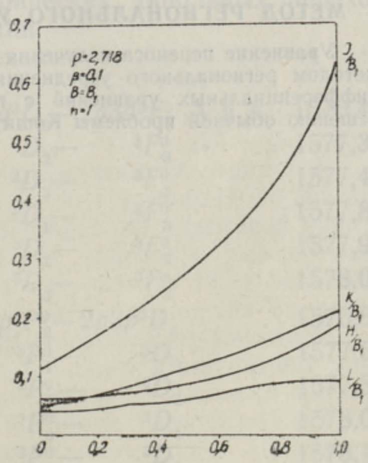

Fig. 3.

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KIIRGUSE LEVIKU VÕRRANDI LAHENDAMISEST REGIONAALSE KESKMESTAMISE MEETODIL

Kiirguse leviku võrrand sfäärilise täheatmosfääri juhul lahendatakse regionaalse keskmestamise meetodil, kusjuures kahes punktis antud ääretingimustega kolmest diferentsiaalvõrrandist koosneva süsteemi lahendamine taandatakse üldistatud Riccati teisenduse abil tavalise Cauchy probleemi lahendamiseks.

Т. ВИЙК

МЕТОД РЕГИОНАЛЬНОГО УСРЕДНЕНИЯ ДЛЯ ПЕРЕНОСА ИЗЛУЧЕНИЯ

Уравнение переноса излучения в случае сферической звездной атмосферы решается методом регионального усреднения. При этом решение получаемой системы из трех дифференциальных уравнений с граничными условиями в двух точках сводится к решению обычной проблемы Коши при помощи обобщенного преобразования Риккати.