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QUANTUM EMISSION CAUSED BY OPTICAL NUTATION

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Abstract. Quantum emission of a dielectric plate, caused by optical nutation under resonance pulse excitation is considered. The emission appears due to periodic time dependence of the refractive index and is connected with the transformation of the initial destruction operators of photons to a linear combination of the final destruction and the creation operators. The spectrum of the emission is described by a narrow Gaussian at $\Omega/2$, where Ω is the Rabi frequency. The process considered allows one to achieve remarkable conversion of the visible radiation to the infrared one.

Key words: photons, optical nutation, Rabi oscillation, zero-point energy, Hawking emission.

1. INTRODUCTION

In this communication we consider the emission of a photon which follows optical nutation excited in a medium by a strong resonant light pulse [1,2]. The mechanism of the emission has a pure quantum origin and is connected with a periodic change of the zero-point energy of the medium $E^{(0)}$ in the time t. Such $E^{(0)}(t)$ dependence arises from the corresponding oscillatory change of the refractive index in time. The frequency of the emitted radiation equals $\Omega/2$ (where Ω is the Rabi frequency) and for strong pulse excitation it can be in the infrared.

The mechanism of photon generation considered is connected with the transformation of field operators in time: the initial destruction operators are linear combinations of the final destruction and creation operators. As a result, the initial zero-point state, being the zeroth state for the initial destruction operators, is not the zeroth state for the final destruction operators. It means that in the final state there appear photons. (Besides, if initially there were some photons, then finally there appeared additional photons with opposite wave vectors [3].) The mechanism mentioned has an analogy with the Hawking mechanism of the emission of photons by a gravitationally collapsing star (so-called Black Hole emission) [4] and with that of photon emission by an accelerating mirror or the registration of photons by an accelerating photon detector (so-called Unruh radiation), see e.g. [5–7]. It is also analogous to the quantum emission of a medium under an abrupt change of the refractive index in time considered in [3, 8]. Another analogous process in solids is anharmonic decay of a strong local vibration [9] (this vibration causes a quasiperiodic change of the zero-point energy of phonons by that inducing the phonon generation).

2. RESONANT POLARIZATION

We consider a thin dielectric plate doped with atoms (ions), having zero-phonon transition at ω_0, and suppose that the plate is excited by a strong light pulse with the wave front parallel to the plate. We use the second quantization representation in which the quantity under consideration is the linear field operator $\hat{\psi}$. This operator satisfies the same Maxwell equation as the classical field amplitude ψ [5]. The latter equation reads [1, 2]

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\psi = -n_0^{-2}\frac{\partial^2}{\partial t^2}P^{nl} \ . \tag{1}$$

Here $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $\psi \equiv \psi(t,x,y)$, c is the light velocity, n_0 is the refractive index of the non-excited plate, P^{nl} is the nonlinear polarization of the unit area of the plate. In the case of resonant excitation $P^{nl} = N_a S_x D/d$, where N_a is the number of atoms in the unit area, d is the plate's thickness, S_x is the x component of the vector of the quasispin $\vec{S}$ satisfying the Bloch equation [1, 2]

$$\dot{\vec{S}} = \vec{\Omega}\times\vec{S}\ , \quad \vec{\Omega} = (\kappa E, 0, \omega_0)\ , \tag{2}$$

$\kappa = 2D/\hbar$, E is the strength of the electromagnetic field, D is the dipole matrix element. E has two parts: $E = E_0 + \psi$, where $E_0 = \mathcal{E}_0\cos\omega_0 t$ describes the strong light pulse with the mean frequency ω_0, and ψ is the weak part, which describes the considered field with the frequency $\omega \approx \Omega/2$, where $\Omega = \kappa\mathcal{E}_0 - 0 << \omega_0$. Analogously, the quasispin has two parts: $\vec{S} = \vec{S}_0 + \vec{\sigma}$. The quasispin $\vec{S}_0$ satisfies Bloch equation (2) with E replaced by E_0 and equals (in rotating wave approximation $(\Omega/\omega_0)^2 << 1$) [1, 2]

$$\vec{S}_0 = (\sin\Theta\sin\omega_0 t, -\sin\Theta\cos\omega_0 t, -\cos\Theta)\ ,$$

where

$$\Theta = \kappa\int_{-\infty}^{t}\mathcal{E}_0(\tau)d\tau\ .$$

The vector $\vec{\sigma}$ satisfies the equation

$$\dot{\vec{\sigma}} = \vec{\Omega}_0 \times \vec{\sigma} + \vec{\sigma}_0 \,, \tag{3}$$

where $\vec{\Omega}_0 = (\kappa E_0, 0, \omega_0)$, $\vec{\sigma}_0 = (0, \kappa\psi\cos\Theta, \kappa\psi\sin\Theta\cos\omega_0 t)$. We have to find the linear (with respect to ψ) part of σ_x. From (3) it follows that

$$\ddot{\sigma}_x = -\omega_0^2\sigma_x + \omega_0\kappa\psi\cos\Theta - \omega_0\kappa E\sigma_z \,. \tag{4}$$

Let us consider the term $\sim \sigma_z$. In the rotating wave approximation $\vec{\Omega}_0 \approx \vec{\Omega}^+ = (\Omega\cos\omega_0 t, \Omega\sin\omega_0 t, \omega_0)$ one gets

$$\dot{\sigma}_z \approx \Omega v + \kappa\psi\sin\Theta\cos\omega_0 t \ ,$$

$$\dot{v} \approx -\Omega\sigma_z - \kappa\psi\cos\Theta\cos\omega_0 t \,,$$

where $v = -\sigma_x\sin\omega_0 t + \sigma_y\cos\omega_0 t$. Solving the last equations, one finds

$$\sigma_z \approx \omega_0^{-1}\kappa\psi\sin\Theta\sin\omega_0 t$$

(up to terms $\sim (\Omega/\omega_0)^2$). One can see that the term $\sim \sigma_z$ in (4) has the small multiplier Ω/ω_0; besides, it oscillates fast $\sim (\sin 2\omega_0 t)$. Consequently, this term gives very small contribution and can be neglected. This gives

$$\sigma_x \approx \omega_0^{-1}\kappa\psi\cos\Theta$$

(up to terms $\sim (\Omega/\omega_0)^2$, $\Omega = \kappa\mathcal{E}_0$) and

$$P^{nl} = (N_a D/d)(S_{0x} + \omega_0^{-1}\kappa\psi\cos\Theta) \,. \tag{5}$$

3. QUANTUM PARAMETRIC RESONANCE

In the case under consideration Eq. (1) has the plane wave solutions: $\psi = \exp(i\vec{k}\vec{r})\varphi_k$, where φ_k satisfies the equation

$$\left(\frac{\partial^2}{\partial t^2} + c^2k^2\right)\varphi_k = -p\frac{\partial^2}{\partial t^2}\varphi_k\cos\Theta, \tag{6}$$

$p = 2D^2N_a/n_0^2\hbar\omega_0$ describes the amplitude of time variation of the dielectric constant. Here we take into account that the term $\sim S_{0x}$ does not depend on x, y, and therefore does not contribute to the plane-wave solution with $\vec{k} \neq 0$. By introducing $\chi_\omega = (1 + p\cos\Theta)\varphi_k$, one obtains the following equation for χ_ω:

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\omega^2}{1 + p\cos\Theta}\right)\chi_\omega = 0 \,. \tag{7}$$

One can see that χ_ω satisfies the wave equation with the quasiperiodically time-dependent refractive index $n \sim n_0(1+\cos\Theta)^{1/2}$.

Let us consider an almost rectangular excitation pulse with the duration $t_0 >> \Omega^{-1}$ which reaches the plate at $t = 0$. At that Ω is constant for $0 < t < t_0$ and $\Omega = 0$ elsewhere. Then Eq. (7) reduces to the one describing the oscillator with locally periodically time-dependent frequency. In this case the so-called parametric resonance phenomenon takes place: solutions of (7) with $\omega \approx \Omega/2$ grow exponentially with time, see e.g. [7, 10]. If one chooses $\varphi_k = \chi_\omega = \exp(-i\omega t)$, $t < 0$, and supposes that the characteristic duration of the ends of the pulse τ_0, ($\tau_0 << t_o$) satisfies the adiabatic switching-on and -off conditions $\tau_0 >> \Omega^{-1}$, then for $p << 1$, $t > t_0$ one obtains $\varphi_k = \chi_\omega/(1+p)$,

$$\chi_\omega = \frac{1}{2}[(e^{\gamma_\omega t_0} + e^{-\gamma_\omega t_0})e^{-i\omega t} + (e^{\gamma_\omega t_0} - e^{-\gamma_\omega t_0})e^{i\omega t}] , \quad t > t_0 , \tag{8}$$

where

$$\gamma_\omega^2 = (\omega p/2)^2 - (\Omega - 2\omega)^2 > 0 . \tag{9}$$

Let us return now to field operators. We are interested in time dependence of the operator which satisfies the following initial condition:

$$\hat{\psi} = \exp(i\vec{k}\vec{r} - i\omega t)\hat{a}_{\vec{k}} , \quad t < 0 . \tag{10}$$

Here $\hat{a}_{\vec{k}}$ corresponds to the initial operator of the destruction of a photon with the wave vector $\vec{k}$. Taking into account solution (8) for χ_ω and bearing in mind that the time dependence $\exp(-i\omega t)$ corresponds to the destruction operators while the time dependence $\exp(i\omega t)$ corresponds to the creation operators, we get

$$\hat{\psi} = e^{i\vec{k}\vec{r}}[\hat{b}_{\vec{k}} ch(\gamma_\omega t_0)e^{-i\omega t} + \hat{b}^+_{\vec{k}} sh(\gamma_\omega t_0)e^{i\omega t}] , \quad t > t_0 , \tag{11}$$

which means that

$$\hat{a}_{\vec{k}} = ch(\gamma_\omega t_0)\hat{b}_{\vec{k}} + sh(\gamma_\omega t_0)\hat{b}^+_{-\vec{k}} . \tag{12}$$

Here $\hat{b}_{\vec{k}}$ and $\hat{b}^+_{-\vec{k}}$ correspond to the final ($t > t_0$) destruction and the creation operator (we take into account that the space dependence $\exp(i\vec{k}\vec{r})$ corresponds to the creation operator of a photon with the wave vector $-\vec{k}$).

Note that in the case of parallel mirrors, normal to the mirrors wave vectors $\vec{k}$ and $-\vec{k}$ should not be distinguished. For such $\vec{k}$, formula (12) corresponds to the squeezing transformation. At that, if $\gamma_\omega t_0 >> 1$, the degree of squeezing is high. This conclusion is in agreement with [11] where it was shown that repeating jumps between two frequencies of harmonic oscillator generate fast increasing squeezing.

4. QUANTUM EMISSION

Now one can find the number $N_{\vec{k}}$ of photons with the wave vector $\vec{k}$ generated by the unit surface of the plate:

$$N_{\vec{k}} = < i|(\hat{b}^{+}_{\vec{k}}\hat{b}_{\vec{k}} - \hat{a}^{+}_{\vec{k}}\hat{a}_{\vec{k}})|i> =$$

$$= sh^2(\gamma_\omega t_0)(1 + N^{(0)}_{\vec{k}} + N^{(0)}_{-\vec{k}}) - sh(2\gamma_\omega t_0)Re < i|\hat{a}_{\vec{k}}\hat{a}_{-\vec{k}}|i > , \qquad (13)$$

where $|i>$ is the initial state and $N^{(0)}_{\vec{k}} = < i|\hat{a}^{+}_{\vec{k}}\hat{a}_{\vec{k}}|i >$ is the number of photons with the wave vector $\vec{k}$ in the initial state. The last term in (13) differs from zero only for special-type two-photon-correlated initial states and will later be neglected.

If the initial photons are absent ($N^{(0)}_{\vec{k}} = 0$), the spectrum of the generated photons equals

$$W(\omega) = SN_k\rho(\omega) = \pi S\omega c^{-2} sh^2(\gamma_\omega t_0) , \qquad (14)$$

where S is the surface of the excited area of the plate, $\rho(\omega) = \pi\omega c^{-2}$ is the density of modes. If $Q \equiv \Omega t_0 p/2 >> 1$, the shape of the spectrum of the generated photons is a Gaussian with a small width:

$$W(\omega) \approx \frac{\pi\Omega S}{2c^2}\exp\left[Q - \frac{2t_0^2}{Q}(\omega - \Omega/2)^2\right] . \qquad (15)$$

Here we take into account that for $d \sim \lambda = 2c\Omega^{-1}$ the amplitude of the dielectric constant $p \sim fC$, where $f \leq 1$ is the oscillator strength of the resonant transition, C is the relative concentration of resonant atoms ($C \leq 1$). Therefore the dispersion ($\sim$ width) of spectrum (14) is small: $\sigma \sim f\Omega C/8Q^{1/2} << \Omega/2$.

If the excitation pulse is not a rectangular one, in (12) the pulse duration t_0 should be replaced by t_{0eff}, the duration of resonance $|\omega - \Omega/2| \leq \sigma$. Supposing the pulse has a Gaussian shape, one gets $\Omega = \Omega_{max}\exp(-(1/2t - t/t_0/2)^2)$ and $\sigma \sim (\Omega_{max} fC/t_{0eff})^{1/2}$. Equating $\Omega_{max} - \Omega = 2\sigma$, one finds $t_{0eff} \sim t_0(fC/\Omega t_0)^{1/5}$, and $Q = (\Omega t_0)^{4/5} \cdot (fC)^{6/5}$.

The total number of the generated photons is of the order

$$N_{tot} \sim \frac{\pi fCS\Omega^2}{8c^2Q^{1/2}}e^Q . \qquad (16)$$

One can see that N_{tot} essentially depends on the amplification parameter $Q \sim \Omega t_{0eff} fC$. To estimate Q and N_{tot}, we take a light pulse ~ 1 J with the duration $t_0 \sim 10^{-7}$ sec. Then for $S \sim 0.01$ cm^2, one gets $\Omega \sim 3 \cdot 10^{12}$ sec^{-1} and $d \sim 10^{-2}$ cm. Taking now $fC = 10^{-3}$, one gets $Q \sim 35$ and $N_{tot} \sim 10^{13}$. For a twice higher concentration and the same d we get a nonrealistic value $N_{tot} \sim 10^{28}$ (from the energy conservation law it follows that here N_{tot} cannot exceed 10^{21}). This means that, in fact, the working thickness is less than $\sim \lambda$.

Above the existing variation of the zero-phonon transition, frequency ω_0 from centre to centre was not taken into account. This variation (which is called inhomogeneous broadening) leads to the variation of Ω. One can neglect this variation if it remains smaller than σ. Taking into account that $\Omega = (\kappa^2 \mathcal{E}_0^2 + \Delta^2/16)^{1/2}$, where Δ is the variation of ω_0, one finds that the effect of the variations mentioned is negligible if

$$(\Delta/\Omega)^2 << 4fCQ^{-1/2} . \qquad (17)$$

In good crystals $\Delta \sim 10^{10} - 10^{11}$ sec^{-1} and in the considered case of $\Omega \sim 3 \cdot 10^{12}$ sec^{-1}, $Q = 35$, $fC = 10^{-3}$ condition (17) is fulfilled.

Above the effects connected with the spontaneous emission of the excited atoms were not taken into account either. In the case considered these effects are weak, while the energy loss due to the emission ($\sim \hbar\omega_0 Ne\gamma t_{0eff} \sim 10^{-3}$ J, $\gamma \sim 10^8$ sec^{-1} is the radiative decay rate, $Ne \sim 10^{16}$ is the number of atoms in the excited volume Sd) is small in comparison with the energy of the macroscopic polarization oscillations (~ 1 J).

If $N^{(0)}_{\vec{k}_0} \neq 0$ (i.e. there were initial photons with the wave vector $\vec{k}_0$), then, in addition, $N^{(0)}_{\vec{k}_0} sh^2(\gamma_\omega t_0)$ photons with the wave vector $\vec{k}_0$ and the same number of photons with the wave vector $-\vec{k}_0$ will be generated. This means that a) initial photons enhance the photon generation process considered; b) the creation of photons with the opposite (to $\vec{k}_0$) wave vector takes place. We call the latter process a dynamical phase-conjugated reflection (analogously to the corresponding static nonlinear optical process [12, 13]; see also [3], where the dynamical process was called photon–antiphoton conversion). We note also that the processes mentioned ($\sim N^{(0)}_{\vec{k}} + N^{(0)}_{-\vec{k}}$) allow us to use a resonator to promote the photon generation process considered.

Finally we note that inversion oscillations in a dielectric medium affect the phonon system and therefore can also cause the generation of phonons. Expression (14) obtained here holds also for the spectrum of generated phonons, if one replaces the oscillator strength f by the relative change of the elastic constants on electronic transition and the photon density of states $\rho(\omega)$ by the phonon density of states $\rho_{ph}(\omega)$. The latter is much larger than the former one ($\rho_{ph}(\omega) >> \rho(\omega)$). On the other hand, the feedback is much easier to achieve for photons than for phonons. Therefore, it depends on the experimental conditions, which process will prevail.

We may conclude that the considered quantum emission caused by the optical nutation allows us to achieve remarkable conversion of visible photons to the infrared ones.

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OPTILISE NUTATSIOONI KVANTKIIRGUS

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On uuritud dielektrilise plaadi kvantkiirgust optilise nutatsiooni korral, mida ergastatakse resonantse valgusimpulsiga. Kiirgus tekib tänu murdumisnäitaja perioodilisele muutumisele ajas, mis muundab footonite algoleku tekkeoperaatorid lõppoleku tekke- ja kaooperaatorite lineaarkombinatsiooniks. Kiirgusspektril on kitsa Gaussi joone kuju keskmise sagedusega $\Omega/2$, kus Ω on Rabi sagedus. Vaadeldud protsess lubab märgatava efektiivsusega muundada nähtavat valgust infrapunaseks.

КВАНТОВОЕ ИЗЛУЧЕНИЕ ПРИ ОПТИЧЕСКОЙ НУТАЦИИ

Владимир ХИЖНЯКОВ, Наталья ИВАНОВА

Рассматривается квантовое излучение диэлектрической пластинки при оптической нутации, возбуждаемой резонансным импульсом света. Причиной излучения является периодическая временная зависимость показателя преломления, приводящая к преобразованию начальных операторов рождения фотонов в линейную комбинацию конечных операторов рождения и уничтожения. Спектр излучения имеет форму узкой гауссовой линии со средней частотой $\Omega/2$, где Ω – частота Раби. Рассмотренный процесс позволяет с заметной эффективностью преобразовывать видимое излучение в инфракрасное.