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THE THRESHOLD THEORY AND DYNAMIC PROGRAMMING IN TOWN PLANNING

It is shown here that if the threshold theory of B. Malisz [1, 2] is mathematically formulated, a dynamic programming problem arises. In order to take into account the time-dependence of urban development costs a new numerical dynamic programming technique is developed. The mathematical formulation of the threshold theory opens up a prospect of considering a large amount of preliminary information and allows to determine the optimal urban development policy with the aid of electronic computers.

1. Introduction

Complexity of the problems which arise in urban planning as well as the great amount of information which is to be taken into account in solving them, makes the development of adequate urban planning techniques vitally necessary. It holds particularly true for the countries where an essential part of capital investments are allocated by governmental authorities.

One of the new techniques in town planning is the so-called threshold theory worked out by B. Malisz [1, 2]. That theory is based on complex quantitative estimation in money values of all urban areas which can be developed during the planning period. The stress has been put on the fact that towns in their spatial growth encounter physical limitations which can be overcome only by large costs. Therefore the urban development costs curve has a stepwise character that must be taken into consideration by comparing alternative urban development policies. In recent years the threshold theory has been used in various countries, including the USSR [3].

At the same time some advances have been made in the field of mathematical model-building for urban planning. Methodological aspects of such an approach have been considered by various authors [4–11]. In some of the models the transport costs of inhabitants are used as an objective function [12–15]. Since there are other development costs which are not less important than the transport costs, these models are somewhat limited. Less complicated optimization problems can be solved by the aid of simple linear models [16–21]. Application of the dynamic programming techniques sometimes enables to take into account nonlinear relationships between the development rate and housing costs [22, 23]. In order to work out optimal long-term plans dynamic linear models can be used [22, 24, 25].

In this paper the relationships between the threshold theory and dynamic programming is considered. An improved dynamic programming technique for town planning is developed.

2. Threshold theory and "orthodox" dynamic programming

Let us consider the following problem. In a town a certain amount of dwelling space is to be erected. A number of areas are available for development purposes. The development costs depend upon the area as well as upon the amount of dwelling space built in the area. The task consists in finding the policy by which the total cost of housing development will be minimum.

It is essential that all kinds of costs which may considerably influence the development policy should be taken into account. Special care should be taken to calculate costs of engineering networks, transport accomodations, etc. In order to relate initial and running costs, one must be converted to the equivalent of the other; usually it is common to convert the initial costs to their annual equivalent so that this can be added to the running costs. Therefore the so-called converted costs g must be used, which are calculated from the formula

$$g = E_n K + e, \quad (1)$$

where E_n — the coefficient of efficiency of inital costs (in town planning $E_n = 0.1$ is used), K — initial costs, e — running costs.

Social factors which can be put into money value must also be included into converted costs, e. g. the loss of time of inhabitants on trips [25–27].

Towns in their spatial growth encounter physical limitations, these being due to topographical reasons or to the technology of various public utility networks. They can be overcome by large costs. Confronted with a threshold, a town has the tendency to keep within the threshold line. The threshold theory is based on estimating the threshold costs indispensable for "opening" new urban territory, and on calculating the threshold capacity (expressed in the number of new inhabitants or in the amount of new dwelling space). This enables to define the sequence in which various adjacent areas should be developed.

If it is sufficient to consider in every area only one housing type, the comparison of areas according to their converted costs (1) may prove sufficient to fix the sequence of their development. Such an approach characterizes the threshold theory [1, 2] and has been used also in [28]. However, in reality one has to consider various housing types, differing e. g. in the number of storeys.

It is known [29] that considering an area in isolation the housing costs increase with the increase of number of storeys. The other side of the coin is that in multi-storey housing the housing density may be increased which will be accompanied by the growth of threshold capacity. So the threshold costs of the next area can be postponed and sometimes the development of less economic areas can be avoided.

The consideration of various housing types makes the problem of urban development much more complicated. We are going to consider this problem with the aid of dynamic programming methods.

The dependence of converted costs $f_i(x_i)$ of a unit of dwelling space in the area i upon the amount of dwelling space x_i can be approximately expressed by the following hyperbolic law

$$f_i(x_i) = a_i + \frac{b_i}{x_i} \quad (\text{for } x_i > 0). \quad (2)$$

Here a_i and b_i are constants.

The total housing costs $F_i(x_i)$ can be expressed in the form

$$F_i(x_i) = f_i(x_i) \cdot x_i = a_i x_i + b_i(x_i), \quad (3)$$

where

$$b_i(x_i)=0 \qquad \text{when } x_i=0,$$

$$b_i(x_i)=b_i \qquad \text{when } x_i>0.$$

In terms of the threshold theory $b_i(x_i)$ are to be considered as threshold costs.

It is to be mentioned that in the threshold theory only large costs necessary to "open" a new area are considered as threshold costs [2]. Yet practically the development of any new area demands certain amount of investment which is almost independent of the amount of dwelling space. Therefore we are going to take into account also comparatively small threshold costs as well as the differences in costs which are proportional to the amount of dwelling space. Such an approach increases considerably the number of data. Since we are going to put our problem into a mathematical form and solve it by means of electronic computers, the problem of the number of data is not so critical. Besides that, now it is possible to analyze the urban development plan in greater detail.

The optimal urban development policy can be determined by solving the following problem:

$$\min F=\min_{x_i} \sum_{i=1}^{n} F_i(x_i), \tag{4}$$

subject to

$$\sum_{i=1}^{n} x_i=Q, \tag{5}$$

$$0 \leqslant x_i \leqslant q_i, \tag{6}$$

where q_i is the limit capacity of the area i, n is the number of areas which are considered and Q is the total amount of dwelling space to be erected.

The problem (4) to (6) may be considered as a multistage decision process whose solution is given by the following recurrence relation:

$$\varphi_k(Q)=\min_{0\leqslant x_k\leqslant q_k}[F_k(x_k)+\varphi_{k-1}(Q-x_k)] \qquad (k=1,\,2,\ldots,\,n), \tag{7}$$

where

$$\varphi_1(Q)=F_1(Q)$$

and $\varphi_k(Q)$ denotes the minimal total development costs when k areas are developed.

In practice usually the $F_i(x_i)$ are much more complicated functions than (3). Therefore wellknown numerical techniques of dynamic programming [30–32] must be used to solve the problem (4) to (6).*

The same method is applicable when various housing types are considered. In this case for every value of x_i the least costly type of housing must be chosen. Denoting $F_{ij}(x_i)$ the development costs in area i when housing type j is used, we have

$$F_i(x_i)=\min_j F_{ij}(x_i). \tag{8}$$

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* It is to be noted that since the problem (4) to (6) can be solved numerically by dynamic programming techniques there is no need in this case to use complicated heuristic procedures as proposed in [33].

In the latter case it must be taken into account that the upper limit of the amount of dwelling space in an area depends on the housing type too.

In areas where renewal measures are considered x_i denotes the increase in dwelling space by slum clearance.

The drawback of the model considered above is in that it is static. Since urban development plans are usually worked out for a comparatively long time-period it is essential to take into account the time-dependence of development costs (e. g. the additional costs connected with slum clearance diminish as the physical condition of old houses gets worse) as well as to use discounting in the objective function. That requires some improvement of the dynamic programming model described above.

3. An improved dynamic programming technique

It is very difficult to include the time-factor into the previous model in its general form since in dynamic programming the growth of the number of dimensions arouses serious technical difficulties in solving the problem. Therefore we are going to make a simplification assuming that we know already the optimal sequence of the development of areas. We shall also assume that only one area is developed at a time and that the growth rate of the dwelling space is constant. The latter assumption has no principal meaning.

The task is to determine the values x_i so that the discounted development costs of all the areas are minimal and the given increase in dwelling space is obtained.

In this case the development costs in an area i depend on the amount of dwelling space x_i as well as on the time interval t_i when the development of the area begins, i. e.

$$F_i = G_i(t_i, x_i). \qquad (9)$$

Since the rate of growth of the dwelling space is given beforehand, the time-interval t_i depends on the amount of dwelling space already built in areas 1, 2, . . . , $i-1$:

$$t_i = t_i\left(\sum_{j=1}^{i-1} x_j\right). \qquad (10)$$

From (9) and (10) we obtain

$$F_i = F_i\left(\sum_{j=1}^{i-1} x_j,\ x_i\right) \qquad (i=2, 3, \ldots, n), \qquad (11)$$

$$F_1 = F_1(x_1).$$

To take into account discounting, F_i must comprise the discounted development costs of the area i when in the previous areas an amount $\sum_{j=1}^{i-1} x_j$ of dwelling space has been erected and the amount x_i of dwelling space is erected in the area i. All the costs must be discounted relative to the same fixed year, e. g. relative to the first year of the planning period.

Our task is to determine the optimal development rate for every area. This problem can be formulated as follows:

$$\min F = \min \sum_{i=1}^{n} F_i\Big(\sum_{j=1}^{i-1} x_j,\ x_i\Big) \tag{12}$$

under the conditions

$$\sum_{i=1}^{n} x_i = Q, \tag{13}$$

$$0 \leqslant x_i \leqslant q_i. \tag{14}$$

It is to be mentioned that the problem (12) to (14) is formally analogous to a problem considered in [37].

In (14) the q_i may also depend on the value of $\sum_{j=1}^{i-1} x_j$. In addition, it must be borne in mind that the development of an area must begin at once after the development of the previous area is finished.

The solution of this problem can be obtained by the following recurrence equation:

$$\varphi_k(Q) = \min_{0 \leqslant x_k \leqslant q_k} [F_k(Q-x_k,\ x_k) + \varphi_{k-1}(Q-x_k)] \qquad (k=2, 3, \dots, n), \tag{15}$$

$$\varphi_1(Q) = F_1(Q).$$

The essential difference between formulae (7) and (15) is in that in the latter case the functions F_k depend also on the amount of dwelling space erected in previous areas, i. e. on the value of

$$Q - x_k = \sum_{i=1}^{k-1} x_i.$$

Using eq. (15) it must be remembered that in the case under consideration the sequence of the areas is predetermined. The latter was not important when eq. (7) was used.

We are going to show on an example how a well-known numerical algorithm of solving "orthodox" dynamic programming problems can be modified for the case under consideration.

4. An example

Let there be three areas ($i=1, 2, 3$) which can be developed in the given order, the upper limits for their development being $q_1=q_3=40$, $q_2=50$. Since the development of the first area must begin in the first time-interval (if it is at all reasonable to develop the area 1) then $F_1(x_1)$ doesn't depend on time. The values of the $F_1(x_1)$ are given in Table 1.

Table 1

$F_1(x_1)$

x_1	0	10	20	30	40
$F_1(x_1)$	0	2.0	3.5	4.9	6.2

The development costs of the second and third area depend on the amount of dwelling space that has been already built in previous areas, since that determines the time-interval when their development begins. These

Table 2

$F_2(x_1, x_2)$

x_1 \ x_2	0	10	20	30	40	50
0	0	2.4	3.9	5.3	6.5	7.5
10	0	2.2	3.7	5.1	6.3	7.3
20	0	1.9	3.3	4.7	5.8	6.9
30	0	1.6	3.0	4.4	5.6	6.6
40	0	1.5	2.9	4.3	5.5	6.5

Table 3

$F_3(x_1+x_2, x_3)$

x_1+x_2 \ x_3	0	10	20	30	40
0	0	2.2	3.7	5.1	6.3
10	0	2.1	3.6	5.0	6.2
20	0	2.0	3.5	4.9	6.1
30	0	1.9	3.4	4.8	6.0
40	0	1.8	3.3	4.7	5.9
50	0	1.7	3.2	4.6	5.8
60	0	1.6	3.1	4.5	5.7
70	0	1.4	2.9	4.2	5.3
80	0	1.3	2.8	3.9	4.9
90	0	1.2	2.7	3.8	4.8

Table 4

$F_{1+2}(x_1, x_2)$

x_1 \ x_2	0	10	20	30	40	50
0	0	2.4	3.9	5.3	6.5	7.5*
10	2.0*	4.2	5.7	7.1	8.3	9.3
20	3.5*	5.4	6.8	8.2	9.3	10.4*
30	4.9*	6.5	7.9	9.3	10.5	11.5*
40	6.2*	7.7	9.1*	10.5	11.7	12.7*

data are given in Tables 2 and 3. For instance, from Table 2 we find that if in the first area 30 units of dwelling space are built then the discounted developing costs in the area 2, when $x_2=40$, are 5.6. Likewise from Table 3 it is to be seen that when 70 units of dwelling space are built in the first two areas then the discounted costs of 20 units in the third area equal 2.9.

It is to be mentioned that the diminishing of costs F_2 (or F_3) by growing x_1 (or x_1+x_2) may be due to discounting as well as to the diminishing of the value of old houses in renewal areas, etc.

Let us consider now the case when the first two areas are developed. The sums of the discounted costs for developing these two areas for all possible combinations of x_1 and x_2 are given in Table 4. The values of F_{1+2} (x_1, x_2) have been calculated by the formula

$$F_{1+2}(x_1, x_2) = F_2(x_1, x_2) + F_1(x_1). \tag{16}$$

The values of $F_1(x_1)$ and $F_2(x_1, x_2)$ are taken from Tables 1 and 2.

As an example, from Table 4 it can be seen that if $x_1=30$ and if after that in the second area 40 units of dwelling space are erected, the total discounted costs equal 10.5.

For every given Q the optimal development policy can be found by the aid of formula (15), which in this case takes the form

$$\varphi_2(Q) = \min_{0 \leqslant x_2 \leqslant 50} [F_2(Q-x_2, x_2) + F_1(Q-x_2)]. \tag{17}$$

The optimal developing policy for every given Q can be found on the basis of the Table 4, seeking the minimal element on the corresponding diagonal. Minimal values of F_{1+2} (x_1, x_2) are denoted with asterisks. For istance, if $Q=60$, then the optimal policy is $x_1=40$ and $x_2=20$. In this case the total discounted costs equal 9.1.

Table 5

$\varphi_2(x_1+x_2)$

x_1+x_2	10	20	30	40	50	60	70	80	90
$\varphi_2(x_1+x_2)$	2.0	3.5	4.9	6,2	7,5	9.1	10.4	11.5	12.7
x_1	10	20	30	40	0	40	20	30	40
x_2	0	0	0	0	50	20	50	50	50

In Table 5 for every value of x_1+x_2 the corresponding value of φ_2 (x_1+x_2) is given. Table 5 gives also values of x_1 and x_2 corresponding to the optimal policy.

Now Table 6, where total development costs of all the three areas are given, can be calculated. It is assumed that for every given value of x_1+x_2 the optimal values of x_1 and x_2, which are given in Table 5, are used. The values of F_{1+2+3} (x_1+x_2, x_3) are calculated from the formula

$$F_{1+2+3}(x_1+x_2, x_3) = = F_3(x_1+x_2, x_3)+\varphi_2(x_1+x_2), \quad (18)$$

using Tables 3 and 5.

Table 6

$F_{1+2+3}(x_1+x_2, x_3)$

x_1+x_2 \ x_3	0	10	20	30	40
0	0	2.2	3.7	5.1	6.3
10	2.0*	4.1	5.6	7.0	8.2
20	3.5*	5.5	7.0	8.4	9.6
30	4.9*	6.8	8.3	9.7	10.9
40	6.2*	8.0	9.5	10.9	12.1
50	7.5*	9.2	10.7	12.1	13.3
60	9.1*	10.7	12.2	13.6	14.8
70	10.4*	11.8	13.3	14.6	15.7
80	11.5*	12.8	14.3	15.4*	16.4*
90	12.7*	13.9*	15.4*	16.5	17.5*

For every given Q optimal development policy can be found from the formula (15) which can now be written as follows:

$$\varphi_3(Q) = \min_{0 \leqslant x_3 \leqslant 40} [F_3(Q-x_3, x_3)+\varphi_2(Q-x_3)]. \quad (19)$$

Table 7

Q	0	10	20	30	40	50	60	70	80	90	100	110	120	130
$\varphi_3(Q)$	0	2.0	3.5	4.9	6.2	7.5	9.1	10.4	11.5	12.7	13.9	15.4	16.4	17.5
x_1+x_2	0	10	20	30	40	50	60	70	80	90	90	90/80	80	90
x_3	0	0	0	0	0	0	0	0	0	0	10	20/30	40	40

Optimal development policies in Table 6 are denoted with asterisks. On the basis of Table 6, Table 7 has been calculated where for every value of Q the minimal development costs $\varphi_3(Q)$ and corresponding values of x_1+x_2 and x_3 are given.

The solution of the problem is now easily obtained with the aid of Tables 7 and 5. For example, if $Q=120$ we get from Table 7 that $x_1+x_2=80$, $x_3=40$, and $\varphi_3(120)=16.4$. Further, Table 5 gives that the optimal policy is $x_1=30$ and $x_2=50$. So the problem has been solved.

It is to be noted that it is possible that a number of optimal policies exist. E. g. if $Q=110$ there are two optimal policies of urban development: 1) $x_1+x_2=90$, $x_3=20$, 2) $x_1+x_2=80$, $x_3=30$. In both cases we have $\varphi_3(110)=15.4$.

5. Heuristic approach to the optimization of urban development policy

The method described above can be easily programmed for an electronic computer. Programs for the "orthodox" dynamic programming problem expressed by formulae (4) to (6) can also be used if these are supplemented with a special program to take into account the time-dependence of functions F_i.

There are various possibilities to use the improved dynamic programming technique. As it was already pointed out, the threshold theory gives the sequence of development of the areas which is at least quite near to the optimal sequence. But that theory leaves open the question of what amount of dwelling space must be built in every area. The proposed method enables to solve this problem for a given sequence of areas taking into account the time-dependence of development costs. The problem can be solved also for various sequences of the areas and the policy with least development costs can be chosen.

Since the number of all possible sequences of n areas equals $n!$, there is no chance to make the calculations for all possible sequences, except, perhaps, for the case when $n \leqslant 4$. Therefore various heuristic procedures must be worked out to get the approximately optimal sequence of areas.

The following heuristic approach may be recommended to determine the sequence of areas. Comparing the development costs of areas by a certain Q in the case when $t_i = 1$, the area with least development costs must be fixed as the first area. Now the optimal development rate of the first area must be assumed. Generally it is reasonable to use the maximal capicity. Further the development costs of other areas must be compared assuming that their development begins immediately after the first area is fully developed. The most economic area must be fixed as the second area and the procedure goes on until the summary given capacity of the considered areas exceeds somewhat the required amount of dwelling space. Now the dynamic programming problem can be solved and optimal development rates for all the areas can be determined.

For determining the optimal urban development policy the following planning technique may also be used. It is common to work out urban development plans in two stages: stage plans (for about 5 years) and long-term plans (for about 25 years) [34, 35]. It is clear that considering a longer planning interval it is first of all essential to work out stage plans which to some extent take into account the further possibilities for urban development. Every 5 year the planning procedure must be repeated to work out a new long-term plan and a corresponding stage plan. Such a planning procedure is also typical of other branches of economy and is called floating integral planning [36].

In floating integral planning the most important role play the costs of the first years of the planning interval, which is expressed in the discounting of costs in the objective function.

Let us choose three most economic areas at the beginning of the planning interval and let us determine the optimal policy of their development by solving the dynamic programming problems for all $3! = 6$ possible sequences at a certain Q. As a result of the first step the first area in the sequence of minimal value of the objective function is fixed as the first area. Now all other areas are compared and three of them, the most economic after the development of the first area, already fixed, is finished, are chosen. The dynamic programming problem for these three areas is solved and the first area in the sequence with minimal costs is fixed as the second area to be developed. The procedure is continued till the sequence of all the areas is determined. At the end a dynamic programming problem with *n* areas is to be solved to get the optimal development rate for every area when all the areas are considered at the same time.

This procedure corresponds to the ideas of floating integral planning and requires the solving of 6*n* dynamic programming problems with 3 areas and one dynamic programming problem with *n* areas.*

There are certainly other possibilities to determine the sequence of areas.

However, in practice it often happens that the most difficult problem to solve is the determining of the optimal sequence of development of the areas. In this case a linear dynamic model can be used which has been worked out previously [22, 24, 25].

It is to be noted that similarly to the threshold theory the proposed dynamic programming technique may be used by planning a single town as well as by working out a development plan for a given region.

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* We assume that the number of available areas is greater than *n*, and that it is reasonable to consider the possibility of developing *n* areas.

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H. ABEN

LÄVEDE TEOORIA JA DÜNAAMILINE PROGRAMMEERIMINE LINNADE PLANEERIMISEL

Artiklis näidatakse, et B. Maliszi lävede teooria matemaatiline formuleerimine viib dünaamilise programmeerimise ülesandele. Et arvestada märgitud ülesande lahendamisel üksikute rajoonide linnaehitusiike kulude sõltuvust ajast ning opereerida sihifunktsioonis diskonteeritud kuludega, täiustatakse dünaamilise programmeerimise ülesannete numbrilise lahendamise «ortodoksset» metoodikat. Vaadeldakse mitmeid heuristilisi võtteid linnaehituslike otsuste optimiseerimiseks.

Х. АБЕН

ТЕОРИЯ ПОРОГОВ И ДИНАМИЧЕСКОЕ ПРОГРАММИРОВАНИЕ ПРИ ПЛАНИРОВКЕ ГОРОДОВ

В статье показывается, что при математическом формулировании теории порогов Б. Малиша возникает задача динамического программирования. Для учета зависимости градостроительных затрат отдельных районов от времени, а также для рассмотрения в качестве целевой функции дисконтированных затрат усовершенствуется известная методика численного решения задач динамического программирования. Рассматриваются различные эвристические методы оптимизации градостроительных решений.