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HAWKING PROCESS IN A VIBRONIC SYSTEM: RELAXATION OF STRONG VIBRATION

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Abstract. Relaxation of a strongly-excited local vibration caused by cubic anharmonic coupling with phonons is considered. The strong vibration is described in the classical limit while the phonons are considered quantum-mechanically. In this approximation the effect of the strong mode on phonons consists in the time dependence of elastic constants. As a result the zero-point energy of phonons also changes in time, causing the creation of phonons. The mechanism of phonon generation has an analogy with the Hawking mechanism of the Black Hole emission. The proposed theory allows a description of the two-phonon relaxation of the local mode for an arbitrary cubic anharmonicity and provides a nonexponential decay law for the initial relaxation stage. For the weak anharmonicity the decay law obtained is in agreement with the one given by quantum perturbation theory.

Key words: anharmonic interaction, local mode damping, phonon generation, Hawking mechanism.

INTRODUCTION

Electronic transitions in vibronic systems are often accompanied by strong excitation of local vibrations. Time evolution of such vibrations is determined by anharmonic interaction with other vibrational modes. The interaction causes an excitation of these modes and a local mode damping. In this communication, we propose a theoretical description of this damping process.

An essential peculiarity of the problem under consideration is the high degree of the initial excitation of the local mode. In this case the standard quantum perturbation theory may not be applicable for the mode damping. The classical description is not applicable either, as the initial state of the modes which became excited on the damping of the local mode, is nonclassical (at least at sufficiently low temperatures). A possible approach

to the problem is to use the classical description of a strongly excited local mode and the quantum description of all other, initially nonexcited modes. In such an approach the effect of the classical motion of the local mode on other modes consists in the time dependence of elastic constants.

Here we will consider the cubic anharmonicity $\sim x_i x_j x_l$, with x_l being a classical time-dependent coordinate of a strongly excited local mode. In the case considered the zero-point energy of the system changes in time, causing the creation of real phonons. The underlying mechanism of phonons' generation has an analogy with the Hawking mechanism of emission by a gravitationally collapsing star (so-called Black Hole emission) [1, 2] and with the mechanism of the emission of photons by an accelerated mirror or the registration of photons by an accelerated photon detector (so-called Unruh radiation), see e.g. [2–5]. In all these cases the time dependence of the zero-point energy causes a transformation of the initial destruction operators to the linear combination of creation and destruction operators in time. As a result the initial zero point state, being the zeroth state for the initial destruction operators, is not the zeroth state for the final destruction operators. It means that in the final state there appear photons (phonons).

In spite of the analogy mentioned there are still some essential differences in the problem considered and in the Hawking and the Unruh problem. The time dependence of the zero-point energy is quasiperiodic in our problem, while it is aperiodic in the Hawking and the Unruh problem. The physical reason of the time dependence is also different: in our case it comes from the anharmonic interaction of phonons with a strongly excited mode, while in the Hawking and the Unruh problem it is associated with a nonuniform reference frame in which the quantum field is considered. Below we will show that the considered mechanism of a local vibration relaxation in a weak coupling limit gives the well-known result of the quantum perturbation theory for the two-phonon anharmonic decay. The proposed theory allows also the description of the mentioned relaxation for an arbitrary cubic anharmonic interaction.

1. EQUATIONS FOR COORDINATE OPERATORS OF PHONONS

In the case of anharmonic interaction, $V_l = \frac{1}{2}\sum_{ii'} w_{ii'} x_i x_{i'} x_l$, with the local mode coordinate x_l being a function of t ($x_l = Q(t)$), the phonon coordinates $x_i, x_{i'}$ satisfy the following equation of motion:

$$\frac{\partial^2 x_i(t)}{\partial t^2} + \omega_i^2 x_i(t) + Q(t)\sum_{ii'} w_{ii'} x_{i'}(t) = 0\,; \quad i, i' = 1, 2, \ldots N\,. \qquad (1)$$

Here ω_i is the frequency of the phonon i, N is the number of phonon modes (N is of the order 10^{23}). We suppose that $Q(t) = 0$ for $t < 0$, and $Q(t) \to 0$ for $t \to \infty$. The first condition means that the local mode was excited

at $t = 0$, the second one accounts for the local mode relaxation. The interaction constants $w_{ii'}$ can be factorized as follows:

$$w_{ii'} = (e_i w e_{i'}) \equiv \sum_{n=0}^{n_0} e_{in} w_{nn'} e_{i'n'} , \quad \sum_i e_{in} e_{in'} = \delta_{nn'} .$$

Here n_0 is the number of the configurational coordinates contributing to the considered anharmonic interaction. Usually n_0 is small ($n_0 << N$).

Equations (1) have the same form both in the classical and in the quantum theory; in the latter x_i are linear operators ($x_i \equiv \hat{x}_i$). These operators can be presented in the form

$$\hat{x}_i(t) = (g_i(t)\hat{a}_i + g_i^*(t)\hat{a}_i^+)(\hbar/2\omega_i)^{1/2} , \tag{2}$$

where $g_i(t)$ are the solutions of a set of classical equations of motion

$$\frac{\partial^2 g_i(t)}{\partial t^2} + \omega_i^2 g_i(t) + Q(t) \sum_{i'} (e_i w e_{i'}) g_{i'}(t) \left(\frac{\omega_i}{\omega_{i'}}\right)^{1/2} = 0 \tag{3}$$

satisfying the initial conditions,

$$g_i(t) = \exp(-i\omega_i t + i\varphi_i) , \quad t \leq 0 ,$$

φ_i are arbitrary phases (final expressions should be averaged over φ_i), $\hat{a}_i$, and $\hat{a}_i^+$ are linear operators obeying the usual commutation relations

$$[\hat{a}_i, \hat{a}_{i'}^+] = \delta_{ii'} , \quad [\hat{a}_i, \hat{a}_{i'}] = [\hat{a}_i^+, \hat{a}_{i'}^+] = 0 .$$

These operators have the meaning of destruction ($\hat{a}_i$) and creation ($\hat{a}_i^+$) operators of phonons for $t < 0$, i.e. before the local mode excitation.

The system of differential equations (3) can be presented in the following integral form:

$$g_i(t) = \mathrm{e}^{-i\omega_i t} + \sum_{i'} (\bar{e}_i w \bar{e}_{i'}) \int_0^t \sin(\omega_i(t-\tau)) Q(\tau) g_{i'}(\tau) d\tau , \tag{3a}$$

($\bar{e}_i = e_i/\sqrt{\omega_i}$). By using an iteration procedure, one can find the following solution of these equations:

$$g_i(t) = \mathrm{e}^{-i\omega_i t} + \int_0^t d\tau \sin(\omega_i(t-\tau)) \sum_{i'} (\bar{e}_i D_{i'}(\tau) \bar{e}_{i'}) . \tag{4}$$

Here the $n_0 \times n_0$ matrix $D_i(\tau)$ satisfies the following Volterra-type integral equation:

$$D_i(\tau) = Q(\tau) w \left[I \mathrm{e}^{-i\omega_i \tau} + \int_0^\tau d\tau' G(\tau - \tau') D_i(\tau') \right] , \tag{5}$$

I is the $n_0 \times n_0$ unit matrix, $G(\tau)$ is the $n_0 \times n_0$ matrix with the elements

$$G_{n_1 n_2}(\tau) = \sum_i \bar{e}_{in_1} \bar{e}_{in_2} \sin(\omega_i \tau) \tag{6}$$

being the Green's functions of local dynamics. This matrix function can be calculated by standard methods of lattice dynamics [6, 7]. In an important case of the local mode's coupling with a single configurational mode ($n_0 = 1$) the matrices D and G reduce to usual one-dimensional functions.

2. TIME-DEPENDENT HAMILTONIAN OF PHONONS

With allowance made for expression (2) the Hamiltonian of the phonon system can be presented in the form

$$\hat{H}_p = \hat{H}_0 + \hat{V} ,$$

where

$$\hat{H}_0 = \frac{1}{2}\sum_i \hbar\omega_i[\varepsilon_i(t)(2\hat{a}_i^+\hat{a}_i + 1) + f_i^*(t)\hat{a}_i^{+2} + f_i(t)\hat{a}_i^2] , \tag{7}$$

$$\hat{V} = \frac{\hbar Q(t)}{4}\sum_{ii'}(\bar{e}_i w \bar{e}_{i'})(g_i(t)\hat{a}_i + g_i^*(t)\hat{a}_i^+)(g_{i'}(t) + g_{i'}^*(t)\hat{a}_{i'}^+) , \tag{8}$$

$$\varepsilon_i(t) = \frac{1}{2}(|g_i(t)|^2 + \omega_i^{-2}|\dot{g}_i(t)|^2) , \quad f_i(t) = \frac{1}{2}(g_i^2(t) + \omega_i^{-2}\dot{g}_i^2(t)) . \tag{9}$$

The Hamiltonian $\hat{H}_p$ can be diagonalized by the Bogolyubov-type transformation. Here this transformation will be carried out in two steps: first we diagonalize $\hat{H}_0$ and then $\hat{H}_p$.

$\hat{H}_0$ is diagonalized by the transformation

$$\hat{a}_i = \mu_i\hat{A}_i + \nu_i\hat{A}_i^+ ,$$

where $\hat{A}_i$ and $\hat{A}_i^+$ are new creation and destruction operators, satisfying the usual commutation relations,

$$\mu_i = [(1 + \varepsilon_i/r_i)/2]^{1/2} , \tag{10}$$

$$\nu_i = (f_i^*/|f_i|)[(\varepsilon_i/r_i - 1)/2]^{1/2} , \tag{11}$$

$$r_i = (\varepsilon_i^2 - |f_i|^2)^{1/2} . \tag{12}$$

In the new representation $\hat{H}_0$ and $\hat{V}$ have the form

$$\hat{H}_0(t) = \sum_i \hbar\overline{\omega}_i(t)\left(\hat{A}_i^+\hat{A}_i + \frac{1}{2}\right) ,$$

$$\hat{V}(t) = \frac{1}{4}\hbar Q(t)\sum_{ii'}(\bar{e}_i w \bar{e}_{i'})\hat{\xi}_i\hat{\xi}_{i'} ,$$

where $\overline{\omega}_i = \omega_i r_i$, $\overline{g}_i = g_i\mu_i + g_i^*\nu_i^*$, $\hat{\xi}_i = \overline{g}_i\hat{A}_i + \overline{g}_i^*\hat{A}_i^+$. Considering $\hat{q}_i = \hat{\xi}_i(\hbar/2\overline{\omega}_i)^{1/2}/|\overline{g}_i|$ as new coordinate operators, the Hamiltonian H_p can be presented as follows:

$$\hat{H}_p(t) = \frac{1}{2}\sum_i\left(-\hbar^2\frac{\partial^2}{\partial q_i^2} + \overline{\omega}_i^2 q_i^2\right) + \frac{1}{2}\sum_{ii'}(e_{1i}B(t)e_{1i'})q_i q_{i'} . \tag{13}$$

Here

$$B_{nn'}(t) = Q(t)C^2(t)w_{nn'} , \tag{14}$$

$$e_{1i} = e_i r_i^{1/2} |\overline{g}_i(t)|/C(t) , \tag{15}$$

$$C^2(t) = \sum_i e_i^2 r_i |\overline{g}_i(t)|^2 . \tag{16}$$

Hamiltonian (13) can be diagonalized by the methods of local dynamics (see, e.g. [8]). Here we use these methods in the secondary quantization formulation given in [9]. The diagonalization is achieved with the operators

$$\hat{b}_j = \frac{1}{2} \sum_i \frac{c_{ij}}{(\overline{\omega}_i \Omega_j)^{1/2}} [(\overline{\omega}_i + \Omega_j)\hat{A}_i + (\Omega_j - \overline{\omega}_i)\hat{A}_i^+], \tag{17}$$

which leads to

$$\hat{H}_p(t) = \sum_j \hbar \Omega_j(t) \left(\hat{b}_j^+ \hat{b}_j + \frac{1}{2} \right) . \tag{18}$$

Here $\Omega_j(t)$ are new frequencies satisfying the conditions

$$\sum_i \frac{e_{1in} e_{1in'}}{\Omega_j^2 - \overline{\omega}_i^2} = \sum_j \frac{e_{2jn} e_{2jn'}}{\Omega_j^2 - \overline{\omega}_i^2} = B_{nn'}^{-1} , \tag{19}$$

c_{ij} is the orthogonal matrix of the Dushinsky rotation, which in the considered case is equal to

$$c_{ij} = \frac{(e_{1i} B e_{2j})}{\Omega_j^2 - \overline{\omega}_i^2} , \tag{20}$$

e_{2j} is related to e_{1i} by the "rotation" transformation $e_{2j} = \sum_i c_{ij} e_{1i}$; no explicit expression for e_{2j} is needed below.

3. EQUATION FOR STRONG VIBRATION

One can see that the considered problem of the relaxation of strong local vibration reduces to a system of quantum oscillators with time-dependent frequencies and destruction and creation operators. The latter are the following linear combinations of the initial creation and destruction operators:

$$\hat{b}_j(t) = \sum_i (\overline{\mu}_{ij}(t)\hat{a}_i + \overline{\nu}_{ij}(t)\hat{a}_i^+) . \tag{21}$$

Here

$$\overline{\mu}_{ij} = \frac{c_{ij}}{2(\overline{\omega}_i \Omega_i)^{1/2}} [(\overline{\omega}_i + \Omega_i)\mu_i + (\overline{\omega}_i - \Omega_j)\nu_i^*] ,$$

$$\overline{\nu}_{ij} = -\frac{c_{ij}}{2(\overline{\omega}_i \Omega_j)^{1/2}} [(\overline{\omega}_i - \Omega_j)\mu_i + (\overline{\omega}_i + \Omega_j)\nu_i] .$$

$\overline{\nu}_{ij}$, as well as ν_i remain finite for all $t > 0$ including $t \to \infty$ (when $Q_l(t) \to 0$). This means that the initial zero-point state $|0>$, satisfying the condition $a_i|0> = 0$ for all i, does not remain the zero-point one for $t > 0$. The total number of phonons at the time t in this state equals

$$N(t) = \sum_j <0|\hat{b}_i^+(t)\hat{b}_j(t)|0> = \sum_{ij} |\overline{\nu}_{ij}(t)|^2 . \qquad (22)$$

The total energy of the generated phonons is

$$E_{ph}(t) = \sum_j \hbar\Omega_j(t) <0|b_j^+(t)b_j(t)|0> =$$

$$= \frac{\hbar}{4} \sum_{ij} |c_{ij}|^2 \overline{\omega}_i^{-1} |(\overline{\omega}_i + \Omega_j)\nu_i + (\overline{\omega}_i - \Omega_j)\mu_i|^2 . \qquad (23)$$

Substituting expressions (10)–(12) into (23), one gets

$$E_{ph}(t) = \frac{1}{4}\hbar \sum_{ij} c_{ij}^2 [2\omega_i\varepsilon_i + (\Omega_j^2 - \overline{\omega}_i^2)(\varepsilon_i - 2\mathrm{Re}\ f_i)/r_i\overline{\omega}_i - 2\Omega_j] .$$

Another contribution to the energy of the system comes from the time dependence of the zero-point energy:

$$E_0(t) = \sum_j \hbar\Omega_j(t)/2 - \sum_i \hbar\omega_i/2 .$$

Adding $E_0(t)$ to $E_{ph}(t)$ and taking into account that according to (19)

$$\sum_j c_{ij}^2(\Omega_j^2 - \overline{\omega}_i^2) = (e_{1i}Be_{1i}) ,$$

we find (see also (10)–(12))

$$E(t) = \frac{\hbar}{2} \sum_i \omega_i(\varepsilon_i(t) - 1) +$$

$$+ \frac{\hbar}{4} Q(t) \sum_i (e_i w e_i) |\overline{g}_i(t)|^2 (\varepsilon_i(t) - \mathrm{Re}\ f_i(t))/\omega_i r_i(t) . \qquad (24)$$

This energy should be averaged over the initial phases φ_i. Besides, supposing that the characteristic time of the local mode relaxation essentially exceeds its period, one can average $E(t)$ over the local mode period. As a result, the last term in (24) tends to zero and one obtains the following formula for the energy of the phonon system at the time moment t:

$$\overline{E}(t) = \hbar \sum_i \omega_i(\overline{\varepsilon}_i(t) - 1) =$$

$$= \frac{\hbar}{4} \sum_{ii'} \omega_{i'}^{-1} \left| \int_0^t d\tau (e_i D_{i'}(\tau) e_{i'}) e^{-i\omega_{i'}\tau} \right|^2 \tag{25}$$

(here we take into account that $\sin(\omega_i(t-\tau)$ can be replaced by $(2i)^{-1}\exp(i\omega_i(t-\tau))$).

Total energy of the system consists of the energy of the phonon system $\overline{E}(t)$ and of the local mode energy:

$$E_l(t) = \frac{1}{2}(\omega_l^2 Q_l^2(t) + \dot{Q}_l^2(t)) \,. \tag{26}$$

According to the energy conservation law

$$\overline{E}(t) = E_l(0) - E_l(t) \,. \tag{27}$$

Substituting (25) and (26) into (27), one obtains an equation for the time-dependent coordinate $Q(t)$ of the local mode. This is a transcendental equation, as $D_i(t)$ depend on $Q(t)$ via integral equation (5). In the case of fast relaxation, comparable with the local mode frequency ω_l, one should also allow for the second term $\sim Q(t)$ in (24).

4. WEAK COUPLING OF THE LOCAL MODE WITH PHONONS

As a simple example, we consider a two-phonon damping of the local mode in a weak coupling limit. In this case $D_i(\tau) \approx Q(\tau) w \exp(-i\omega_i \tau) I$ (see Expr.(5)), which leads to the exponential solution of Eq. (27). Indeed, in the case of an exponential decay, $Q(t) = Q_0 \cos\omega_l t \cdot \exp(-\gamma t/2)$, $\gamma << \omega_l$, one gets

$$E_l(0) - E_l(t) \approx \frac{1}{2}\omega_l^2 Q_0^2 (1 - e^{-\gamma t}) \,,$$

$$\overline{E}(t) \approx \frac{\hbar Q_0^2}{16} \sum_{ii'} (e_i w e_{i'})^2 \omega_{i'}^{-1} \left| \int_0^t d\tau e^{i(\omega_l - \omega_i - \omega_{i'})\tau - \gamma\tau/2} \right|^2 \approx$$

$$\approx \frac{\pi\hbar}{16\gamma} Q_0^2 \omega_l \left[\int_0^{\omega_m} d\omega (\overline{e}(\omega) w \overline{e}(\omega_l - \omega))^2 \right] (1 - e^{-\gamma t}) \,,$$

where

$$\overline{e}_n(\omega)\overline{e}_{n'}(\omega) = \sum_i \omega_i^{-1} e_{ni} e_{n'i} \delta(\omega - \omega_i) \,,$$

ω_m is the maximal frequency of phonons. Consequently, in this model the decay is exponential with the rate

$$\gamma \approx \frac{\pi\hbar}{8\omega_l} \int_0^{\omega_m} d\omega (\overline{e}(\omega) w \overline{e}(\omega_l - \omega))^2 \,.$$

This result coincides with a well-known result of the quantum perturbation theory for the two-phonon anharmonic decay of a mode at $T = 0$ [10].

5. ARBITRARY COUPLING; NONEXPONENTIAL DECAY

The proposed theory also allows one to find the two-phonon decay law of a strong local mode beyond the perturbation limit. This will be shown for the case when the local mode interacts with a single configurational coordinate ($n_0 = 1$). In this case integral equation (5) can be presented in the form

$$D_i(\tau) = wQ(\tau)e^{-i\omega_i\tau}\left[1 + w\int_0^\tau d\tau_1 e^{i\omega_i(\tau-\tau_1)}G(\tau-\tau_1)Q(\tau_1) + \right.$$

$$\left. + w\, e^{i\omega_i\tau}\int_0^\tau d\tau_1\int_0^{\tau_1} d\tau_2 G(\tau-\tau_1)G(\tau_1-\tau_2)Q(\tau_1)D_i(\tau_2)\right], \quad (28)$$

where w is the interaction constant. We take $Q(\tau) = Q_{0\tau}\cos\omega_l\tau$ and suppose that the amplitude $Q_{0\tau}$ slowly changes in time (compared to ω_l). In this case the second term in brackets in (28) can be omitted as it oscillates fast. Neglecting the fast oscillating terms also in the third term, one gets an approximate equation

$$D_i(\tau) \approx v_\tau e^{i(\omega_l-\omega_i)\tau} + v_\tau^2\int_0^\tau d\tau_1\int_0^{\tau_1} d\tau_2 G(\tau-\tau_1)G(\tau_1-\tau_2)e^{i\omega_l(\tau-\tau_1)}D_i(\tau_2)\,, \quad (29)$$

where $v_\tau = wQ_{0\tau}/2$. As the dependence of v_τ on τ is low, it can be omitted when solving (29), and restored in the answer. In this approximation the solution of (29) can be found by means of the half-axis Fourier transform:

$$D_i(\omega) = \int_0^\infty e^{-i\omega\tau-\varepsilon\tau}D_i(\tau)d\tau =$$

$$= -iv[\omega + \omega_i - \omega_l - i\varepsilon]^{-1} + v^2 D_i(\omega)G(\omega)G(\omega-\omega_l)\,, \quad \varepsilon\to 0\,, \quad (30)$$

where

$$G(\omega) = \int_0^\infty e^{-i\omega\tau-\varepsilon\tau}G(\tau)d\tau\,. \quad (31)$$

Consequently,

$$D_i(\omega) = -iv[(\omega + \omega_i - \omega_l - i\varepsilon)(1 - v^2G(\omega)G(\omega-\omega_l))]^{-1} \quad (32)$$

and

$$D_i(\tau) = \frac{1}{2\pi}\int_{-\infty}^\infty d\omega D_i(\omega)e^{i\omega\tau} =$$

$$= \frac{v_\tau e^{i(\omega_l-\omega_i)\tau}}{1 - v_\tau^2 G(\omega_i)G(\omega_i-\omega_l)} - \sum_k \frac{e^{i\omega_k\tau}}{R_\tau(\omega_k)(\omega_i+\omega_k-\omega_l)}\,, \quad (33)$$

where $R_\tau(\omega) = v_\tau dG(\omega)G(\omega-\omega_l)/d\omega$, $\omega_k = \mathrm{Re}\,\omega_k + i\Gamma_k$ are the upper poles of the resolvent $1/(1 - v_\tau^2 G(\omega)G(\omega-\omega_l))$. The number and the position of these poles depend on the local mode amplitude $Q_{0\tau}$: for large $Q_{0\tau}$ these poles are present, while for an amplitude smaller than the critical one ($Q_{0\tau} < Q_{cr}$) they disappear.

Here for simplicity we consider a case when only the first term on the right side of (33) is present. In this case the differentiating of (27) over t gives

$$\frac{dE_l(t)}{dt} \approx -\gamma(t)E_l(t)\,, \tag{34}$$

where

$$\gamma(t) \approx \frac{\pi\hbar w^2}{8\omega_l}\int_0^{\omega_m} \frac{d\omega\rho(\omega)\rho(\omega_l-\omega)}{|1-w^2E_l(t)G(\omega)G(\omega-\omega_l)/2\omega_l^2|^2}\,, \tag{35}$$

$\rho(\omega) = \overline{e}^2(\omega)$ is one-phonon density function. One can see that for a small energy E_l the decay rate $\gamma(t) \approx \gamma_0 = \text{const.}$ and the decay is exponential, but for large E_l the rate depends on E_l. Consequently, the decay law of the initially strongly excited local mode is superexponential, becoming exponential in the final stage. In the case of a sufficiently strong initial excitation, the poles ω_k appear in (33) and the decay gets additional channels.

We conclude that according to the proposed theory the relaxation of strong vibration is associated with time dependence of the zero-point energy of the phonons interacting with vibration. The relaxation is nonexponential. Only in the final stage the relaxation becomes exponential.

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REFERENCES

1. Hawking, S. W. Nature (London), 1974, **243**, 30.
2. Birel, N. O., Davies, P. C. Quantum Fields in Curved Space. Cambridge University Press, Cambridge, 1982.
3. Гриб А. А., Мамаев С. Г., Мостепаненко В. М. Вакуумные квантовые эффекты в сильных полях. Энергоиздат, Москва, 1988.
4. Pluminen, G., Müller, B., Griener, W. Phys. Rep., 1986, **134**, 87.
5. Гинзбург В. Л., Фролов В. П. УФН, 1987, **153**, 633.
6. Maradudin, A. A., Montroll, E. W., Weiss, G. H. Theory of Lattice Dynamics in Harmonic Approximation. Academic Press, New York–London, 1963.
7. Economou, E. N. Green's Function in Quantum Physics. Springer-Verlag, Berlin–Heidelberg–New York–Tokyo, 1983.
8. Maradudin, A. A. Theoretical and Experimental Aspects of the Effects of Point Defects and Disorder on the Vibrations of Crystals. Academic Press, New York–London, 1966.
9. Hizhnyakov, V. J. Phys. C: Solid State Phys., 1987, **20**, 6073.
10. Klemens, P. Phys. Rev., 1961, **122**, 443.

HAWKINGI PROTSESS VIBROONSÜSTEEMIS: TUGEVA VÕNKUMISE RELAKSATSIOON

Vladimir HIŽNJAKOV

On vaadeldud tugevasti ergastatud lokaalse võnkumise relaksatsiooni, mille tingib anharmooniline vastakmõju foononitega. Lokaalset võnkumist on käsitletud klassikaliselt, foononeid aga kvantmehaaniliselt. Selles lähenduses põhjustab vastakmõju tugevasti ergastatud võnkumisega foononsüsteemi elastsuskoefitsientide aegsõltuvuse. Tulemusena muutub ajas foononsüsteemi nullenergia, põhjustades foononite genereerimise. Foononite tekkemehhanism on musta augu kiirguse Hawkingi mehhanismi analoog. Käesolev teooria võimaldab kirjeldada lokaalse võnkumise kahefoononist lagunemist meelevaldse kuubilise anharmoonilisuse korral ja näeb ette mitteeksponentsiaalset lagunemist võnkerelaksatsiooni algetapil. Nõrga anharmoonilisuse korral ühtib lagunemisseadus kvantmehaanilise häiritusteooria vastava seadusega.

ХОКИНГОВСКИЙ ПРОЦЕСС В ВИБРОННОЙ СИСТЕМЕ: РЕЛАКСАЦИЯ СИЛЬНОГО КОЛЕБАНИЯ

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Рассматривается релаксация сильно возбужденного локального колебания, обусловленного кубическим ангармоническим взаимодействием этого колебания с фононами. Сильно возбужденное колебание описывается классически, а фононы – квантовомеханически. В этом приближении сильное колебание приводит к зависимости упругих постоянных фононной системы от времени. В результате нулевая энергия фононов также изменяется во времени, что вызывает генерацию фононов. Механизм генерации фононов аналогичен хокинговскому механизму излучения Черных Дыр. Предложенная теория позволяет описать двухфононный распад сильно возбужденного локального колебания при произвольной силе кубического ангармонизма и предсказывает неэкспоненциальный закон распада такого колебания. При слабом ангармонизме полученный закон распада совпадает с законом, даваемым квантовомеханической теорией возмущения.